Quest for Progress

The Math and Design of Idle Games

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Who Am I?

- At Kongregate for 7 years, directing our browser-based F2P virtual goods business
- Producer for AdVenture Capitalist mobile
- Also an indie designer, cofounder of Level Up Labs

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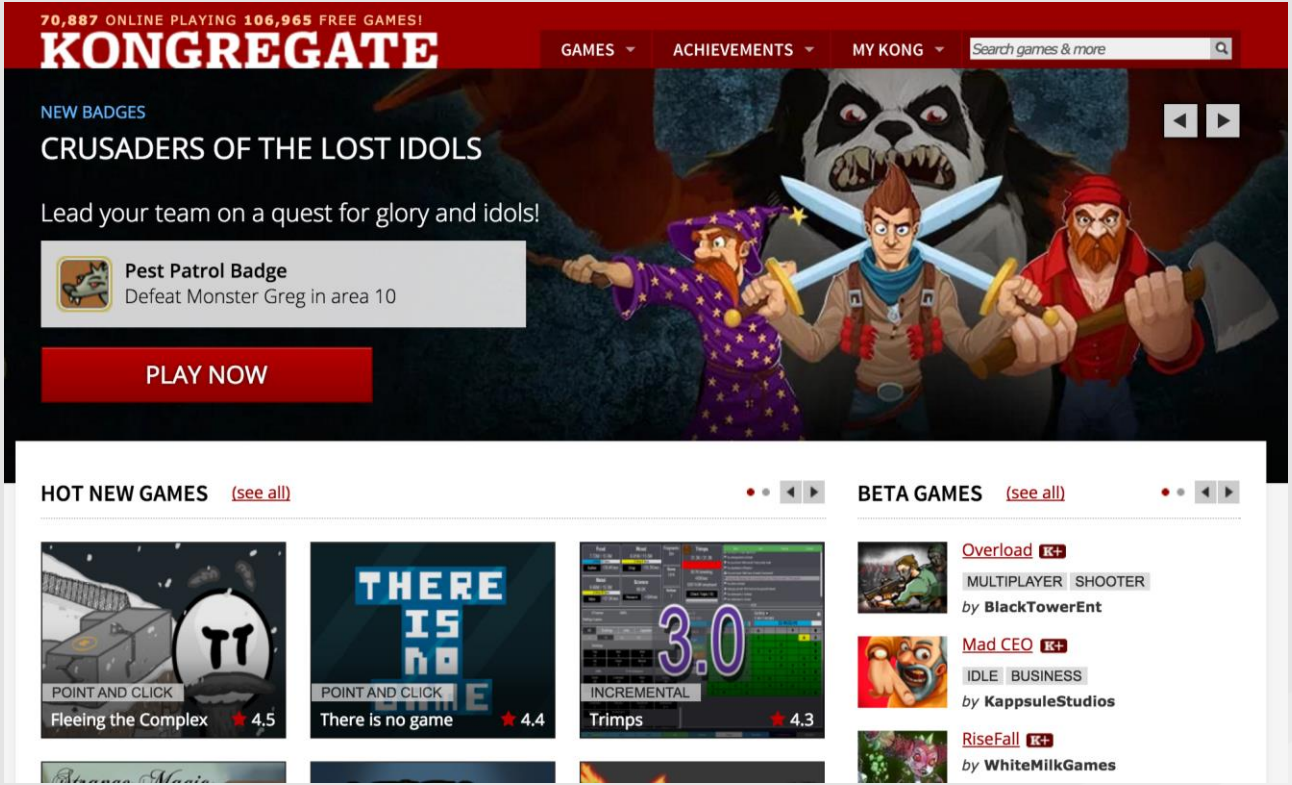

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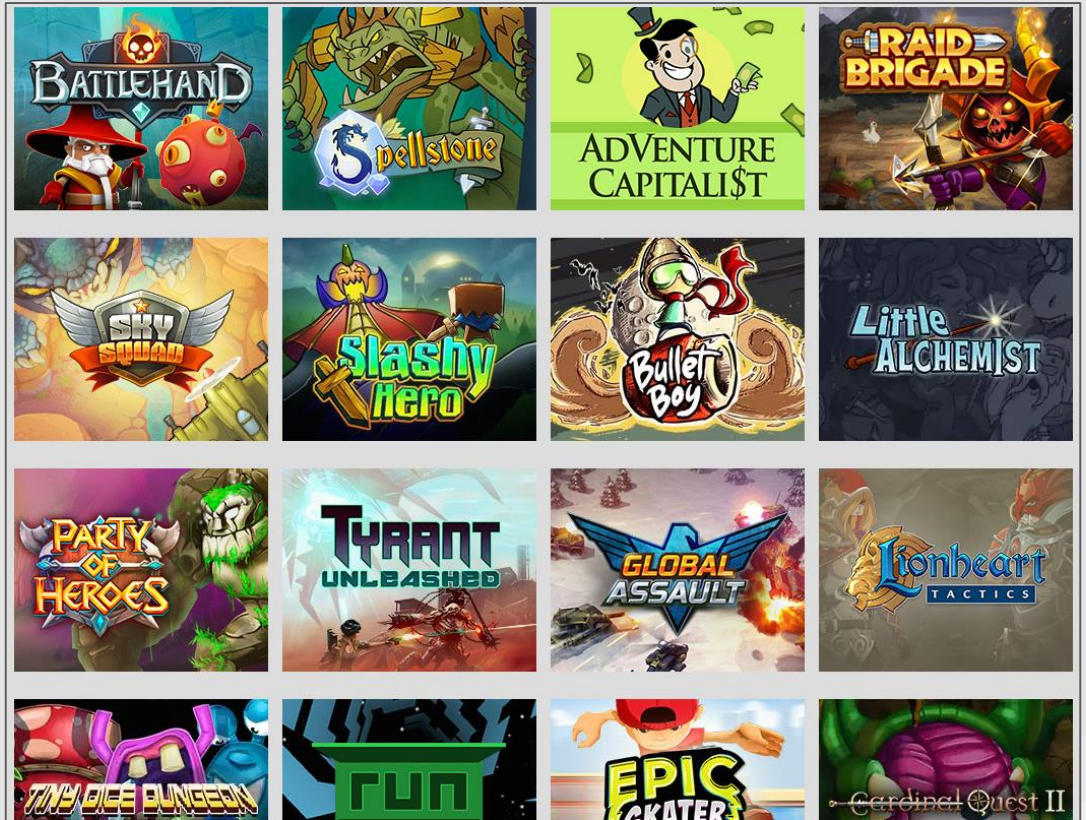


Opening Disclaimers

- This is a design and math talk, not a data and monetization talk
- There is some calculus, but you don’t need to remember how it works
- If you don’t like graphs, numbers, and formulas you’re not going to like this talk

Interactive Charts and Sheets

- All of the graphs generated in this talk come from sheets that are publicly available as both Google Sheets and .xls files
- You can access these sheets here: http://kon.gg/idle-math-spreadsheets

Genre Terminology

- Incremental Games
  - A game in which the primary goal is to continually increase a number
  - Often grows in complexity over time
    - When the scope of a game change substantially it is can referred to as an “unfolding game”
    - Examples: A Dark Room, Candy Box, Frog Fractions

Genre Terminology

- Idle Games
  - Subset of Incrementals
  - Progress or income is made without player interaction
  - Player choices impact growth rates
  - It is expected that players leave the game alone regularly
  - Typically do not have an “end”

Genre Terminology

- Clicker Games
  - Emphasis on clicking or tapping to progress
  - Ex. Feed Your Monster, Football Clicker
  - Can, but does not necessarily, have idle elements
    - Clicker Heroes is an “idle clicker”
  - Physicality of the game can be fun but also tiring

Essential Reading

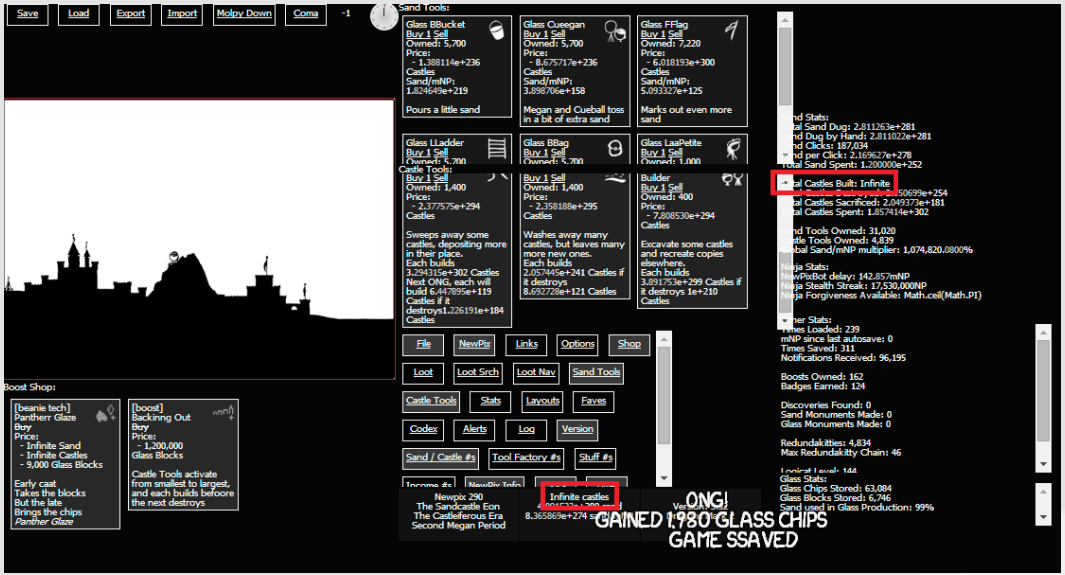
- Cookie Clicker-likes
  - AdVenture Capitalist
  - Realm Grinder
  - Pocket Politics

Essential Reading

- Clicker Heroes-likes
  - Tap Titans
  - Crusaders of the Lost Idols

Essential Reading

- Assorted Others
  - Number (Tyler Glaiel)
  - Swarm Simulator
  - Sandcastle Builder
    - The Dwarf Fortress of idle games



Terminology

- Primary Currency
  - The central number that is being incremented and typically used to purchase most moment-to-moment improvements

Terminology

- Generator
  - The buildings, investments, characters, etc. that a player amasses
  - They automatically generate Primary Currency (or Exchange Currency) over time
  - Costs grow magnitude faster than value

Terminology

- Primary Exchange Currency
  - A special case worth mentioning, in some cases generators produce an exchange currency
  - The most common example is damage / DPS in RPG idlers
    - Damage is not the primary currency but is tied directly to it
    - Gives designer a little more control over primary currency accumulation since you can gate or buff conversion at certain points in the progression

Terminology

- Multiplier
  - Typically a fixed multiple on a generator, or set of generators, in terms of production value or speed
  - These help offset cost growth speed
  - Provide bumps and local victories

Terminology

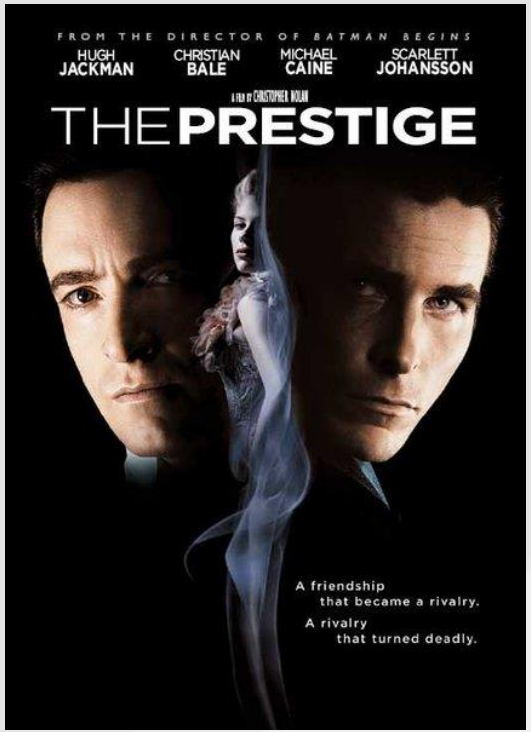
- Multiplier
  - More complex ones might have dependencies on:
    - Its target (e.g. this generator count)
    - Other generators (e.g. total generator count)
    - Meta statistics (e.g. total time played, count of lifetime skills used, lifetime currency earned, seconds since last reset, etc.)
    - External data (e.g. time of day, holidays/events, concurrent player count, etc.)
  - Incentivized ads are often implemented as a multiplier

Terminology

- Active Skill
  - Powerful abilities players can initiate
  - Typically have a long cooldown (5 minutes – a few hours)
  - Increases player agency, letting them burst income or get over a hump
  - In more complex situations, players can strategize around best selection, order, and timing of skill usage

Terminology

- Prestige
  - Player resets game with a bonus to production
  - Similar to the “new game+” concept
- Meta Prestige
  - Prestiges can be nested



Terminology

- Prestige Currency
  - Provides player with Prestige Currency that controls the production bonus
    - This currency typically grows at a slower order of magnitude than Primary Currency – we'll look into examples later
  - In some cases this currency can also be spent for boosts (at the loss of the relative Prestige Currency boost)

Idle Player Motivation Profile

- Quantic Foundry surveyed players of 3 idle games
- 70% identified as “core gamers”, 20% as “hardcore”
- Top motivators were Completion and Power

From Nick Yee’s gamer motivation profiling

Idle Player Motivation Profile

- This can help guide your design process
- Players of this genre want to collection and complete, and they want to grow in power
  - Power growth is central to the genre but you want to make sure players “feel” that growth
  - Can you incorporate collection or completionism into your idle game design?

Super quick review of exp growth curves

- Costs grow exponentially

  $cost_{next} = cost_{base} \times (rate_{growth})^{owned}$

- Production grows linearly

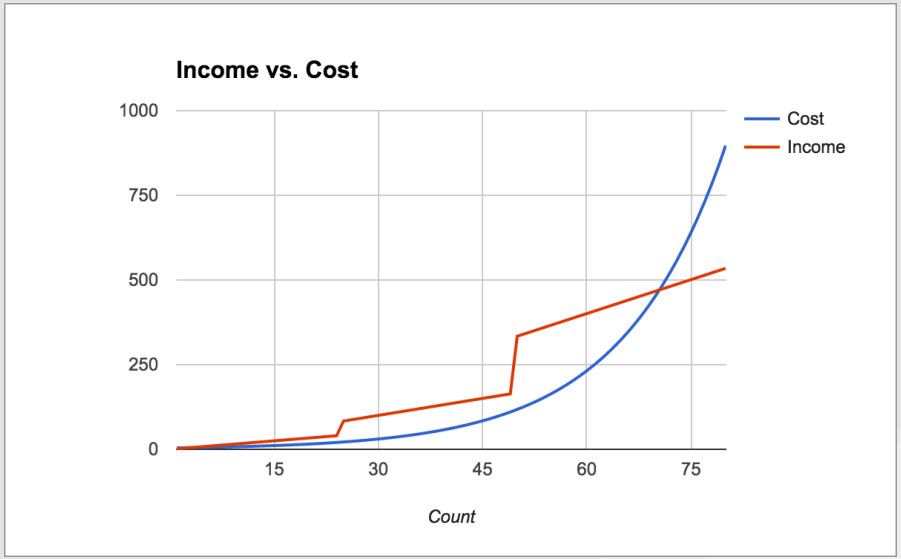
  $prod_{total} = prod_{base} \times n_{owned}$

- Note: x^k is not exponential growth and will **always** be outpaced by k^x (with $k > 1$) eventually.

Exponential Growth Rates in AdCap

- For AdCap, here are the values for a few generators:

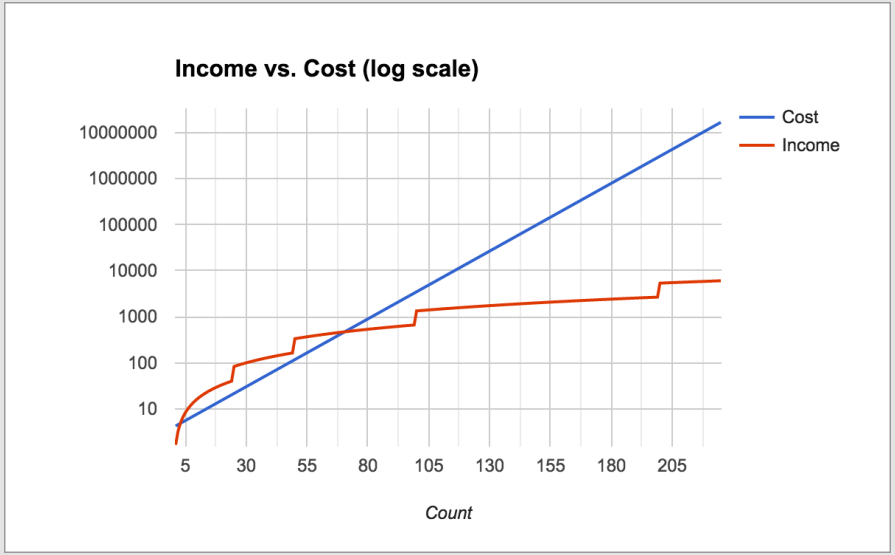
Business	Initial Cost ($)	Coefficient	Initial Time	Initial Revenue	Initial Productivity
Lemonade Stand	3.738	1.07	0.6	1	1.67
Newspaper Delivery	60	1.15	3	60	20
Car Wash	720	1.14	6	540	90
Pizza Delivery	8,640	1.13	12	4320	360
Donut Shop	103,680	1.12	24	51,840	2,160

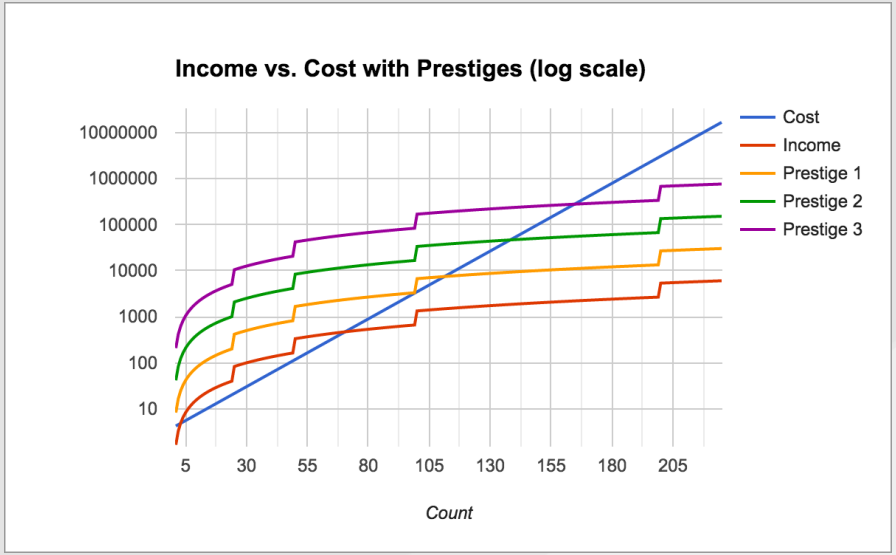


Lemonade Stand growth with doubling at 25, 50 owned

Exponential Growth Rates in AdCap

- More Lemonade Stand graphs, with 2x at every 100 and each prestige giving a 5x boost





Optimal Decisions

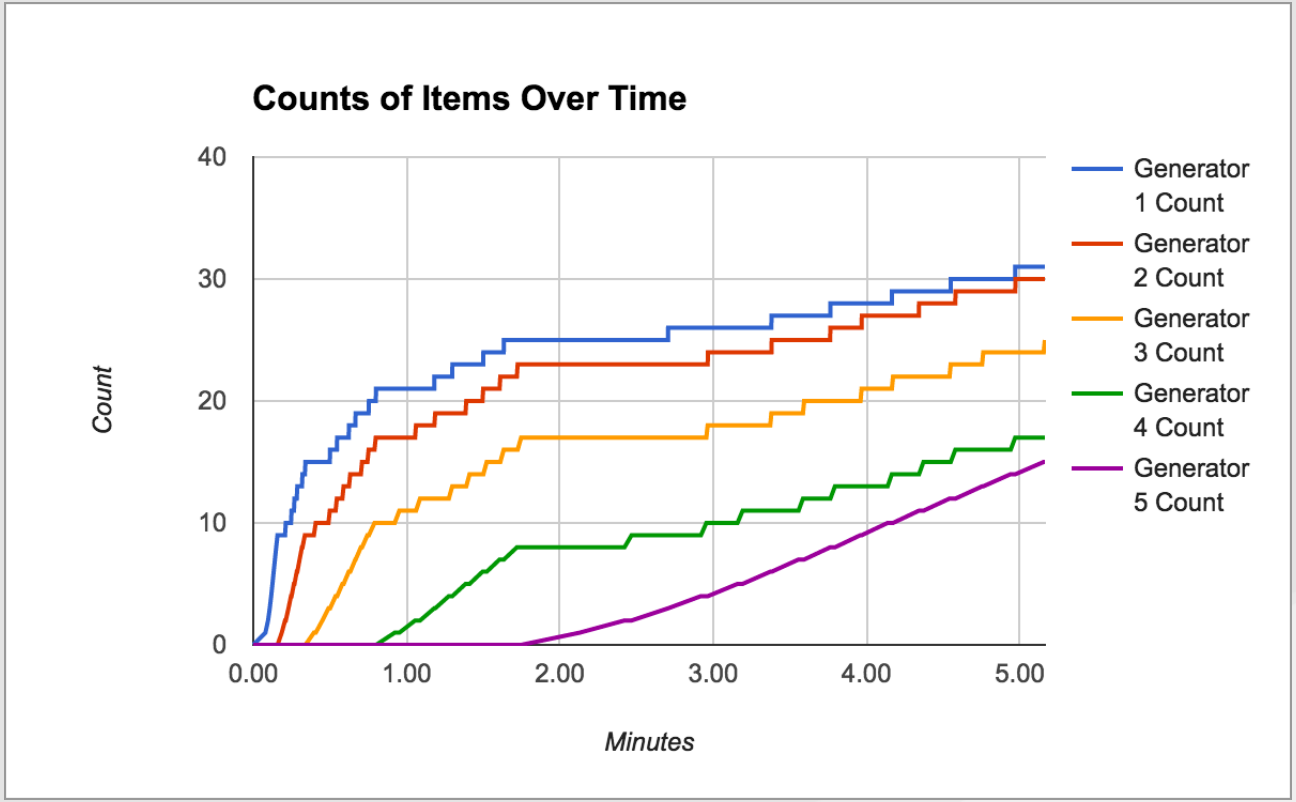
- Due to how costs and value grow, players will regularly be deciding what to buy next
- We can model optimal choice to see if we
  - While players likely won't optimize perfectly, overall patterns will be similar for savvy players
  - Sometimes tiny generators are optimal but negligible in impact – players will most-likely ignore these

Optimal Purchase Decisions

- Do you want optimal purchasing to be an interesting choice for players?
  - If so and you see optimal patterns always favoring the highest tier generator, you may have a balance problem
  - If not then you likely have other game elements that are interesting and don’t need to balance this carefully

Optimal Purchase Decisions

Generator 1	
Base Income	4.5
Starting Cost	4
Cost Mult Factor	1.19
Generator 2	
Base Income	20
Starting Cost	60
Cost Mult Factor	1.15
Generator 3	
Base Income	90
Starting Cost	720
Cost Mult Factor	1.14
Generator 4	
Base Income	360
Starting Cost	8640
Cost Mult Factor	1.13
Generator 5	
Base Income	2160
Starting Cost	103680
Cost Mult Factor	1.1



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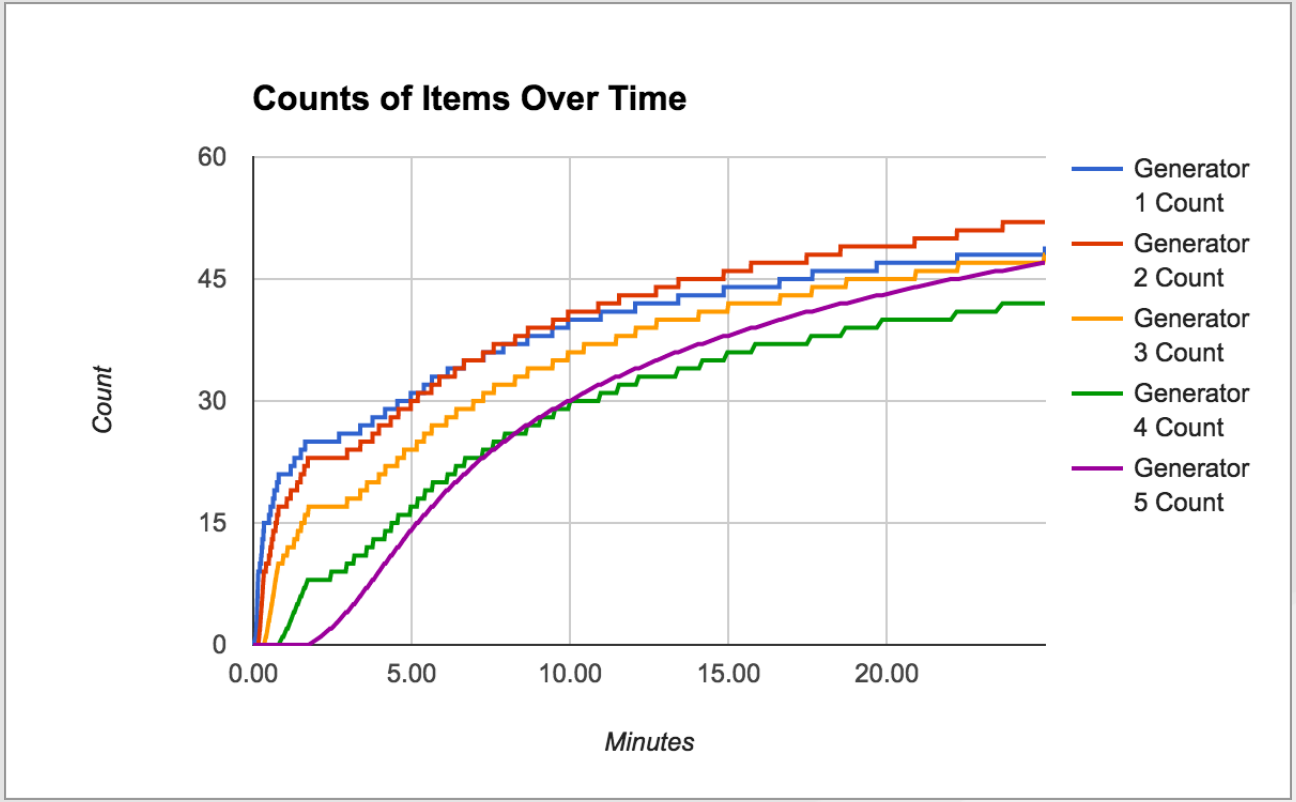


Total Cost of Bulk Purchases

- Bulk-buying of generators is often a necessary function, but what is the cost of such a purchase?
- To buy *n* generators:
  - With base price *b*
  - With exponent *r*
  - Owning *k* already

$$cost = b \cdot \frac{r^k - r^{k+n}}{1 - r}$$

Total Cost of Bulk Purchases

- What's the max number of generators you can buy?
  - With base price *b*
  - With exponent *r*
  - Owning *k* already
  - Having *c* currency

$$max = floor(log_r(r^k - c \cdot \frac{1-r}{b}) - k)$$

Non-Cookie Clicker Growth

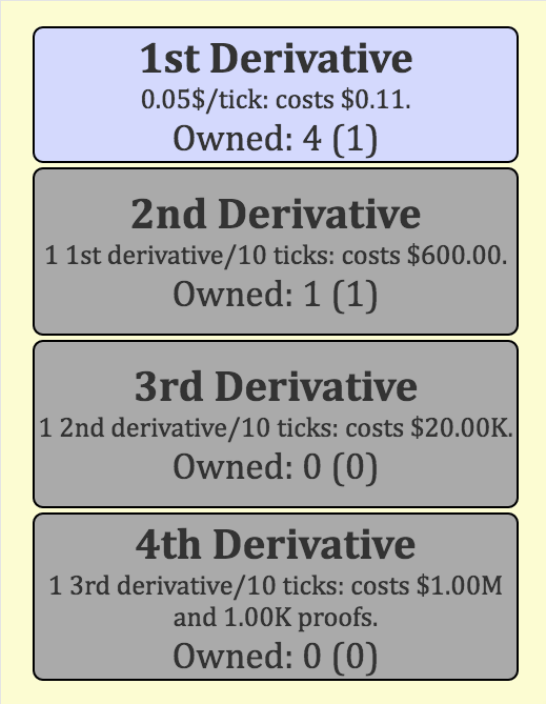
- Cookie Clicker set standard for growth patterns of generators
- Other options exist though and can be explored

Non-Cookie Clicker Growth

- What if generators produced generators?
  - Generator 1 produces Primary Currency
  - Generator 2 produces Generator 1's
  - Generator 3 produces Generator 2's
  - etc.
- These are actually…

Derivatives

- DERIVATIVES!
- Each parent generator is the rate of change of the child
  - Generator 1 = f(x)
  - Generator 2 = f’(x)
  - Generator 3 = f’’(x)
  - etc.



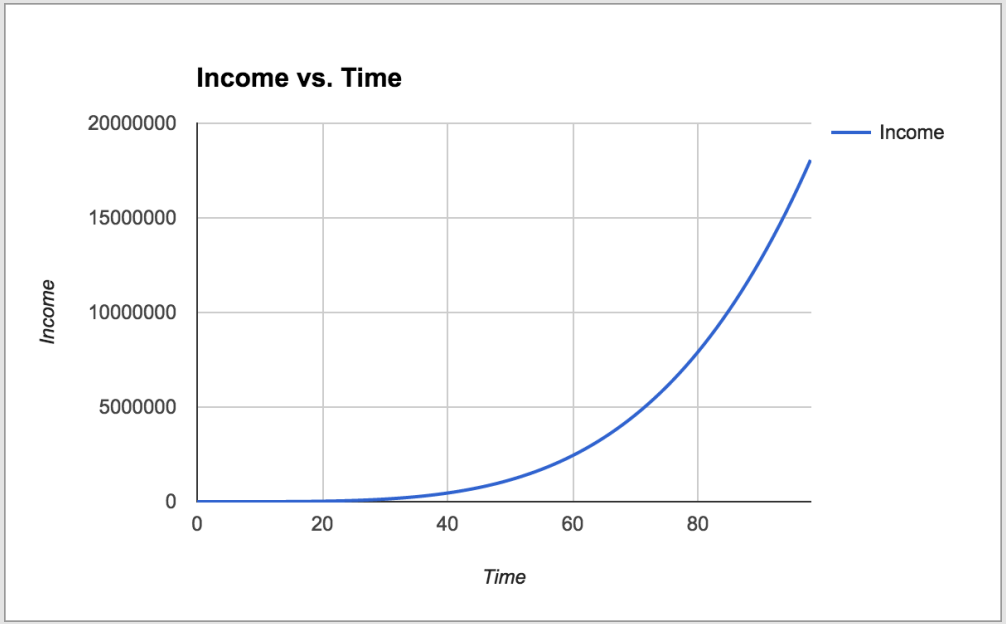
Derivative Clicker

Educational!

- People don’t even realize they’re working with derivatives!
- Great way to conceptualize relationship of higher-order derivatives
  - Physics: Location, Speed, Acceleration, Jerk, …?
  - Here: Each is just the rate of change of the one above it

Derivative Growth In Action

- But what does this actually look like?



Starting Counts	
Tier 0	0
Tier 1	0
Tier 2	0
Tier 3	0
Tier 4	1
Tier 0 (currency generator)	
Base Income	5
Tier 1 (first derivative)	
Tier 0 generation	1
Tier 2 (second derivative)	
Tier 1 generation	1
Tier 3 (third derivative)	
Tier 2 generation	1
Tier 4 (fourth derivative)	
Tier 3 generation	1

Derivative Growth In Action

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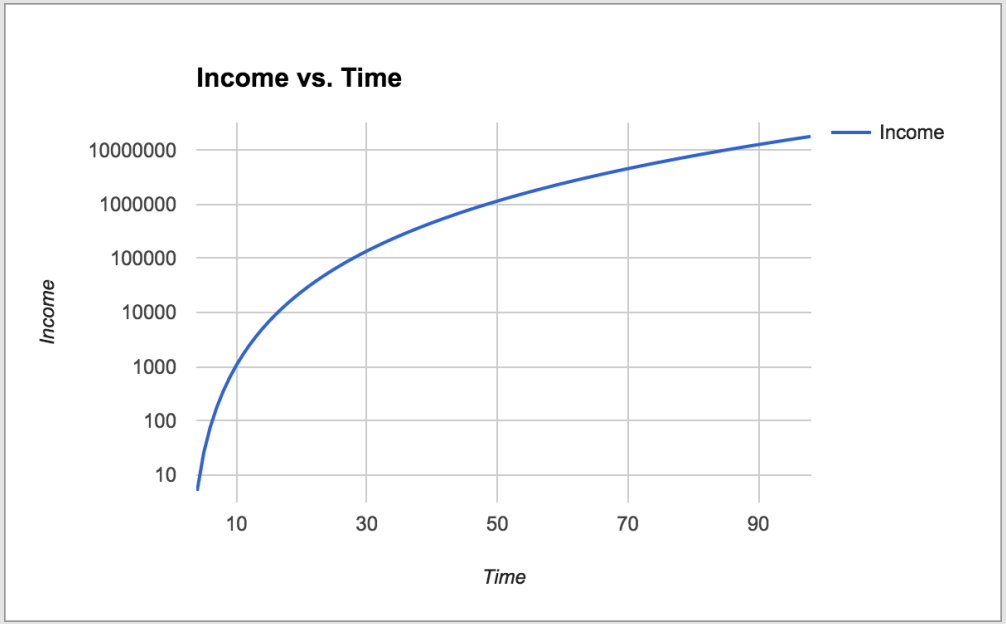

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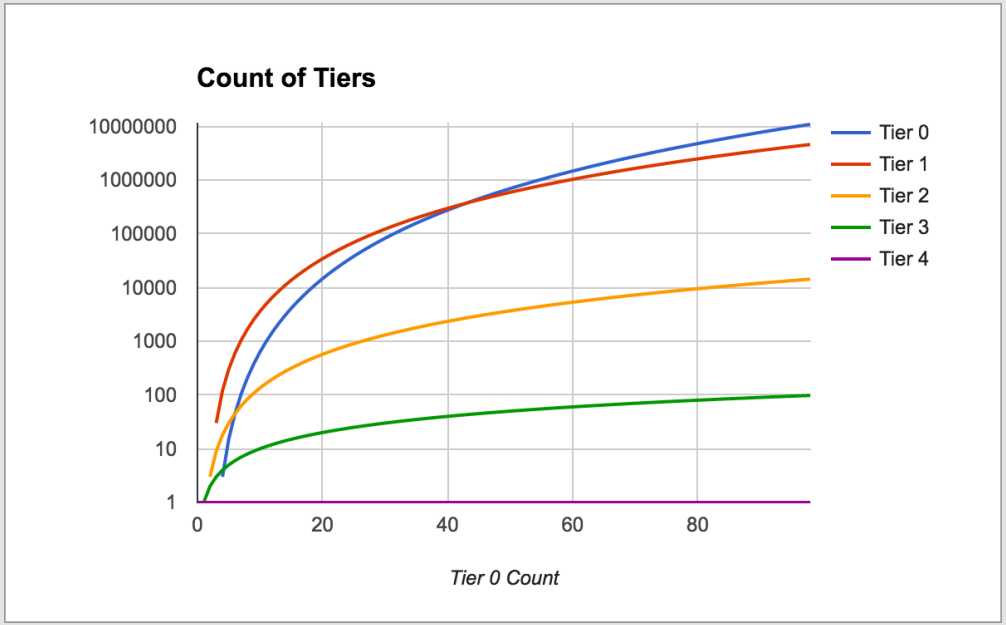


Starting Counts	
Tier 0	0
Tier 1	0
Tier 2	0
Tier 3	0
Tier 4	1
Tier 0 (currency generator)	
Base Income	5
Tier 1 (first derivative)	
Tier 0 generation	0.1
Tier 2 (second derivative)	
Tier 1 generation	10
Tier 3 (third derivative)	
Tier 2 generation	3
Tier 4 (fourth derivative)	
Tier 3 generation	1

Approaching Exponential Growth

- While different in formulation, the growth rate turns out to be similar
  - Let's assume each parent produces 1 child, and we have 4 tiers
  - Tier 4 would be a constant $f(x) = 1$
  - Tier 3 is $\int(1) = x$
  - Tier 2 is $\int(x) = x^2 / 2$
  - Tier 1 is $\int(x^2 / 2) = x^3 / 6$

Approaching Exponential Growth

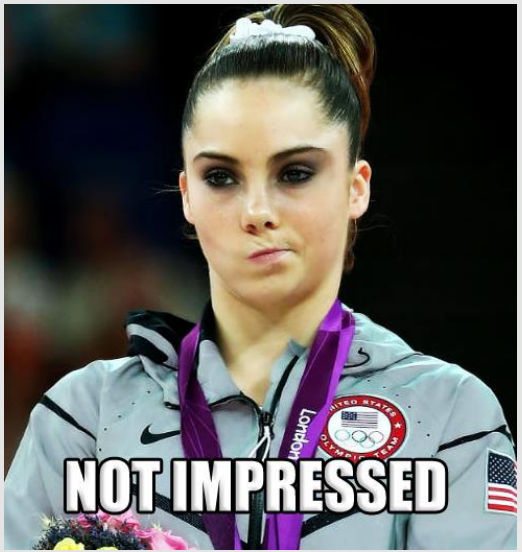
- Anyone remember Taylor series?
- The Taylor series expansion of e^x is…

$$1+\frac{x^1}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\frac{x^5}{5!}+\cdots=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}+\cdots=\sum_{n=0}^{\infty}\frac{x^n}{n!}.$$

- As we get more tiers, we approach actual exponential growth!

Intermission: On Big Numbers

- Think of the biggest number you know of
  - googol? = 10^{100}
  - googolplex? = 10^{googol}



(Special thanks to Eclipse1agg on Reddit)

Intermission: On Big Numbers

- Operator Progression
  - a++ = a + 1 (unary increment)
  - a + b = a++ (b times)
  - a * b = a + a + ... + a (b times)
  - a ^ b = a * a * ... * a (b times)
  - a ↑↑ b = a ^ a ^ … ^ a (b times)
  - a ↑↑↑ b = a ↑↑ a ↑↑ … ↑↑ a (b times)

Intermission: On Big Numbers

- Example Values
  - 3++ = 4
  - 3 + 4 = 7
  - 3 * 4 = 12
  - 3 ^ 4 = 81
  - 3 ↑↑ 4 = 7,625,597,484,987
  - *Note: 3 ↑↑ 5 is > googolplex*
  - 3 ↑↑↑ 4 = ?!?!?!

Intermission: On Big Numbers

- Sample values for ↑↑↑
  - $2 \uparrow\uparrow\uparrow 2 = 2 \uparrow\uparrow 2 = 4$ (always!!)
  - $2 \uparrow\uparrow\uparrow 3 = 256$
  - $3 \uparrow\uparrow\uparrow 2 = 7{,}625{,}597{,}484{,}987$
  - $4 \uparrow\uparrow\uparrow 2$ = a number with over 10^{153} *digits*
  - $2 \uparrow\uparrow\uparrow 4$ = far too big to count digits in scientific notation
  - $3 \uparrow\uparrow\uparrow 3$ = FML

Intermission: On Big Numbers

- Graham's number!
  - $G = g_{64}$
  - $g_1 = 3 \uparrow\uparrow\uparrow\uparrow 3$
  - $g_2 = 3 \uparrow \ldots\{g_1 \text{ times}\}\ldots \uparrow 3$
  - We do know it ends in ...262464195387.
  - Used as an upper bound in a mathematical proof

Keeping All Generators Relevant

- A common problem is keeping smaller generators relevant to the player
  - Production is generally dwarfed, especially with exponential growth of cost
  - This is an even bigger problem for derivative-style growth

Keeping All Generators Relevant

- Do you care if lower generators are relevant for your players?
  - It adds a layer of complexity to both game balance and player understanding and may not be necessary depending on your target audience, play style goals, and other elements
  - If however you do care…

Keeping All Generators Relevant

- Possible Solutions
  - Aggressive multipliers to force relevance
  - Base bonuses on ownership of these generators
    - Even if direct generation is low, impact of purchasing is high
      - Example: Newspapers in AdVenture Capitalist multiply other investments
    - Note: track purchased and earned generators separately in derivative growth systems (Derivative Clicker example)

Keeping All Generators Relevant

- Possible Solutions
  - Tie prestige currency to ownership
    - Clicker Heroes gives one prestige currency for every 2000 generators owned
  - Use ownership multipliers to compound on cheaper generators
    - Clicker Heroes gives a x4 bonus for every 25 of a generator

Prestige Cycles

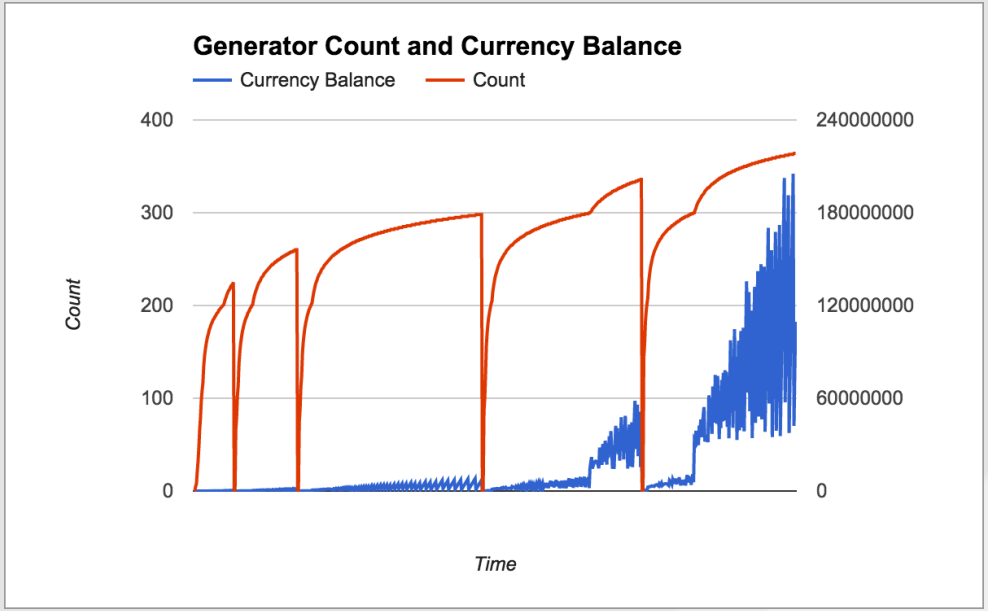
- When will players prestige?
  - Common rule of thumb is to reset when you would gain somewhere in the range of +50% to +200% prestige currency
- What formula will you use for prestige currency?
  - Take a log or a fractional exponent (like square root) to scale back growth rates
  - Ensure that players reach a valuable prestige point regularly

Prestige Cycles

- What does the next cycle look like for players?
  - Will their progress through the early part be quick? This is an important feeling of growth of power.
  - Will they be able to get noticeably farther than last time?
- Do you want the cycles to get faster, slower, or vary?
  - Faster: players will shift prestige point to be a lot higher
  - Slower: could get tiresome, must have a meta prestige too
  - Varied: prestige provides surprises, harder to design

Prestige Cycles

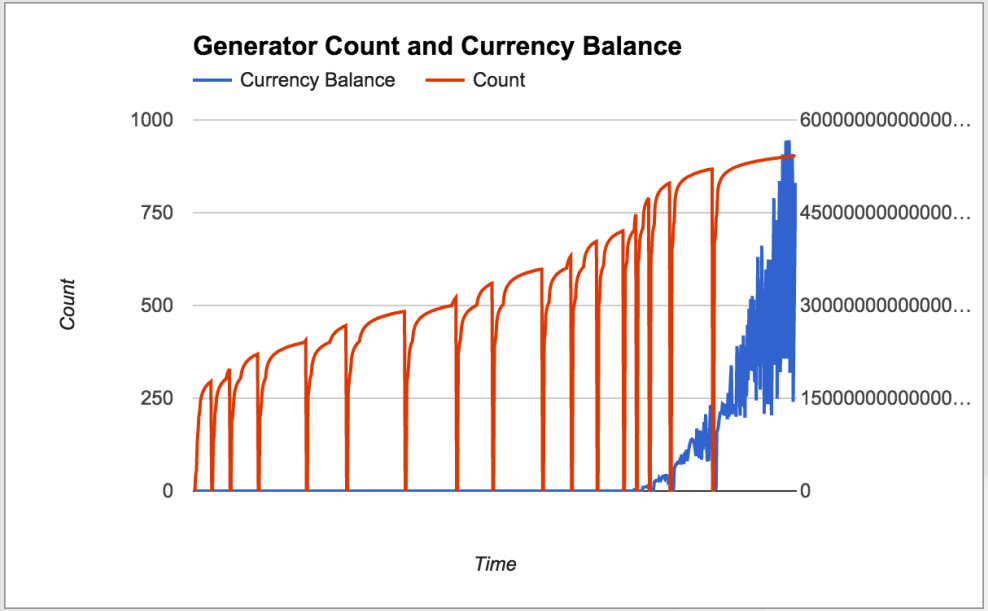
- Simple model of a single generator
  - Can estimate player behavior based on value of generator
  - Multipliers based on number owned allow for variable times between prestiges



Prestige Cycles

- Zooming out we can see a lot of variation in time to prestige

Count Bonus (Non-Cumulative)	
25	2
50	2
100	4
200	4
300	8
400	8
500	16
600	16
700	32



Other Growth Patterns

- Combinatorics
  - Pizza store game, income based on how many different types of pizza you can make
  - ${}_nC_r = n! / (n-r)!r!$
  - Upgrade to use more ingredients per pizza (r) and have larger variety of ingredients to use (n)
- Node connections
  - City-builder with a prestige to found a new city, trade routes between all cities [(n)(n-1)/2]

Notable New Genre Additions

- Egg Inc.
  - Slick, minimalist visual design
  - Egg theme is cute and humorous
  - Dual limiters (egg rate and sale rate) provide for some interesting decisions over time
  - Has a 2 hour limit on offline earning – I think this was a mistake, I churned out myself largely because of this

Notable New Genre Additions

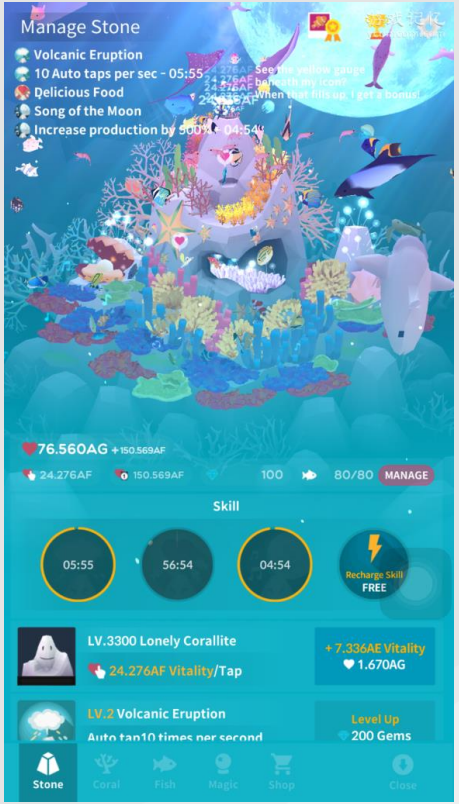
- Nonstop Knight
  - Clicker Heroes-like-like
  - Idle game that looks and feels like a dungeon crawler
  - Armor and hitpoints of hero are important
  - Skills have short cooldowns, playing more like Diablo skills
  - Clever speed-up system to get through early, trivial content when prestiging

Notable New Genre Additions

- War Tortoise
  - Visually very strong with really cool visualization of combat and upgrades
  - Player interaction is powerful and thankfully requires no rapid tapping

Notable New Genre Additions

- AbyssRium
  - Very pretty, relaxing design
  - Milestone multipliers are tied to buying fish, creates a great collection system built into the mechanics



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