GDC 12

Computational Geometry

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GAME DEVELOPERS CONFERENCE
SAN FRANCISCO, CA
MARCH 5-9, 2012
EXPO DATES: MARCH 7-9
2012

What is Computational Geometry?

- Manipulation and interrogation of shapes
- Examples:
  - "What is the intersection of a line and a triangle mesh"
  - "What is the minimum distance separating two objects"
  - "Break a mesh into pieces"

Typical Application in Games

- Interrogation of Geometry
  - Collision detection for physics
  - Proximity triggers for game logic
  - Pathfinding, visibility, and other AI operations
- Manipulation of geometry
  - Creation of game assets in 3ds max/Maya/etc
  - User-generated content (e.g., Little Big Planet)
  - Destruction (Bad Company 2, etc., etc.)

A key takeaway for this talk is that geometry manipulation is no longer reserved for studio artists while producing a game. Geometry is generated and modified at runtime now, and the possibilities to explore this are wide open. There are good ways and bad ways to implement features in games that change the geometry of level and actor geometry. I want to introduce some good practices to game engine programmers working on these types of features, and perhaps to people who are looking to create their own geometry editors.

Some Perspective

- Game levels are usually made of meshes
  - Typically made of triangles
  - Indexed triangle meshes

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2	<-.5, 0, .5>
3	<0, 0, .5>
4	<-.25, 0, 1>
5	<-.5, 1, 0>
6	<0, 1, 0>
7	<-.5, 1, .5>
8	<0, 1, .5>
9	<-.25, 1, 1>

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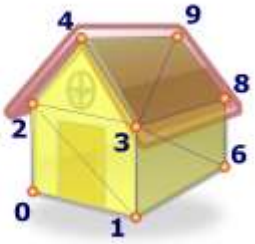

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6	<0, 1, 0>
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8	<0, 1, .5>
9	<-.25, 1, .5>

Triangle Indices
0, 1, 2
1, 3, 2
2, 3, 4
1, 6, 3
3, 6, 8
3, 8, 9
3, 9, 4
...

Some Perspective

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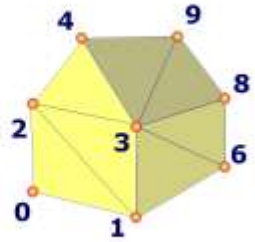

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Triangle Indices
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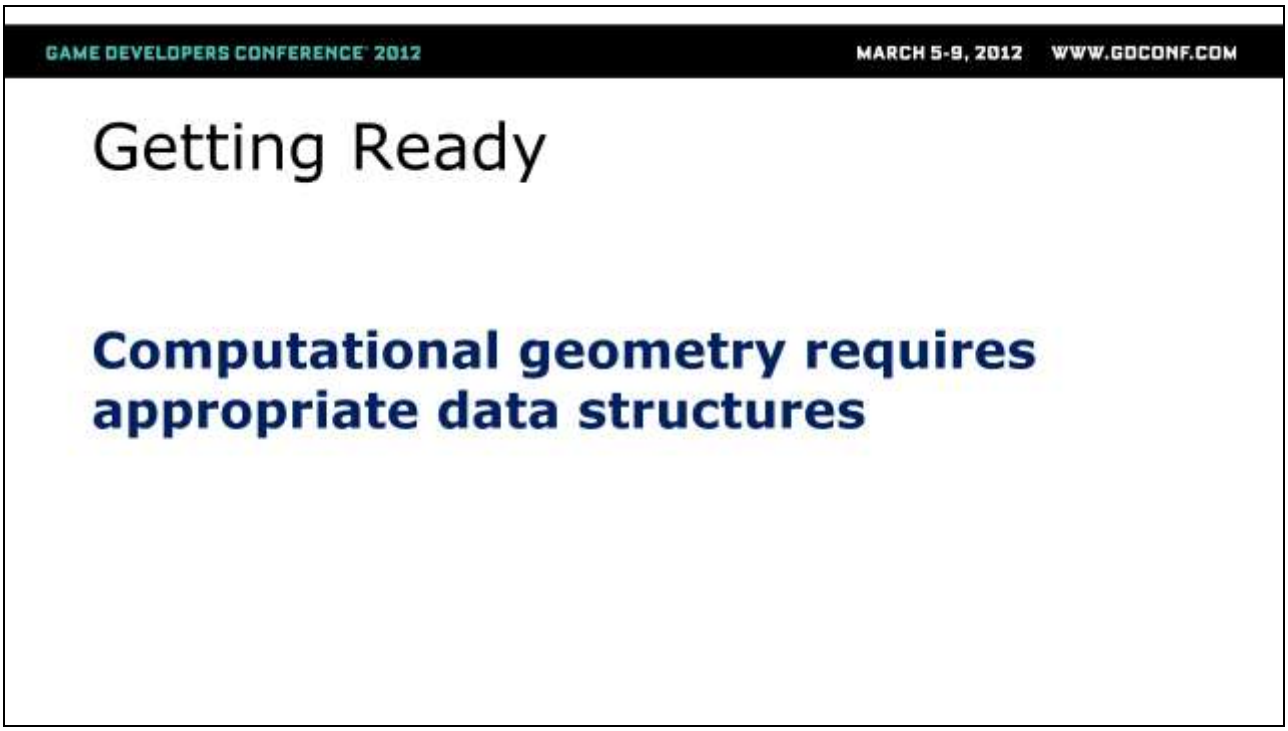


Those linear and indexed buffer representations are optimal for GPU operations (especially if optimally arranged for cache coherence, etc.), but not for general manipulation of geometry.

Categories of Data Structures

- Spatial
  - Find things fast
  - BSP tree, octree, Kd-tree, spatial hashing
  - Etc…
- **Geometry + topology**
  - Change the shape of objects
  - Focus of this talk!

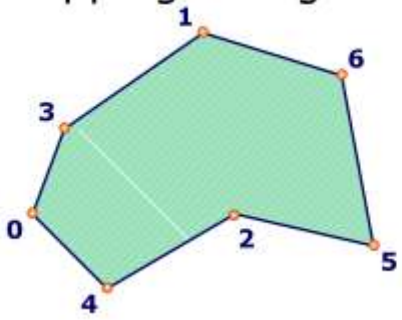
A Computational Geometry Problem

- We have polygons in our game level
- The graphics card requires triangles
  - Triangulation converts polygons into triangles

The Polygon Triangulation Problem

- Intuitive approach: ear-clipping
- Fast approach: monotone decomposition
- Both involve chopping triangles off in a sequence

Number of Polygons:
1



Polygon Indices
6,1,3,0,4,2,5

Monotone decomposition via plane sweep.

The Polygon Triangulation Problem

- Intuitive approach: ear-clipping
- Fast approach: monotone decomposition
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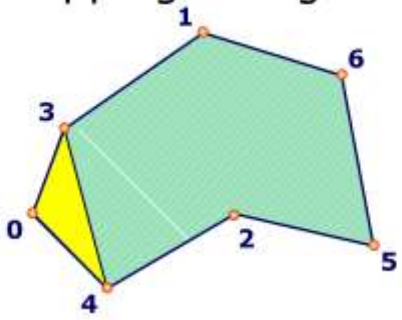
Number of Polygons:
2



Polygon Indices
6,1,3,4,2,5
0,4,3

The Polygon Triangulation Problem

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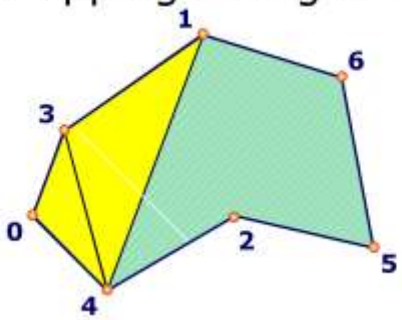
Number of Polygons:
3



Polygon Indices
6,1,4,2,5
0,4,3
4,1,3

The Polygon Triangulation Problem

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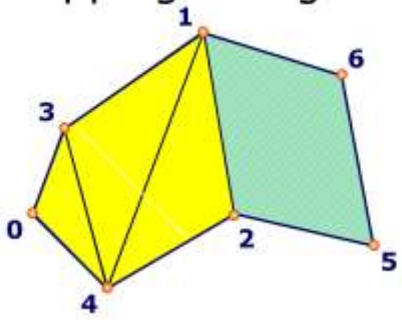
Number of Polygons:
4

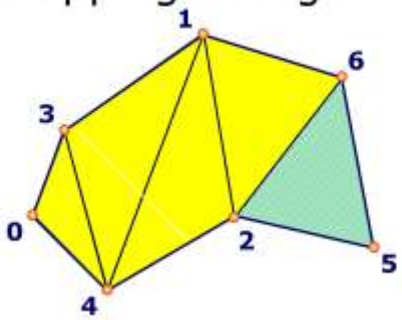


Polygon Indices
6,1,2,5
0,4,3
4,1,3
4,2,1

The Polygon Triangulation Problem

- Intuitive approach: ear-clipping
- Fast approach: monotone decomposition
- Both involve chopping triangles off in a sequence

Number of Polygons:
5



Polygon Indices
6,2,5
0,4,3
4,1,3
4,2,1
2,6,1

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Number of Polygons:
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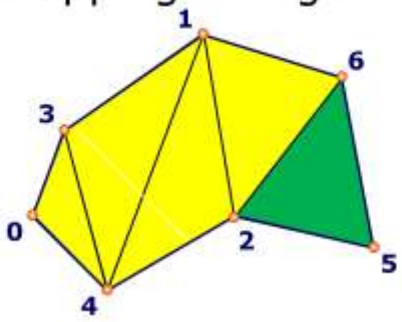


Polygon Indices
6,2,5
0,4,3
4,1,3
4,2,1
2,6,1

The Polygon Triangulation Problem

- Was maintaining the index list convenient?
- NO!
  - Original polygon: 6,1,3,0,4,2,5
  - Final polygon: 2,5,6
  - All the removed points were in the middle of the list!
  - Maintaining the list can be error prone, and slow for complex models
  - Inelegant

Getting Ready

Computational geometry requires appropriate data structures!

(Lets take a look at one)

Triangulation Demo Part 1

Geometric Model Representation

- Geometry describes the shape of model elements (triangles)
- Topology describes how the elements are connected

Manifold Topology

- Each edge joins exactly two faces
  - Model is watertight
- Open edges that join to one face are allowed
- Modeling operation consistency rules
  - "Invariants"

Non-manifold Topology

This is our focus. Simple models with at most two triangles/polygons touching on common edges.

Topological Data Structures

- Enable elegant and fast traversals
  - "Which edges surround a polygon?"
  - "Which polygons surround this vertex?"
- Easy to modify geometry
  - Split an edge or face to add a new vertex
  - Collapse an edge to simplify a mesh

The half edge structure we'll talk about here enables many traversals that complete in linear time, based on the number of results retrieved.

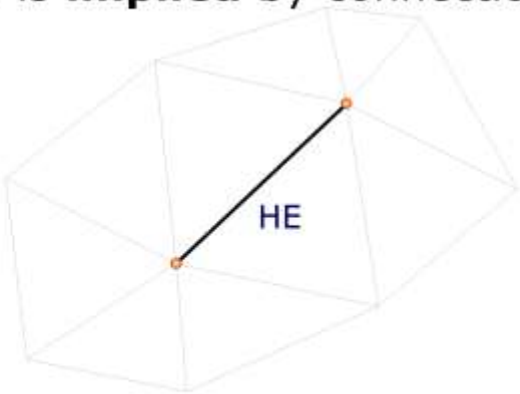
Other Topological Data Structures

- Manifold
  - Winged Edge (Baumgart, 1972)
  - Half Edge (presented here)
  - Quad edge
- Non-manifold
  - Radial edge
  - Generalized non-manifold

Generalized non-manifold is the type of data structure used in computer aided design software. It completely separates geometry and topology, and is much more rigorous that what we need to be concerned with for games. It is also far more difficult to implement. The complete division between geometry and topology makes this quite non-intuitive.

Half Edge Data Structure (HDS)

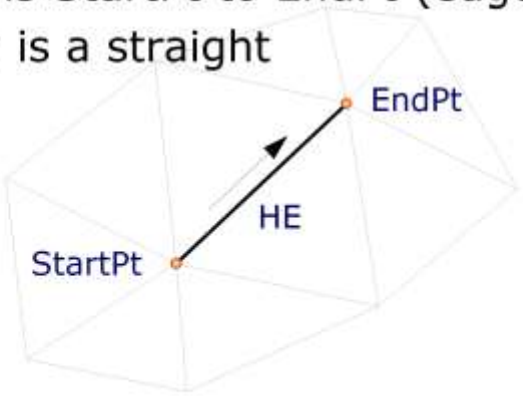
- Basic **topological** element is a half edge (HE)
- Geometry is **implied** by connections*



Half Edge Properties

Half Edge Data Structure (HDS)

- HE **connects** a Start point to an End point
- Traversal is StartPt to EndPt (edge is oriented)
- Geometry is a straight

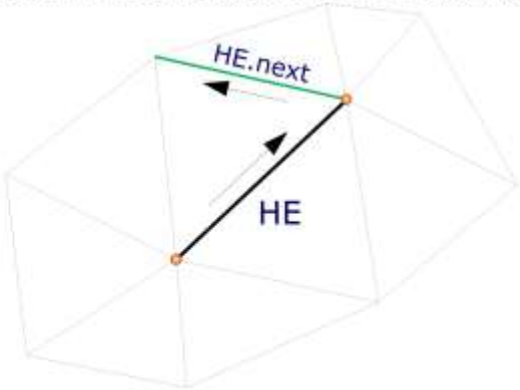


Half Edge Properties
EndPt

The half edge only directly knows about its endpoint, but we'll see that we can get to the start point in constant time.

Half Edge Data Structure (HDS)

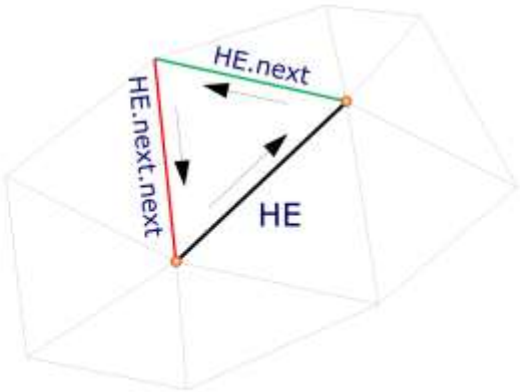
- HE points to next half edge in traversal direction
- Start point of HE.next is HE.EndPt



Half Edge Properties
EndPt
Next

Half Edge Data Structure (HEDS)

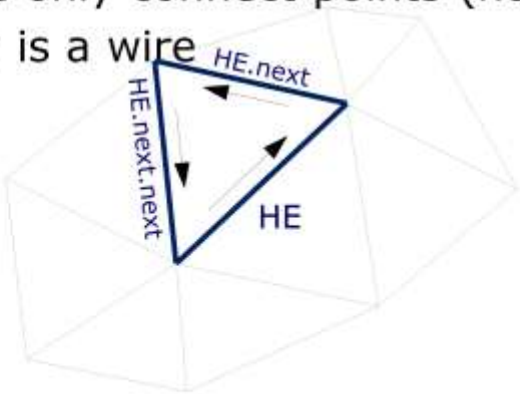
- Traversal directions are consistent



Half Edge Properties
EndPt
Next

Half Edge Data Structure (HEDS)

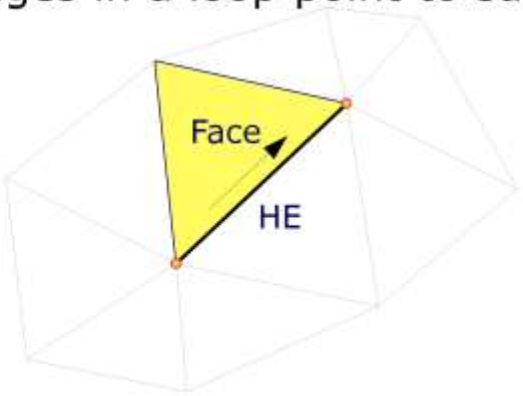
- Note that sequence of half edges forms a loop!
- So far, we only connect points (no polygons yet!)
- Geometry is a wire



Half Edge Properties
EndPt
Next

Half Edge Data Structure (HEDS)

- HE ***may*** point to a face on its **left** side
- All half edges in a loop point to same face

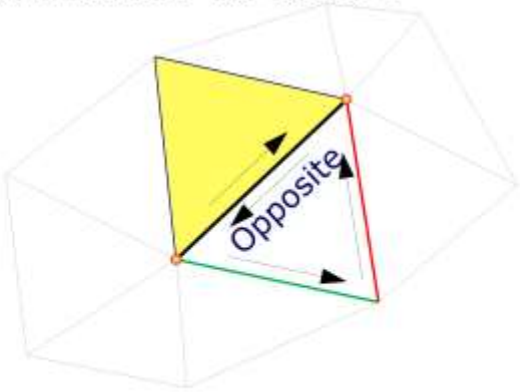


Half Edge Properties
EndPt
Next
Face

The truth is, the face is on the left side only depending on viewpoint. If we look at the half edge from a point-of-view where the loop is traversed in a counterclockwise fashion, the face is on the left of the edge….while walking along the edge we would turn towards the left to see the face. If we looked at this same object from behind, the face would appear to be on the right.

Half Edge Data Structure (HEDS)

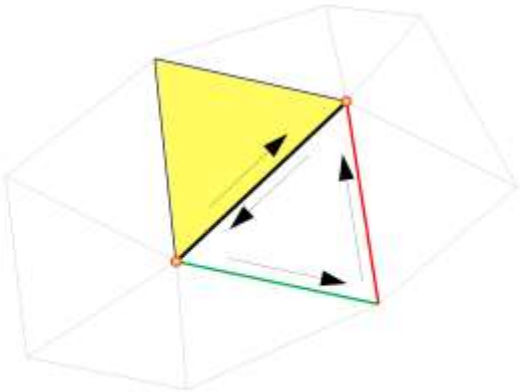
- HE points to its opposite half edge
- Which is attributed as above



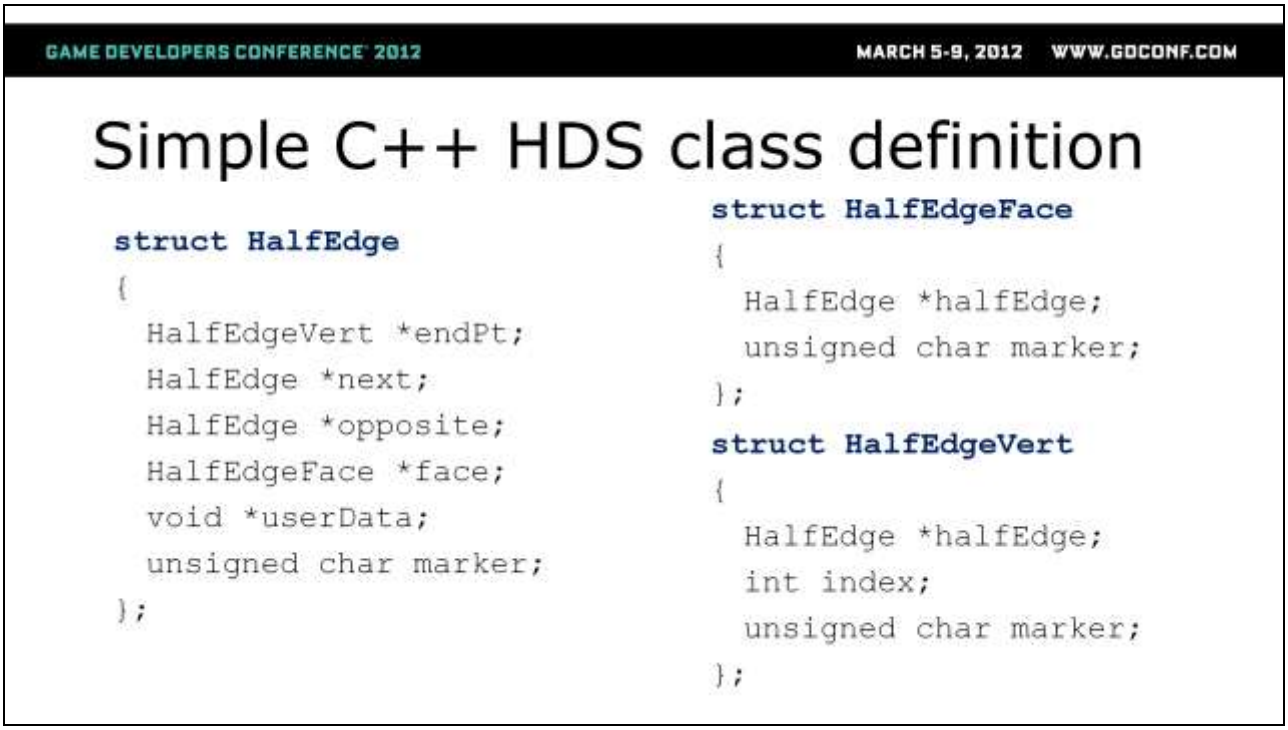
Half Edge Properties
EndPt
Next
Face
Opposite

Half Edge Data Structure (HEDS)

- It is useful to store user data and a marker



Half Edge Properties
EndPt
Next
Face
Opposite
UserData
Marker



We are focusing on the half edge, but typical implementations also define special face and vertex data structures. These enable additional traversals that are useful.

The user data could be assigned to the edge, face, and/or vertex. It could store, for example, texture or UV mapping information.

The marker is useful to aid in traversals. For example, if you want to find he constellation of faces around a given starting face, then traverse around the face's loop. For each vertex around the face, find the ring of faces around that vert, but skip any face that has a marker value of 1. For any as-yet-unmarked face, add it to your list, then set marker = 1 for that face. By using the marker in this way, it indicates that you've already visited a face and so it is already in your output list. You can also use this for Boolean type searches. For example, if you want to find faces connected to vert1, but not to vert2, first find the ring of faces around vert2, and set marker to 1. Then find the ring of faces around vert1, skipping

any face with marker == 1. These are simple examples, but it should be clear that marker can enable rather complex selection logic.

HDS Invariants

- Strict
  - halfEdge != halfEdge->opposite
  - halfEdge != halfEdge->next
  - halfEdge == halfEdge->opposite->opposite
  - startPt(halfEdge) == halfEdge->opposite->endPt
  - There are a few others…
- Convenience
  - Vertex == Vertex->halfEdge->endPt

Simple Traversals

Find vertex loop defined by a half edge

```
IndexList FindVertexLoop(HalfEdge *edge)
{
  IndexList loop;
  HalfEdge *curEdge = edge;
  do {
    loop.push_back(edge.endPt->index);
    curEdge = curEdge->next;
  } while (curEdge != edge);
  return loop;
};
```

Triangulation Demo
Part 2

Simple Traversals
Find vertex loop defined by a half edge

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IndexList FindVertexLoop(HalfEdge *edge)
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};
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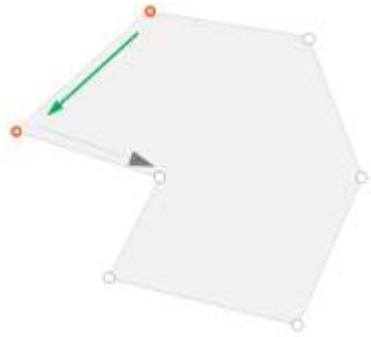

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  } while (curEdge != edge);
  return loop;
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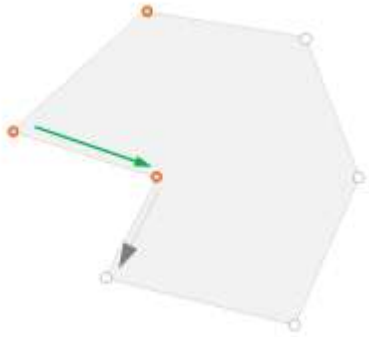

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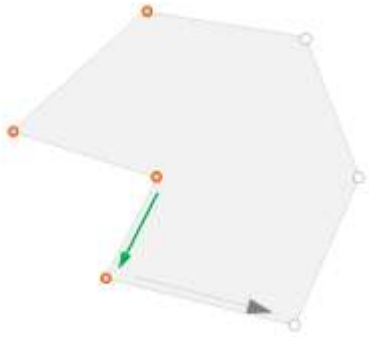
  } while (curEdge != edge);
  return loop;
};
```



Simple Traversals
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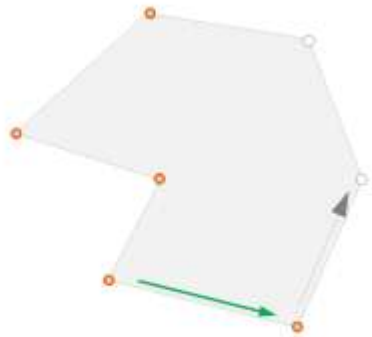
    curEdge = curEdge->next;
  } while (curEdge != edge);
  return loop;
};
```



Simple Traversals

Find vertex loop defined by a half edge

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  HalfEdge *curEdge = edge;
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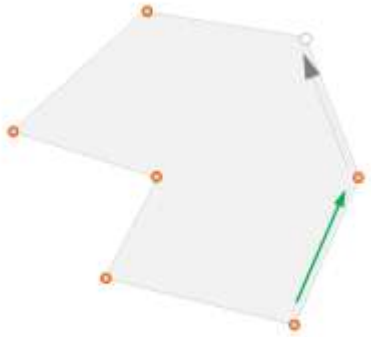
    loop.push_back(edge.endPt->index);
    curEdge = curEdge->next;
  } while (curEdge != edge);
  return loop;
};
```



Simple Traversals

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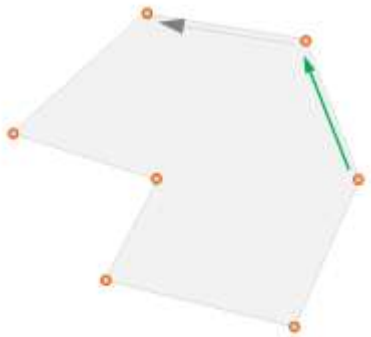
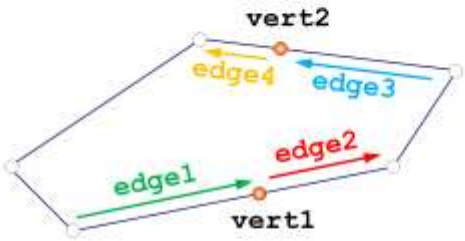


Note that we add each new edge in constant time, so the net cost is O(n), where n is the number of edges in the loop.

Simple Operations

Split a face

```
HalfEdge edge1 = vert1.halfEdge;
HalfEdge edge2 = edge1.next;
HalfEdge edge3 = vert2.halfEdge;
HalfEdge edge4 = edge3.next;
```



*See speaker notes below slide for an important consideration!

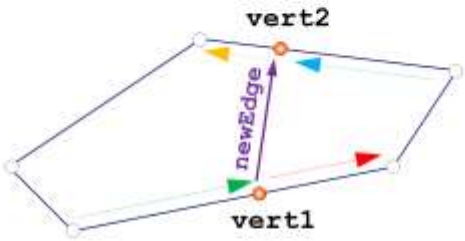
IMPORTANT NOTE: If the face is part of a mesh, then edge1 is not necessarily the only edge whose endPt is vert1. Similarly, edge3 is not necessarily the only edge whose endPt is vert2. **So, in the case of splitting a face in a mesh, it may be necessary to traverse the ring of edges around vert1 (and vert2) to find the edge whose endPt is vert1 (vert2) and whose face is the face of interest.**

Simple Operations

Split a face

```
HalfEdge edge1 = vert1.halfEdge;
HalfEdge edge2 = edge1.next;
HalfEdge edge3 = vert2.halfEdge;
HalfEdge edge4 = edge3.next;
HalfEdge newEdge = new HalfEdge;

edge1.next = newEdge;
newEdge.next = edge4;
newEdge.face = edge1.face;
newEdge.endPt = vert2;
```

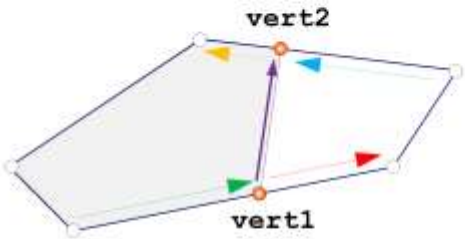


Simple Operations

Split a face

```
HalfEdge edge1 = vert1.halfEdge;
HalfEdge edge2 = edge1.next;
HalfEdge edge3 = vert2.halfEdge;
HalfEdge edge4 = edge3.next;
HalfEdge newEdge = new HalfEdge;

edge1.next = newEdge;
newEdge.next = edge4;
newEdge.face = edge1.face;
newEdge.endPt = vert2;
edge1.face.halfEdge = edge1;
```



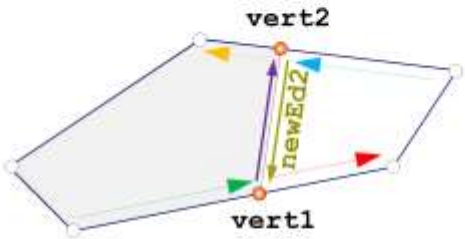
Simple Operations

Split a face

```
HalfEdge newEd2 = new HalfEdge;

newEd2.next = edge2;
newEd2.endPt = vert1;
edge3.next = newEd2;

newEdge.opposite = newEd2;
newEd2.opposite = newEdge;
```

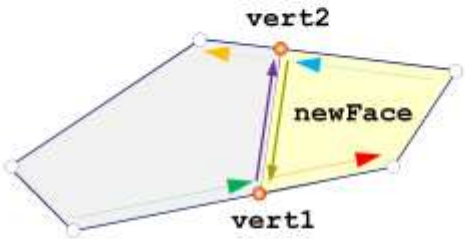


Simple Operations

Split a face

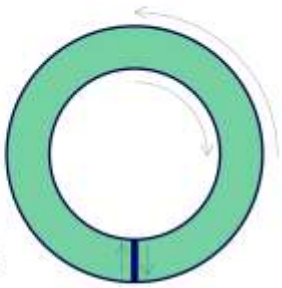
```
newFace = new HalfEdgeFace
newFace.halfEdge = edge2;


HalfEdge *curEdge = edge2;
do {
  curEdge->face = newFace;
  curEdge = curEdge->next;
} while (curEdge != edge2);
```



Triangulation Demo
Part 3

Related Operations



- Cut off an ear/triangle
  - Exactly same as split face
  - Apply recursively to triangulate face
- Insert auxiliary edge
  - Connect inner and outer loops to support holes in faces

What Else Can We Do?

- Split a mesh in half with a cutting plane
  - Step 1: Split edges that cross the plane
  - Step 2: Split faces that share two split edges
  - …

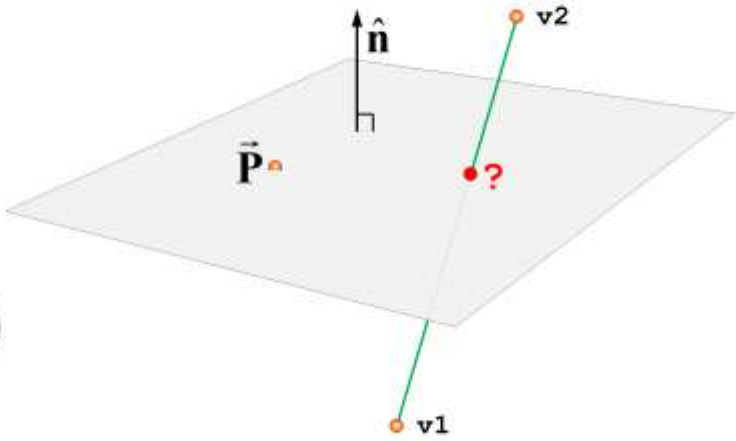
Intersection of edge and plane

- Plane equation

$$\hat{\mathbf{n}} \cdot \vec{\mathbf{P}} = d$$

- Line Equation

$$\vec{\mathbf{P}}_{line} = \vec{v}_1 + t\left(\vec{v}_2 - \vec{v}_1\right)$$

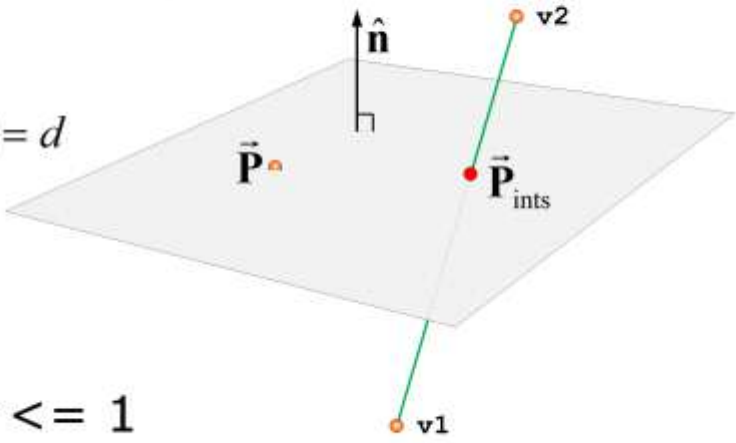


Intersection of edge and plane

- Solve for t

$$\hat{\mathbf{n}} \cdot \vec{v}_1 + t_{\text{ints}} \vec{\mathbf{n}} \cdot \left(\vec{v}_2 - \vec{v}_1\right) = d$$

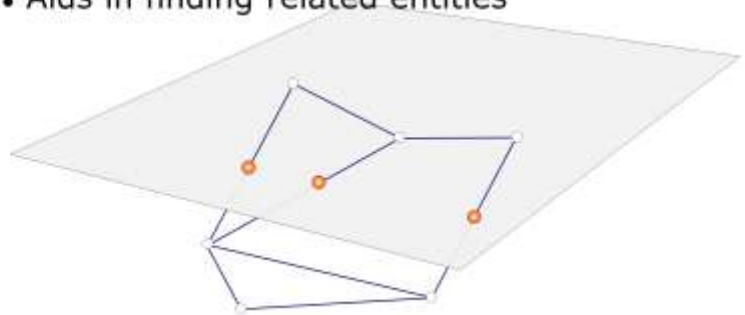
$$t_{\text{ints}} = \frac{(d - \hat{\mathbf{n}} \cdot \vec{v}_1)}{\vec{\mathbf{n}} \cdot \left(\vec{v}_2 - \vec{v}_1\right)}$$



- If $t >= 0$ and $t <= 1$
  - Edge touches plane

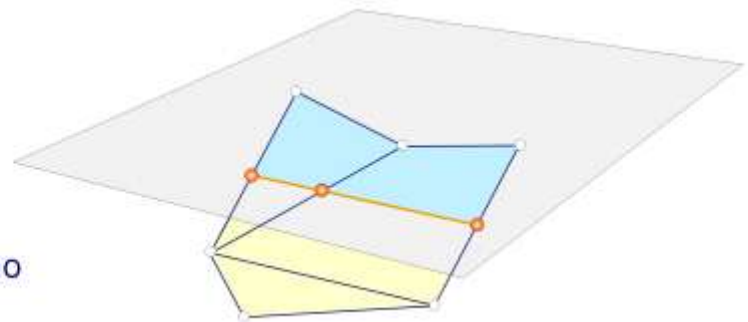
What Can We Do with a B-rep Mesh?

- Split a mesh in half with a cutting plane
  - Step 1: Split edges that cross the plane
    - Use marker variables to tag affected geometry
      - Aids in finding related entities



What Can We Do with a B-rep Mesh?

- Split a mesh in half with a cutting plane
  - Step 2: Split faces that share two split edges

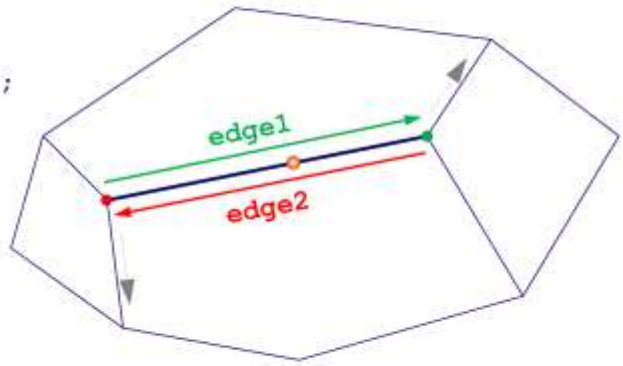


Edge Split Demo
Part 1

Simple Operations

Split an edge

```
HalfEdge edge1;
HalfEdge edge2 = edge1.opposite;
```

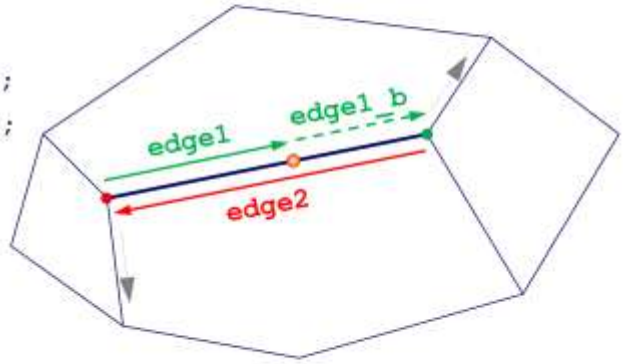


Simple Operations

Split an edge

```
HalfEdge edge1;
HalfEdge edge2 = edge1.opposite;
HalfEdge edge1_b = new HalfEdge;

edge1_b.EndPt = edge1.EndPt;
edge1_b.face = edge1.face;
edge1_b.next = edge1.next;
edge1.EndPt = splitVert;
edge1.next = edge1_b;
edge1_b.EndPt.halfEdge = edge1_b;
```

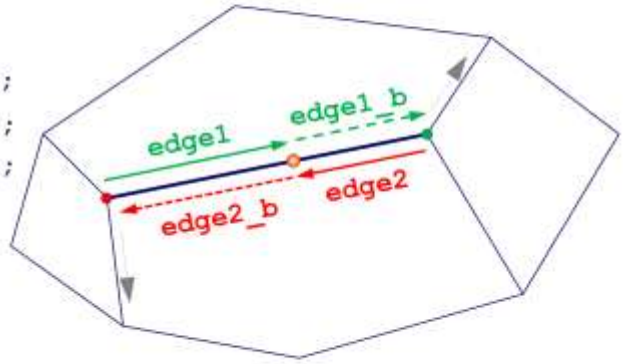


Simple Operations

Split an edge

```
HalfEdge edge1;
HalfEdge edge2 = edge1.opposite;
HalfEdge edge1_b = new HalfEdge;
HalfEdge edge2_b = new HalfEdge;

edge2_b.EndPt = edge2.EndPt;
edge2_b.face = edge2.face;
edge2_b.next = edge2.next;
edge2.EndPt = splitVert;
edge2.next = edge2_b;
edge2_b.EndPt.halfEdge = edge2_b;
```

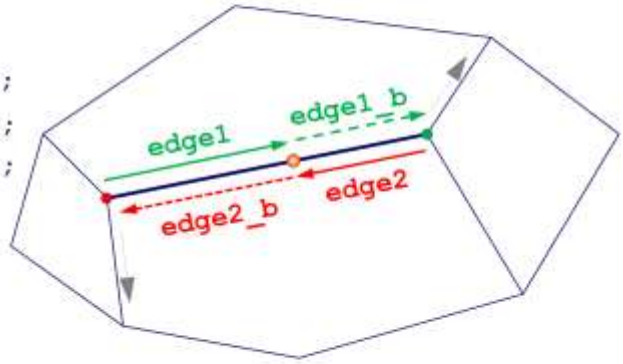


Simple Operations

Split an edge

```
HalfEdge edge1;
HalfEdge edge2 = edge1.opposite;
HalfEdge edge1_b = new HalfEdge;
HalfEdge edge2_b = new HalfEdge;

edge2_b.opposite = edge1;
edge2.opposite = edge1_b;
edge1_b.opposite = edge2;
edge1.opposite = edge2_b;
splitVert.halfEdge = edge1;
```



Other Operations

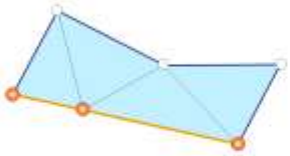
- Remove face(s)
  - Delete HalfEdgeFaces and any related topology that is unused elsewhere
  - Take care to properly RE-connect half edges/verts that are not on open boundary

Other Operations

- Unhook face(s)
  - Same as remove faces but copies removed face and related to another object

What Else Can We Do?

- Split a mesh in half with a cutting plane
  - Step 3: Remove or unhook faces on one side
  - Step 4: Find and cover open boundary loops
  - Step 5: Triangulate the remaining faces



Edge Split Demo
Part 2

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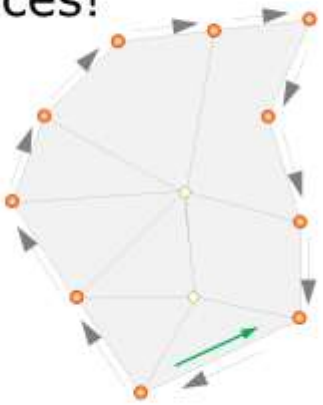
Pop Quiz!

Find the open boundary vertices!

```
IndexList Boundary;
Boundary =
        FindVertexLoop(startEdge->opposite);
```



(But what if the boundary isn't connected properly?)

*See speaker notes below slide for an important consideration!

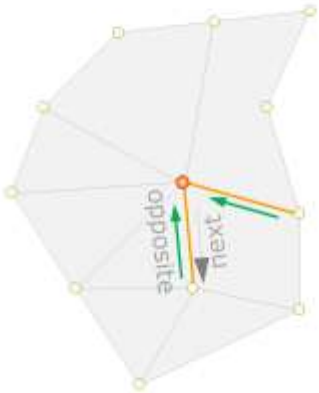
Caution! If the outer edges weren't connected properly to begin with, will have to traverse edge rings (see following slides) for each boundary vertex to locate the boundary edges from the inside. This is more expensive. Best to make sure the data structure is properly created and maintained, in order to extract the best performance.

Simple Traversals
Find edges around a vertex

```
EdgeList FindEdgeRing(HalfEdgeVert *vert)
{
  EdgeList ring;
  HalfEdge *curEdge = vert->halfEdge;
  do {
    ring.push_back(curEdge);
    curEdge = curEdge->next->opposite;
  } while (curEdge != vert->halfEdge);
  return ring;
};
```

Simple Traversals
Find edges around a vertex

```
EdgeList FindEdgeRing(HalfEdgeVert *vert)
{
  EdgeList ring;
  HalfEdge *curEdge = vert->halfEdge;
  do {
    ring.push_back(curEdge);
    curEdge = curEdge->next->opposite;
  } while (curEdge != vert->halfEdge);
  return ring;
};
```



Simple Traversals
Find edges around a vertex

```
EdgeList FindEdgeRing(HalfEdgeVert *vert)
{
  EdgeList ring;
  HalfEdge *curEdge = vert->halfEdge;
  do {
    ring.push_back(curEdge);
    curEdge = curEdge->next->opposite;
  } while (curEdge != vert->halfEdge);
  return ring;
};
```

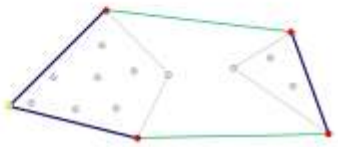
Supposed you needed to find all faces connected to a collection of vertices

You can use the approach shown here to collect faces for each vertex

Use marker values to avoid collecting a given face more than once

What Else Can We Do?

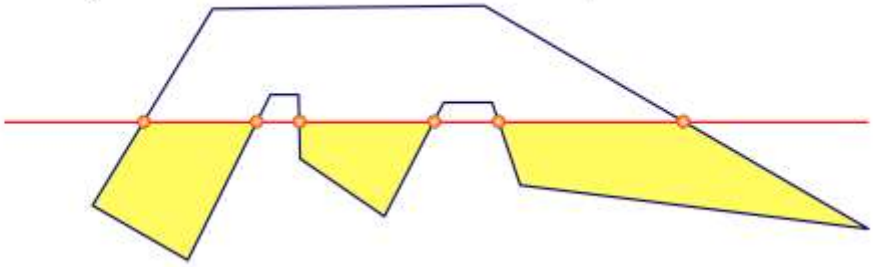
- Generate a convex hull mesh
  - Divide and conquer method is fast
    - O(n log n)



- Role of the half edge data structure
  - Remove interior faces/edges during stitch phase
  - Create new faces between boundary loops to perform the stitch

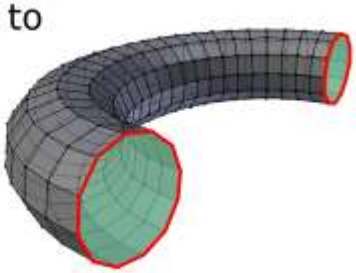
What can go wrong?

- Be careful when clipping concave face
  - Clipping against a plane can generate multiple loops
  - User marker flags to tag start and stop points
  - Recursively traverse to find ears to clip



What can go wrong?

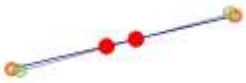
- Some scenarios produce multiple loops
  - Holes in a face
  - Requires additional triangulation logic
  - Nested loops: auxiliary edge to convert to simple polygon
  - Multiple un-nested loops: locate and triangulate each loop separately



*See speaker notes below slide for an important consideration!

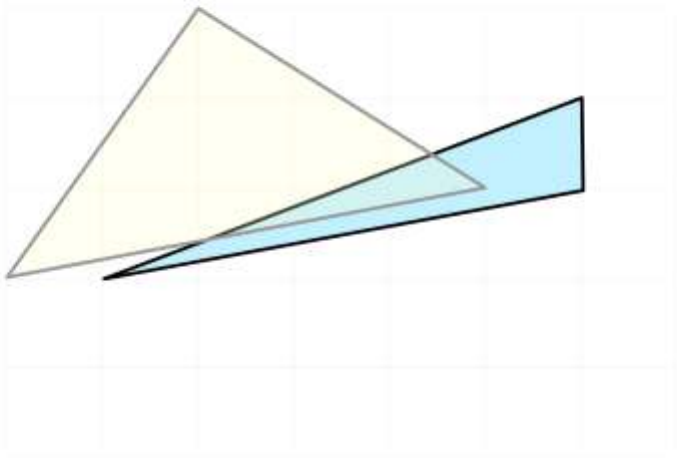
NOTE: It is straightforward to triangulate/cover an open loop that is on a plane. Or one that is approximately planar. If the edges on the loop are not all coplanar, then it is trickier. It may be possible to find some projection plane in which to perform the triangulation connectivity (a plane in which the projection of the edge loop is a simple polygon with all the original edges visible), but a different triangulation will result from different project plane choices. Ultimately, the triangles produced will not be coplanar if the edges were not coplanar, of course.

What can go wrong?



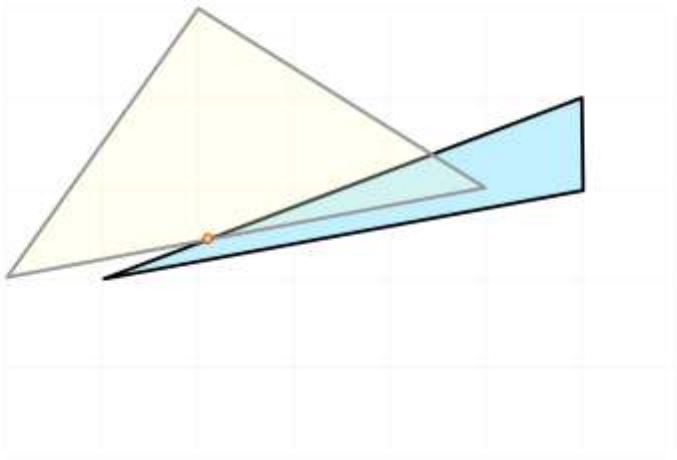
- Tolerance issues
  - Edges not quite collinear
    - Location of intersection point is highly sensitive
  - Points nearly collocated
    - Possible creation of very short/degenerate edges
  - On which side of an edge is a point?/Which face does a ray intersect?

Orientation Inversion



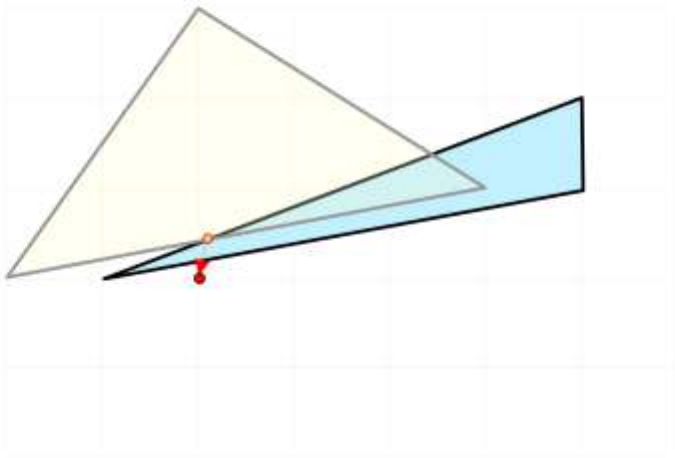
This grid is a portion of the representable floating point numbers. These two triangles are defined by corners that are representable points. Points not lying on the intersection of horizontal and vertical grid lines are not representable. Any unrepresentable number is approximated by a representable number.

Orientation Inversion

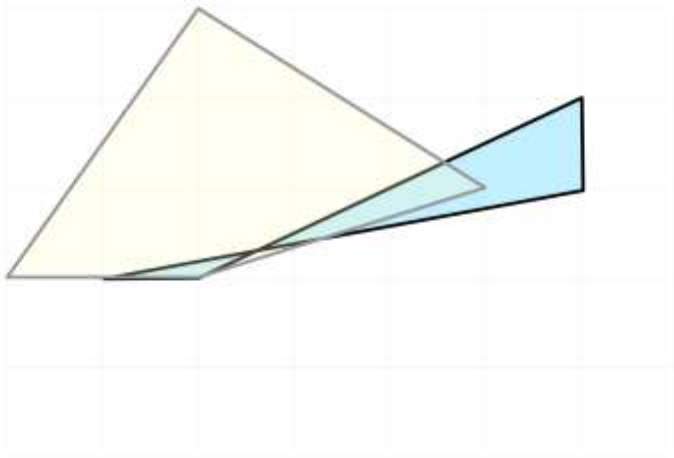


This intersection point is not representable, so the floating point math system will approximate it with the nearest representable number coordinate.

Orientation Inversion

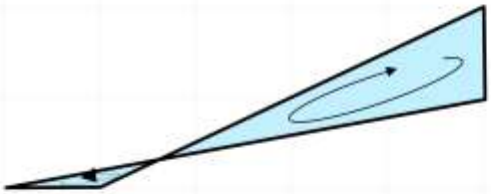


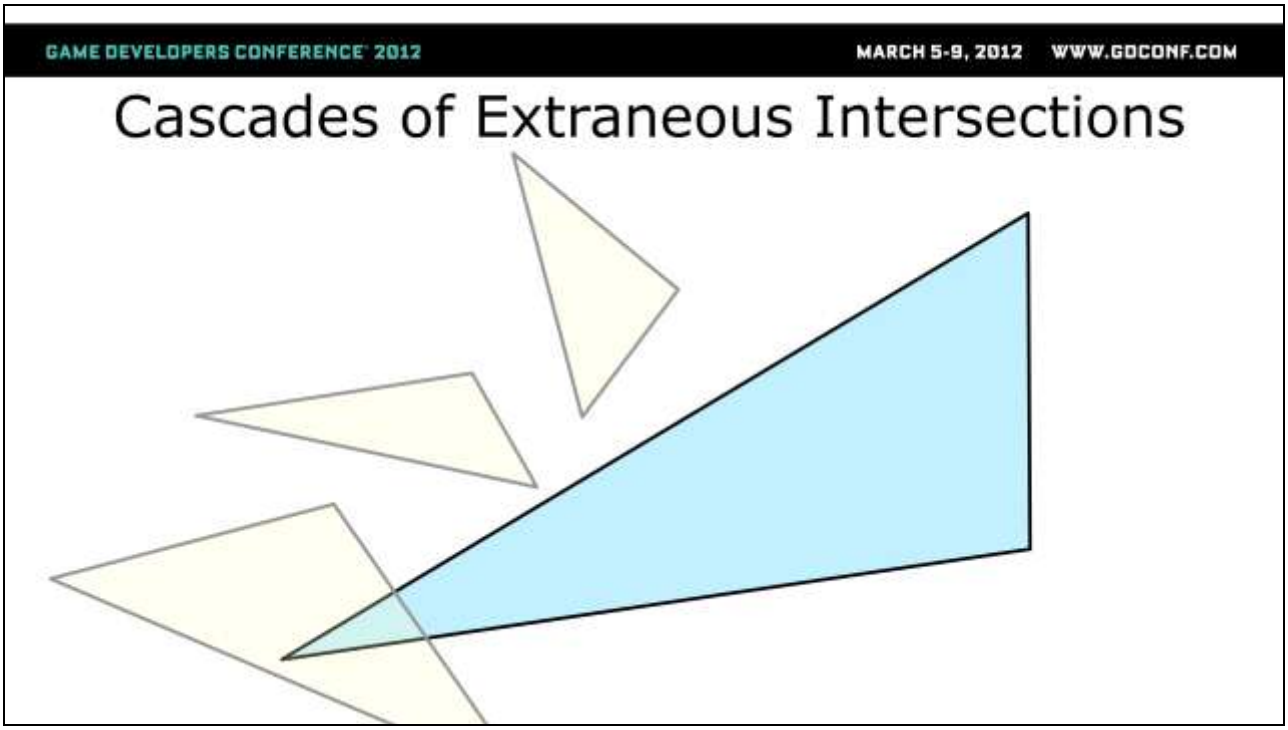
Orientation Inversion



Orientation Inversion

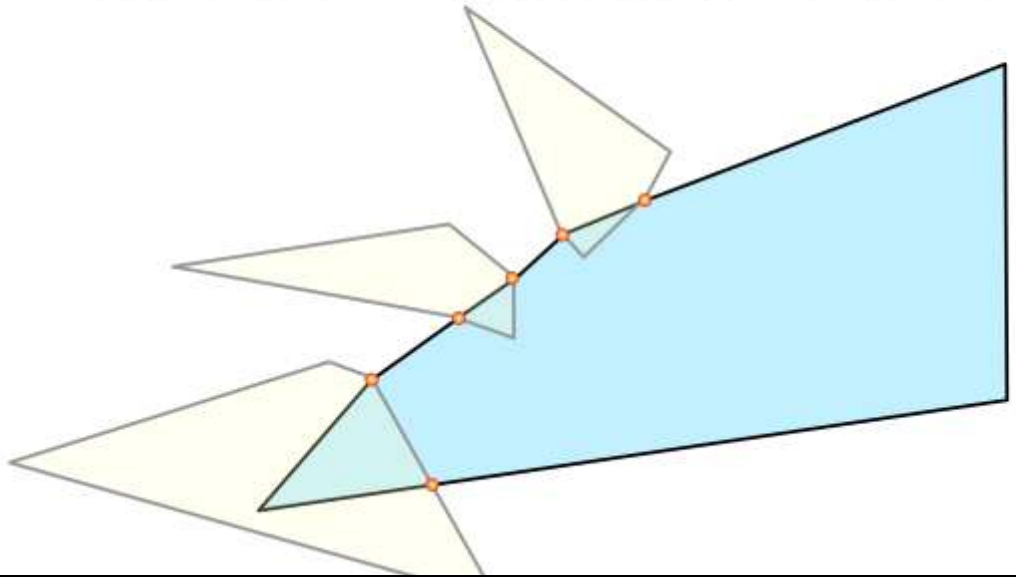
Polygon is no longer simple (it self-intersects) and no longer has a consistent orientation





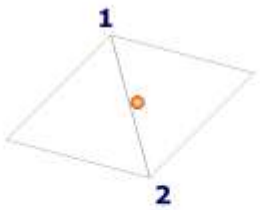
Here, though the floating point grid is not shown, you will see that a single non-representable intersection point can lead to a chain of intersections that aren't present in the original perfect geometry.

Cascades of Extraneous Intersections

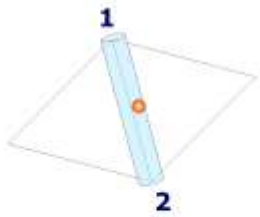


Tolerant Geometry

- Treat edges and points as thick primitives
  - Assign a radius to be used in intersection and proximity testing



Is point on edge?
On left side?
On right side?



Ambiguous answer depends on:
- Edge from 1->2 or 2->1
- Location

Point is **ON** the edge

References and Resources

- Sample code for half edge data structure
  - http://www.essentialmath.com
- These slides
  - See http://www.gdcvault.com after GDC
- References
  - http://www.cs.cornell.edu/courses/cs4620/2010fa/lectures/05meshes.pdf (Shirley & Marschner)
  - http://www.cgafaq.info/wiki/Half_edge_general
    - Nice discussion of invariants
  - http://people.csail.mit.edu/indyk/6.838-old/handouts/lec4.pdf
    - Polygon triangulation

References and Resources

- More references
  - Tolerant geometry and precision issues
    - Christer Ericson, *Real-time Collision Detection*
    - Jonathan Shewchuk's, "Adaptive Precision Floating-Point Arithmetic and Fast Robust Predicates for Computational Geometry"
    - John Hobby, "Practical Segment Intersection with Finite Precision Output" (snap rounding)

Questions?