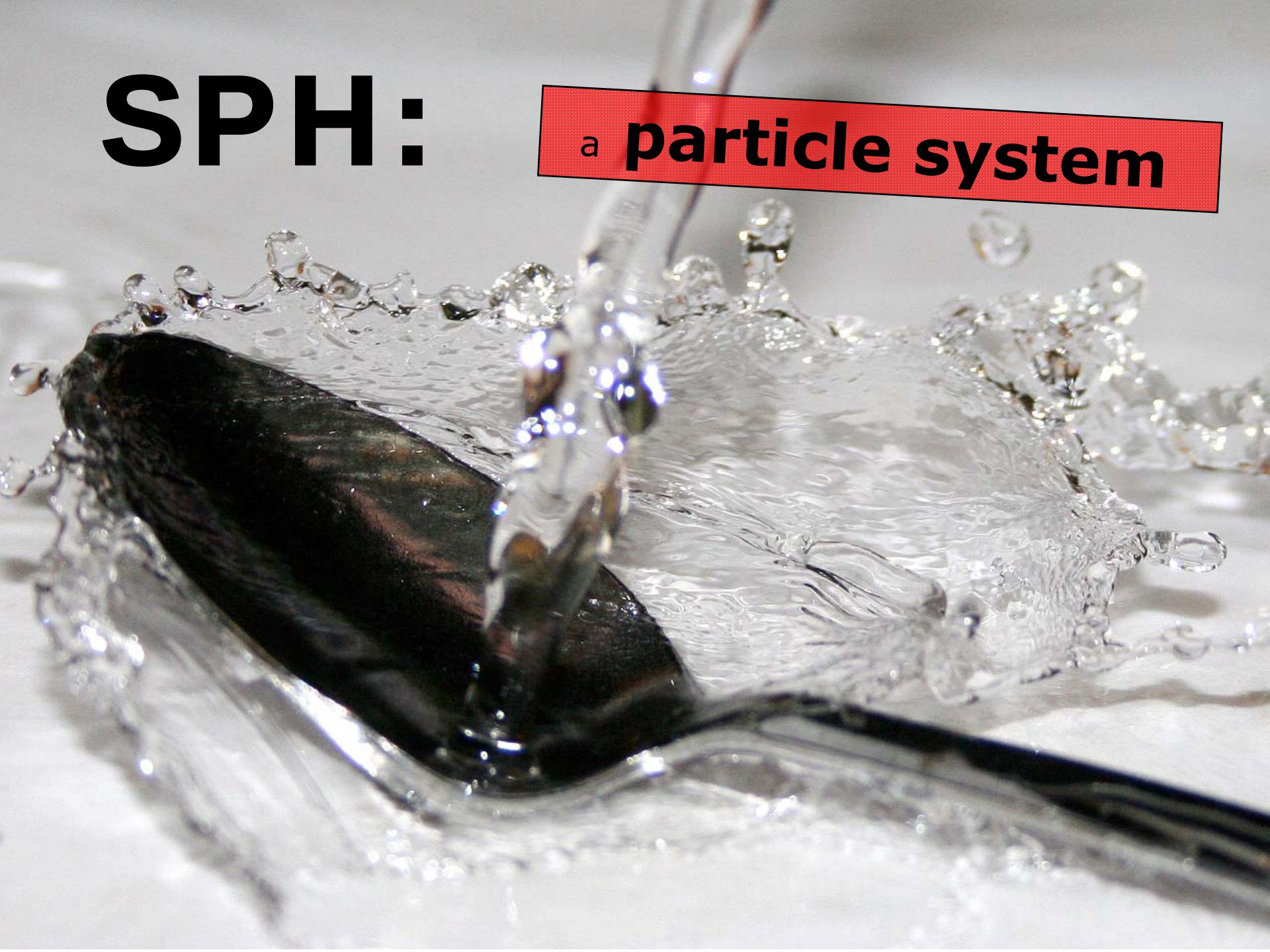
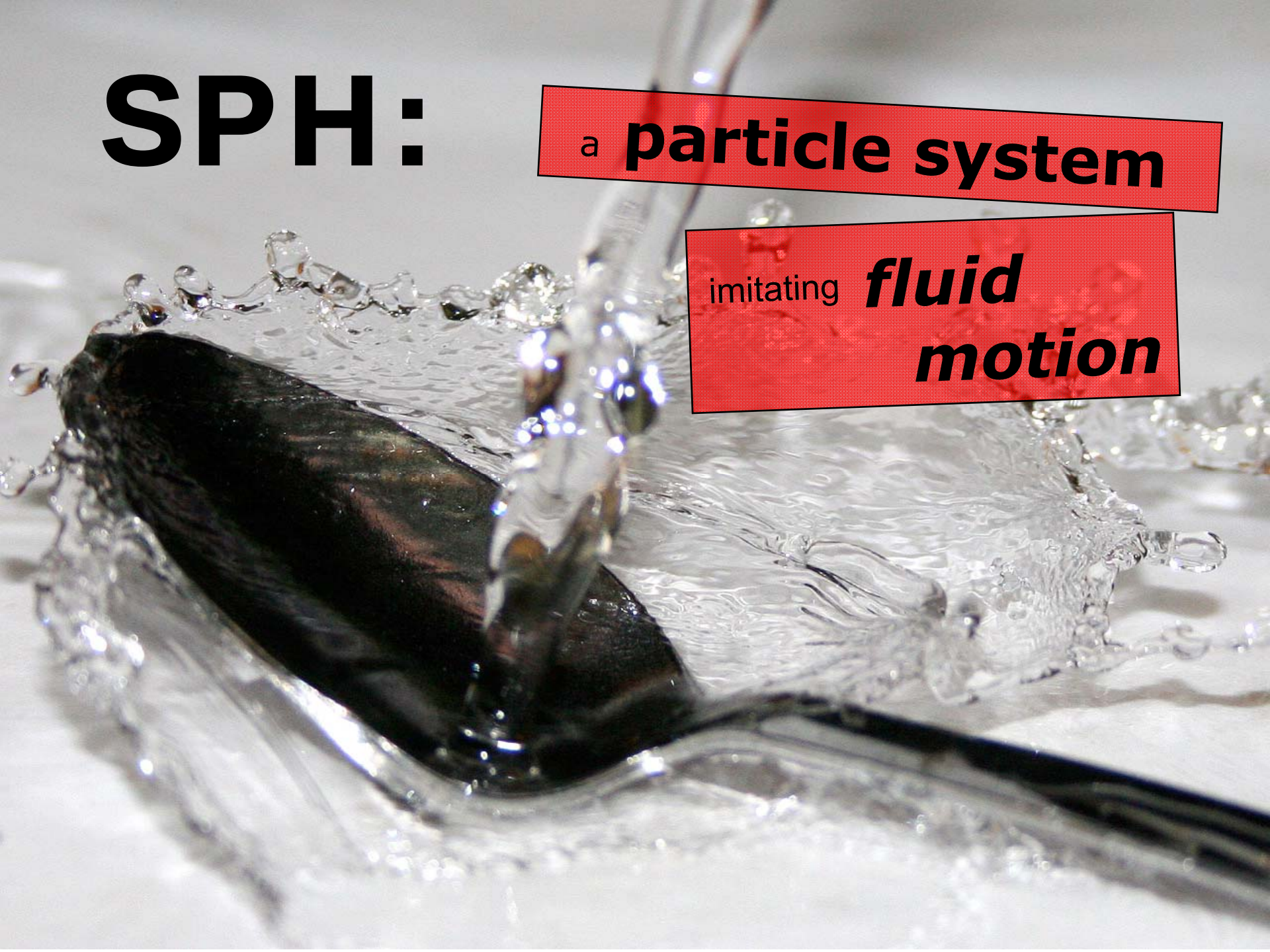
Smoothed Particle Hydrodynamics

by **Kees van Kooten**

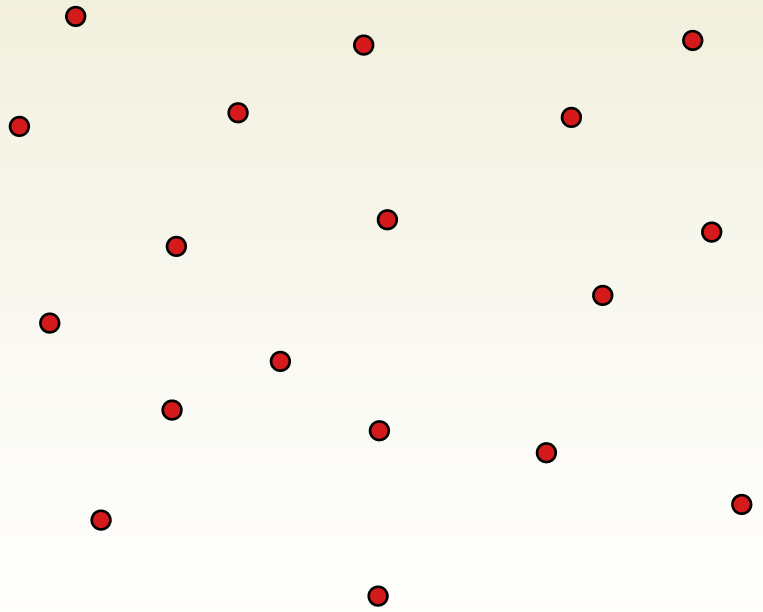
Virtual Proteins

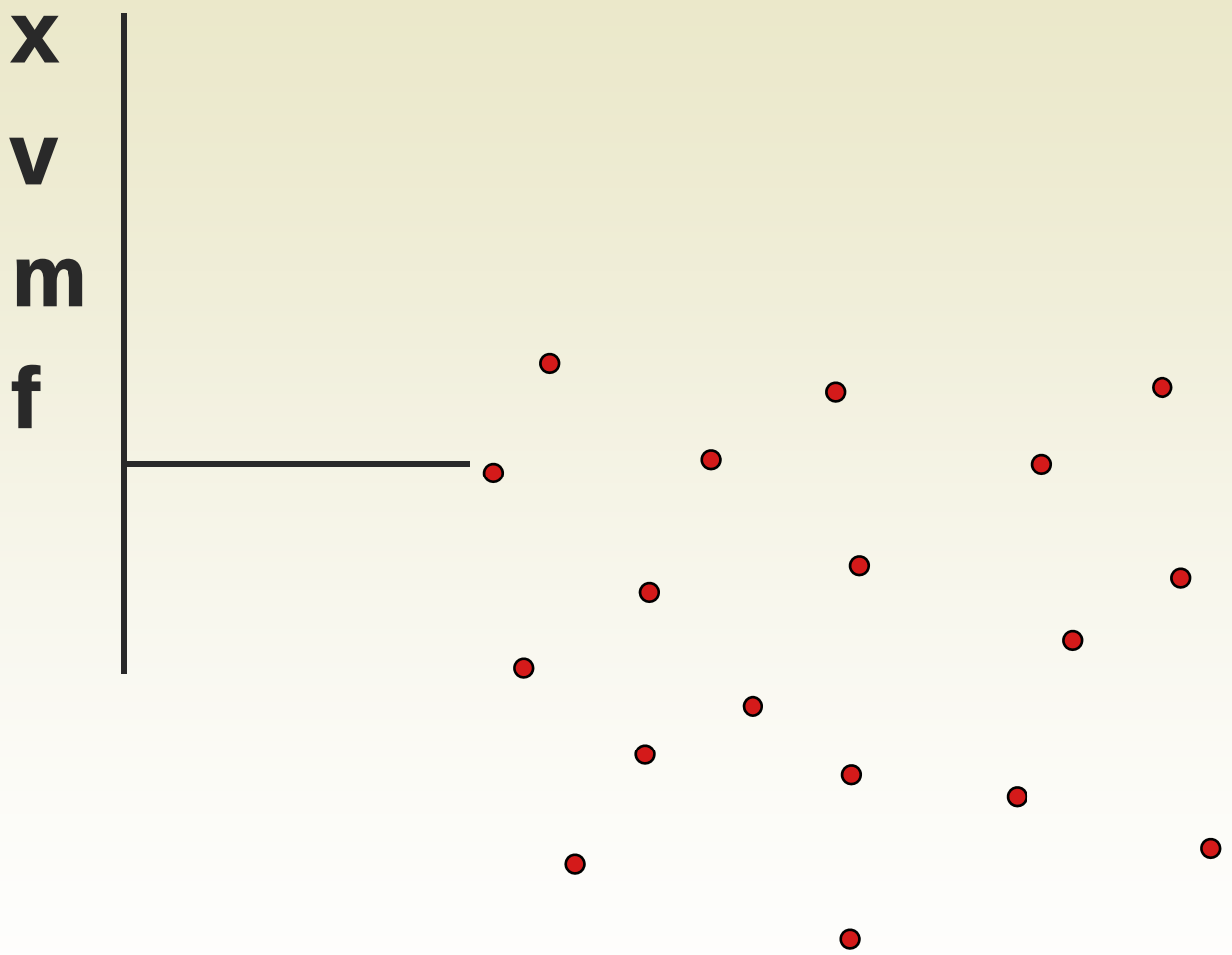

SPH:
a particle system

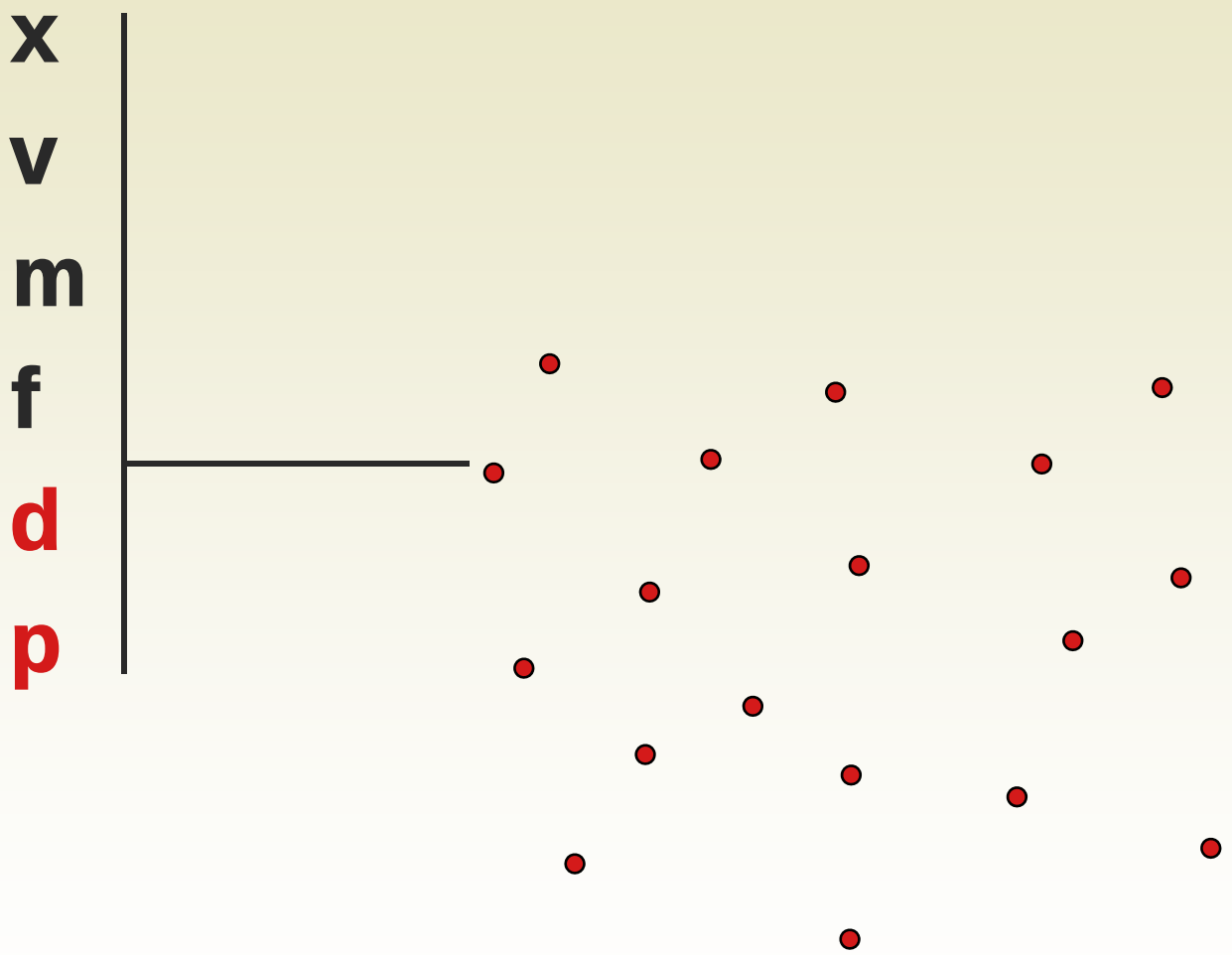

SPH:
a particle system
imitating fluid motion

SPH:

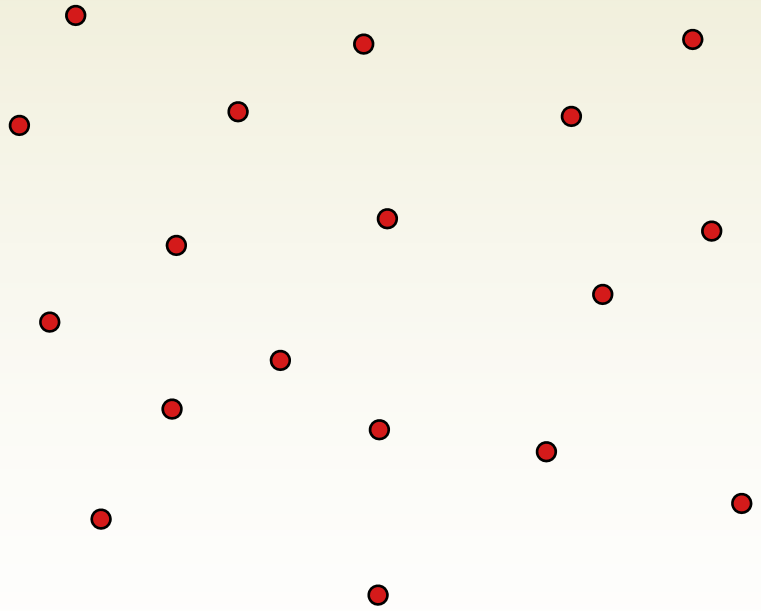
particle system



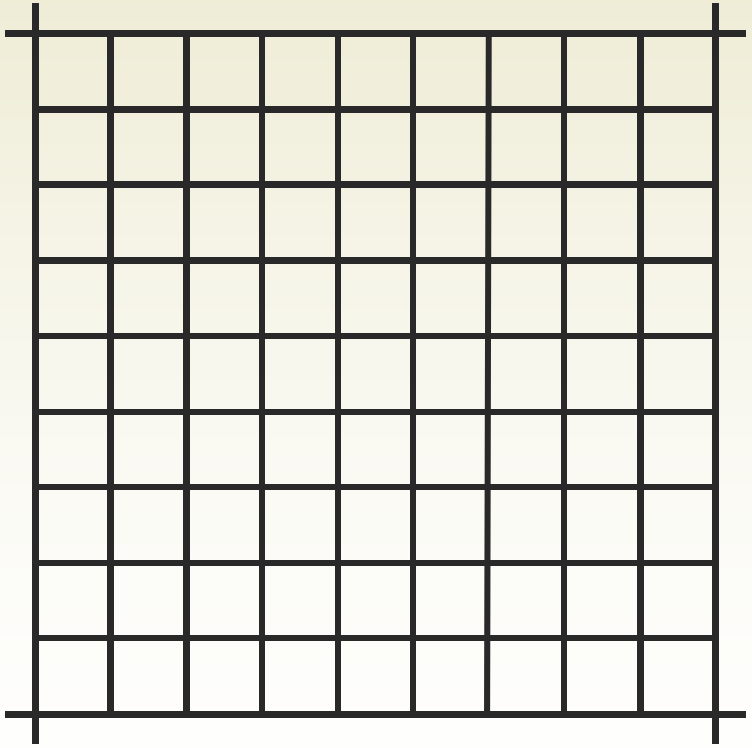

x
v
m
f


x
v
m
f
d
p

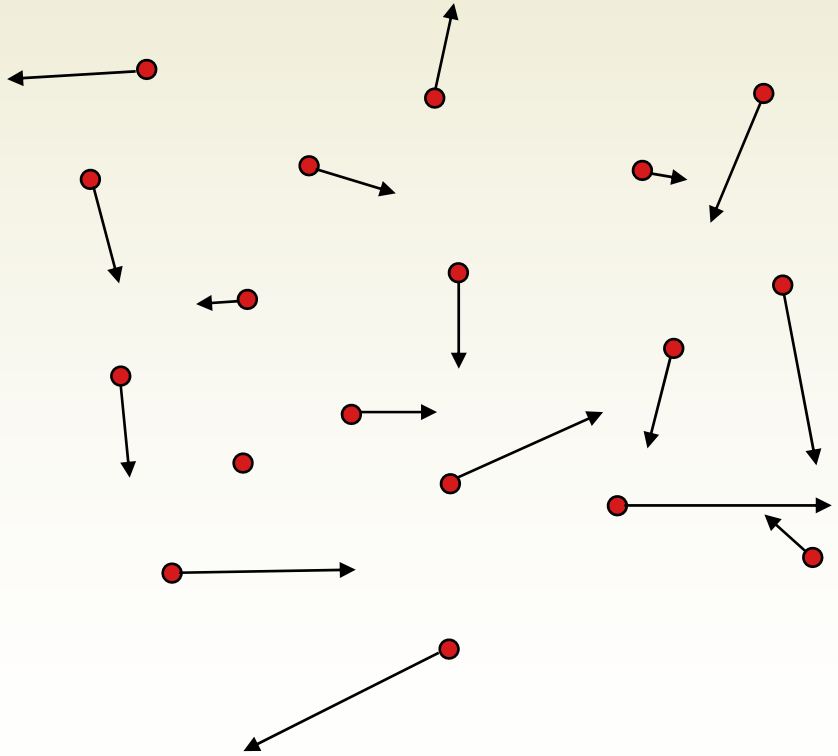
Lagrange



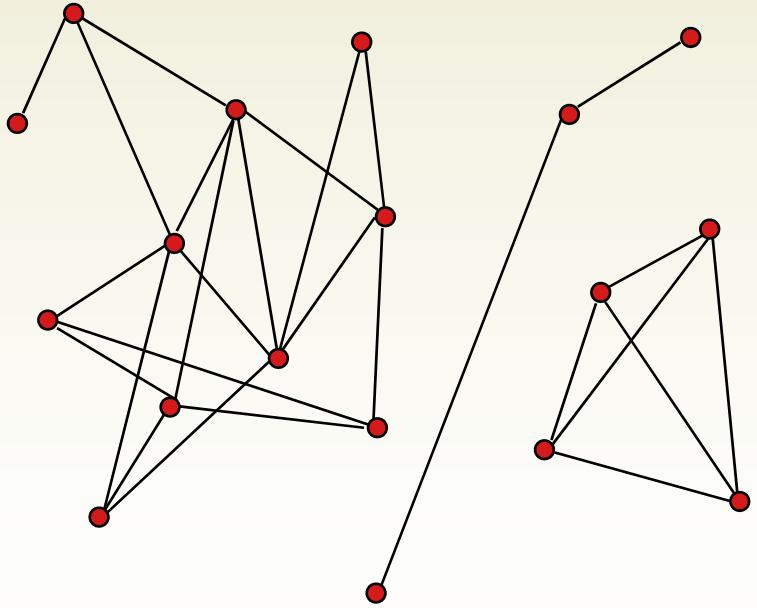
Euler

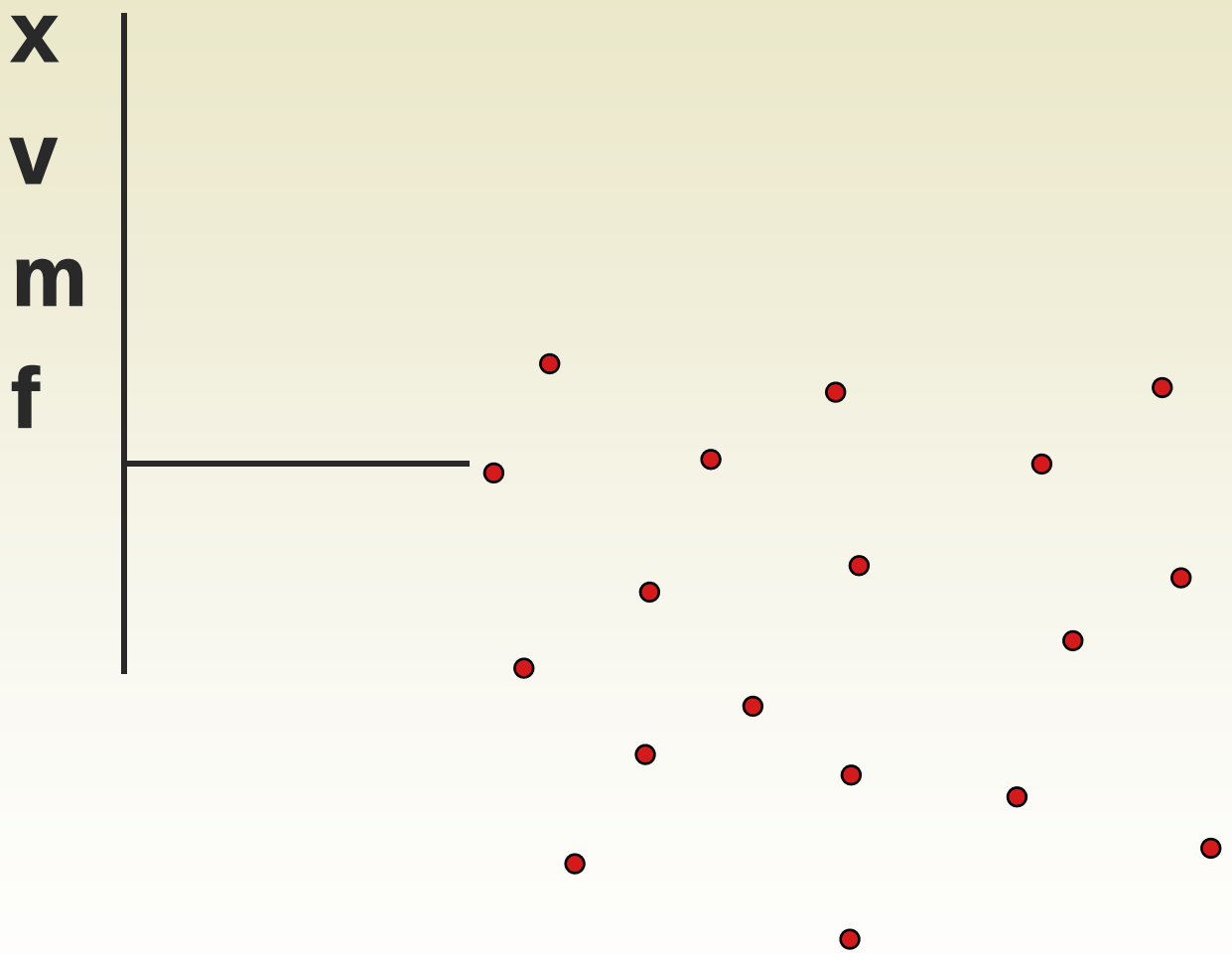


Independence



Dependence




x
v
m
f

$$\frac{dx}{dt} = v$$

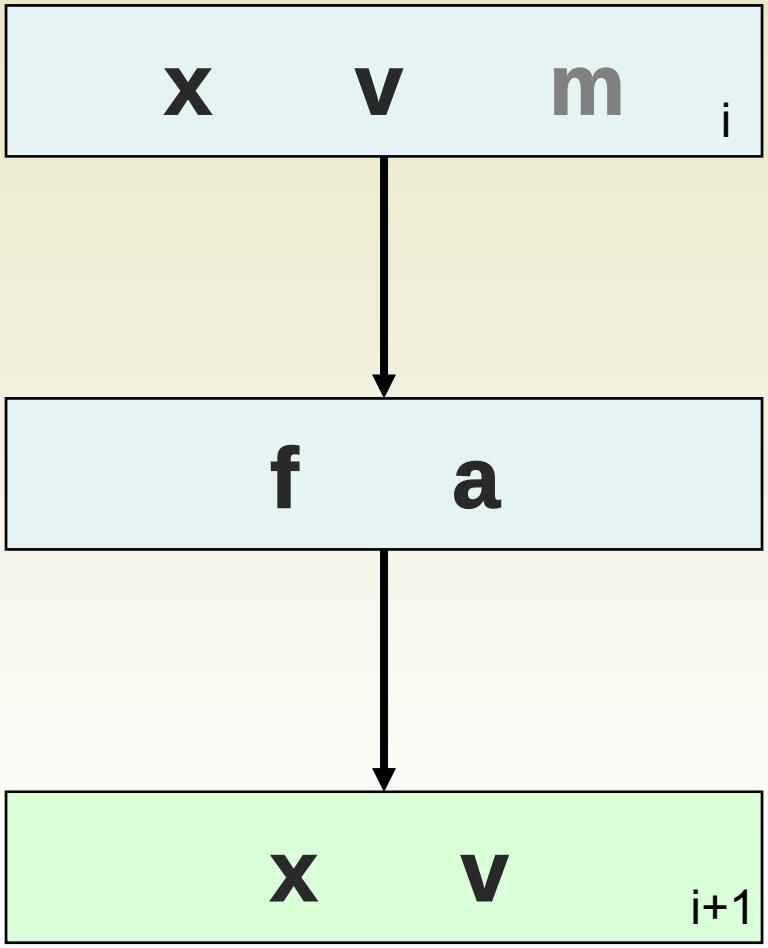
$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = a$$

$$\frac{dx}{dt} = v$$

$$\frac{dv}{dt} = a$$

$$m\,a = f$$


x v m
i
f a
x v
i+1

f

m

v_i

x_i

$\mathbf{f}$

$\mathbf{m}$

$\mathbf{v}_{i+1}$ $\mathbf{v}_i$

$\mathbf{x}_{i+1}$ $\mathbf{x}_i$

$$a = \frac{f}{m}$$

v_{i+1} v_i

x_{i+1} x_i

$$a = \frac{f}{m}$$

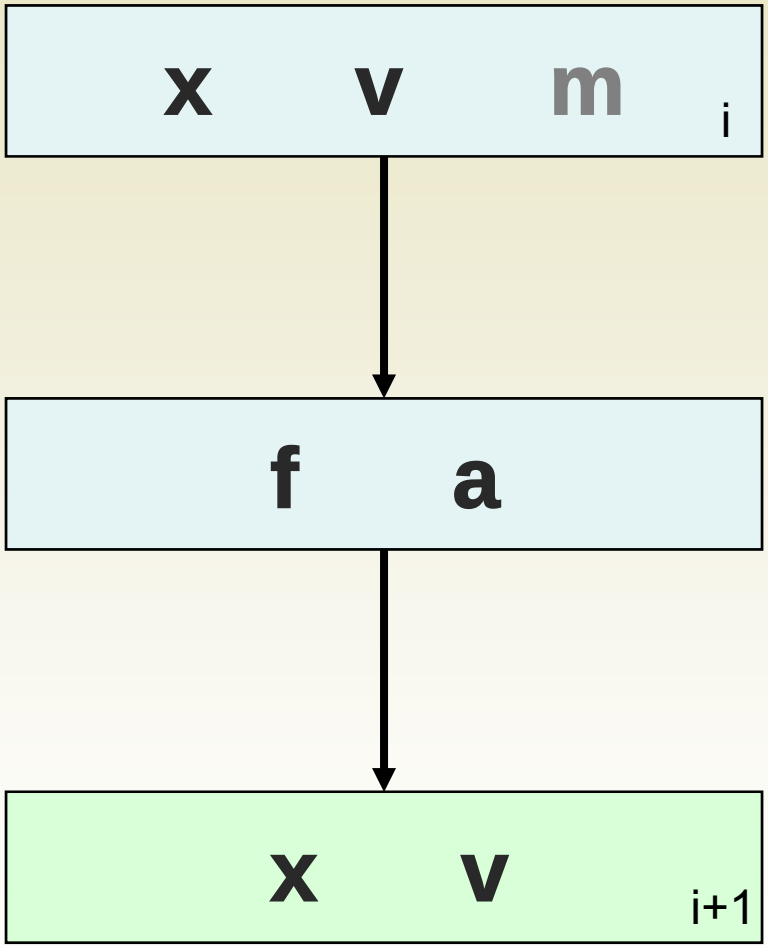
$$v_{i+1} = v_i + a\Delta t$$

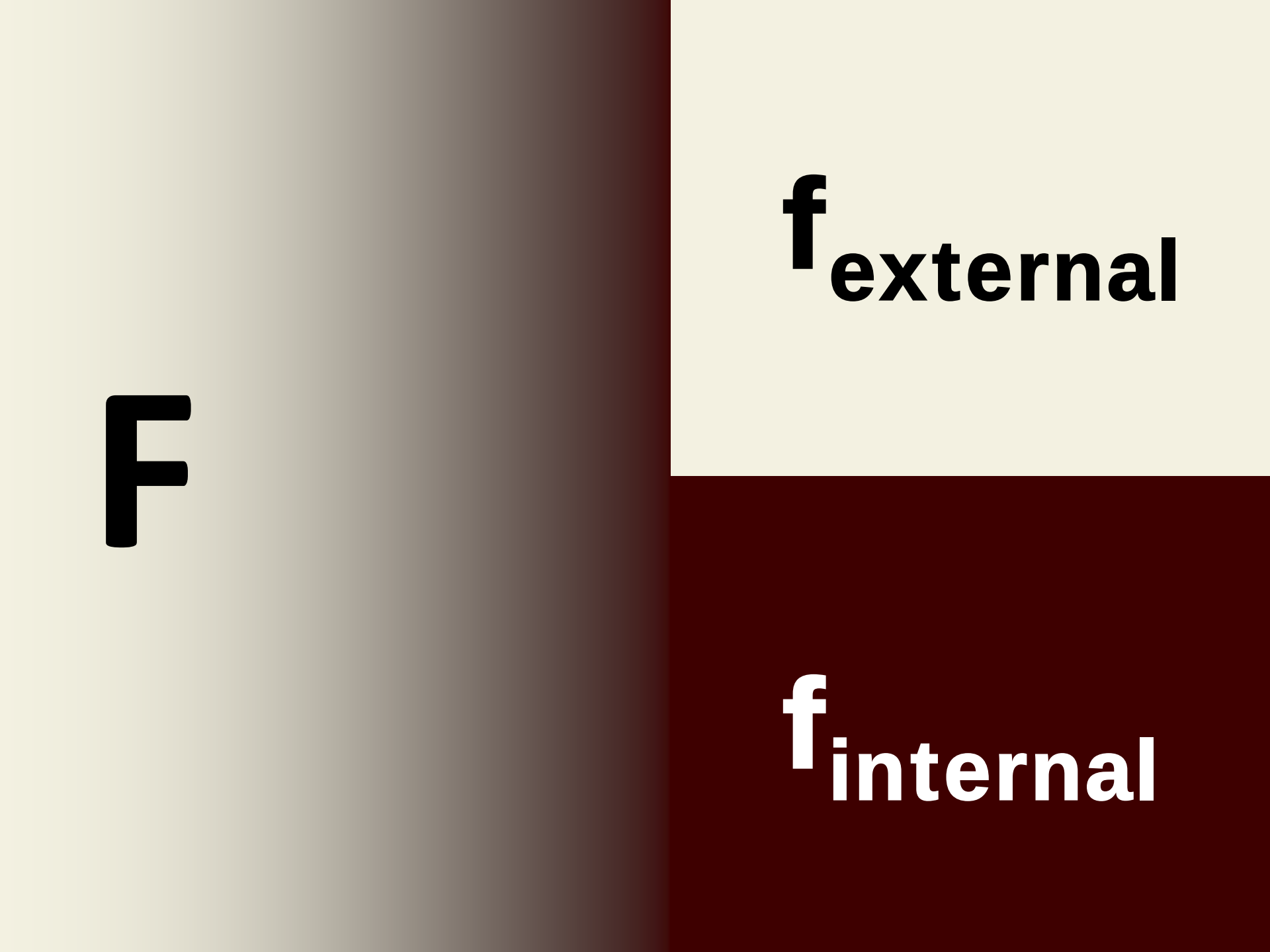
$$x_{i+1} \quad x_i$$

$$a = \frac{f}{m}$$

$$v_{i+1} = v_i + a\,\Delta t$$

$$x_{i+1} = x_i + v_i\,\Delta t$$


x v m
i
f a
x v
i+1


F
f external
f internal

$$f_{internal}$$

$$f_p \sim p$$

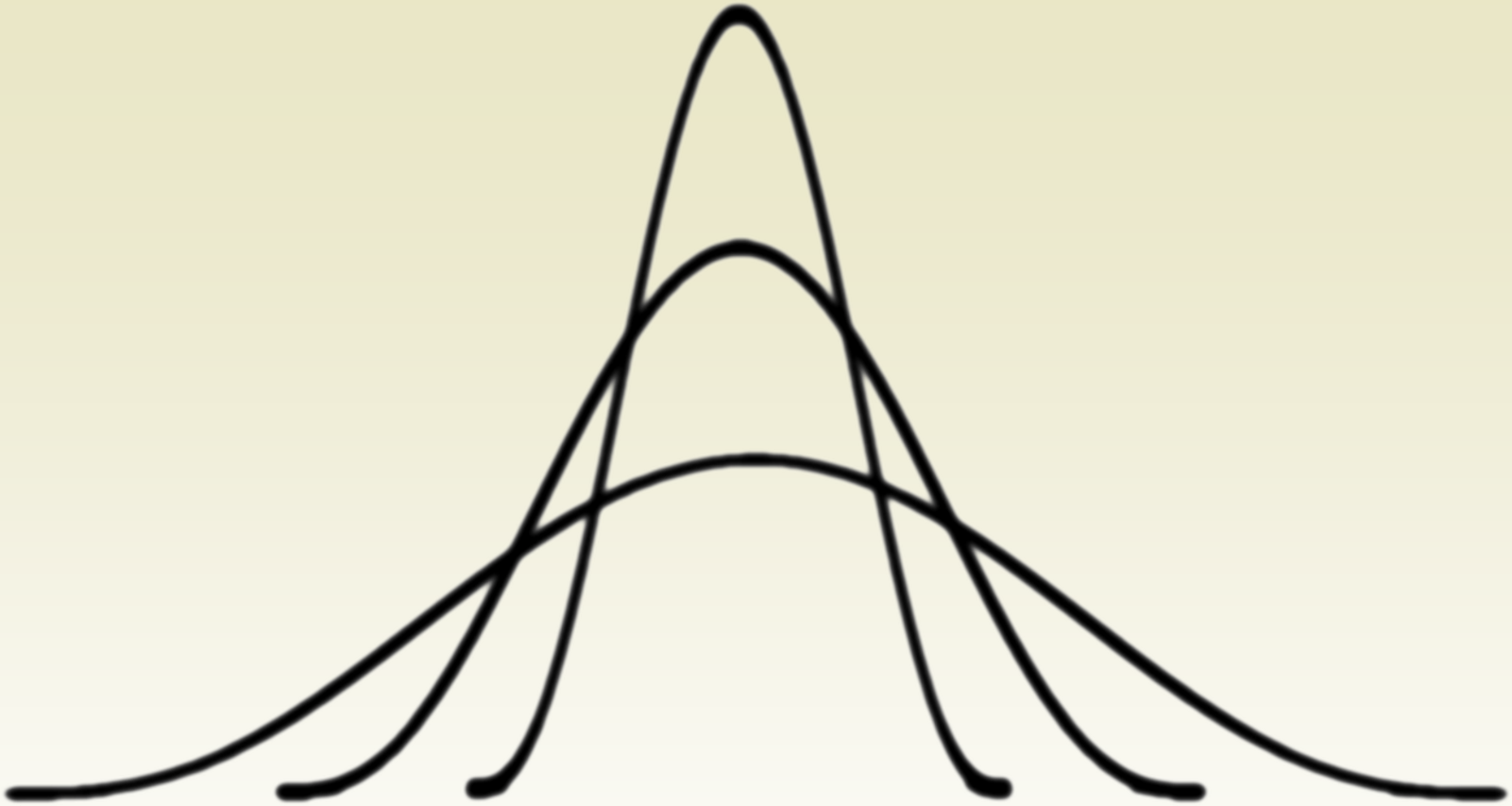
$$f_v \sim v$$

$$f_{internal}$$

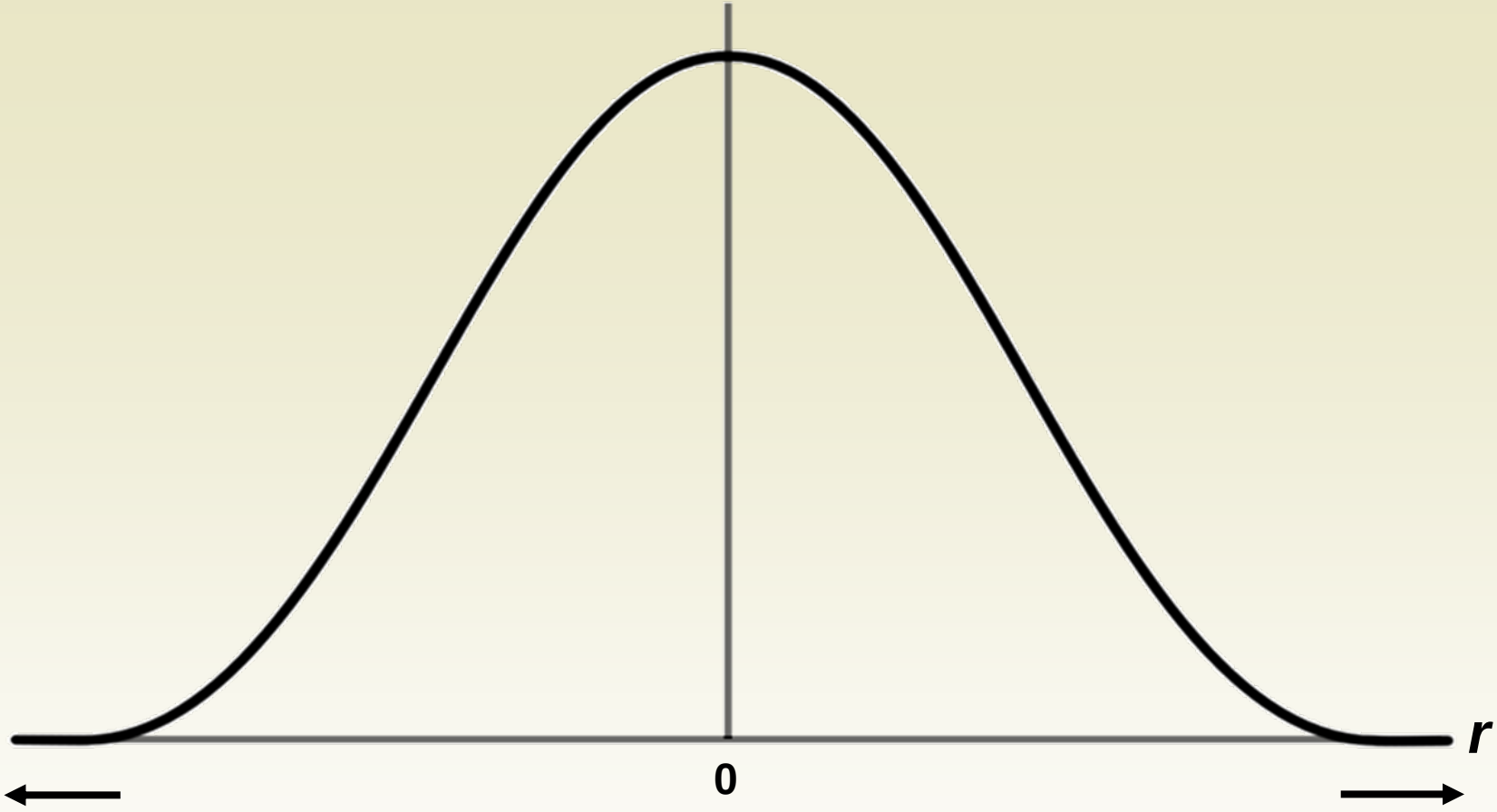
$$\sim d$$

$$f_p \sim p$$

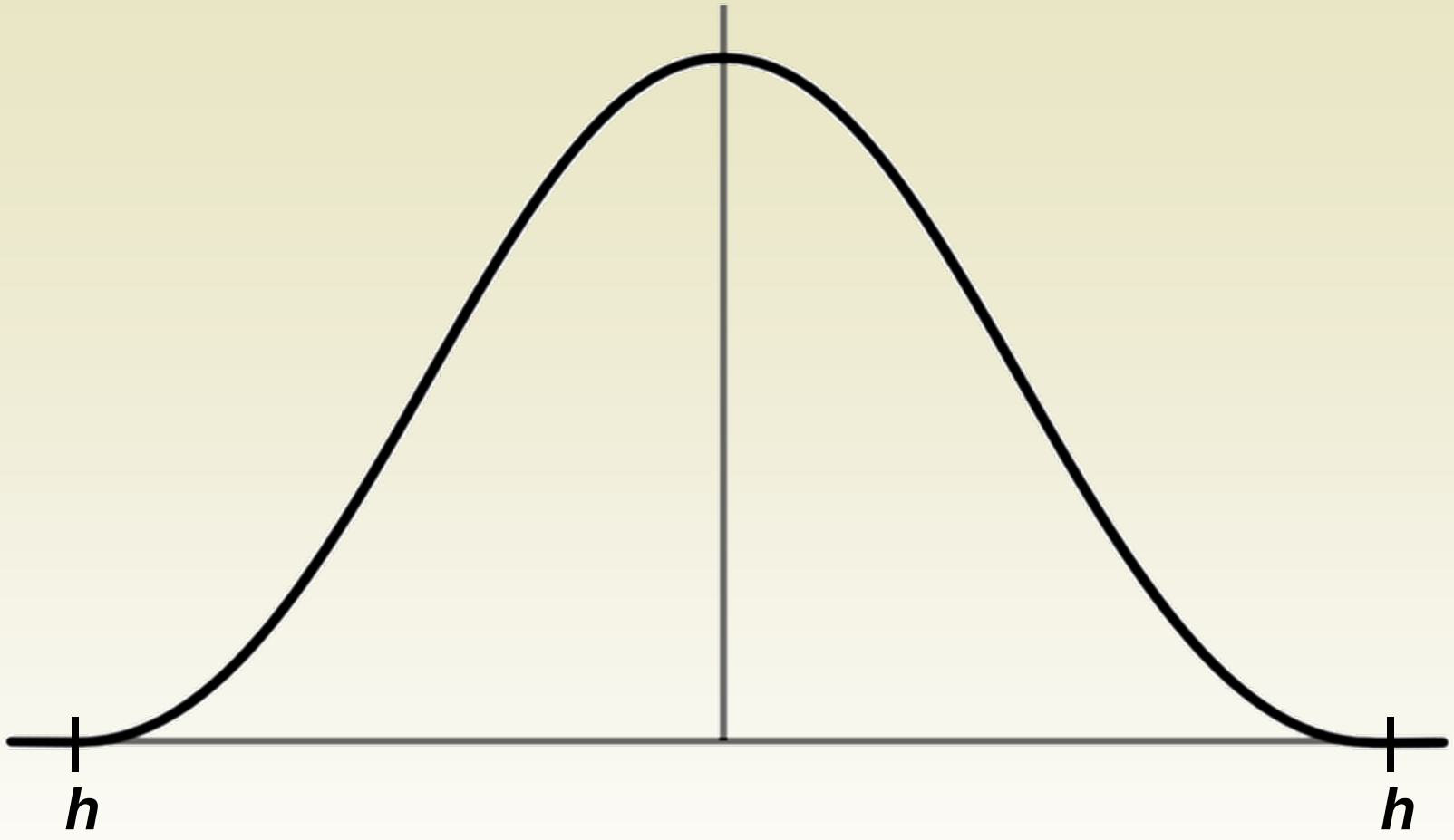
$$f_v \sim v$$



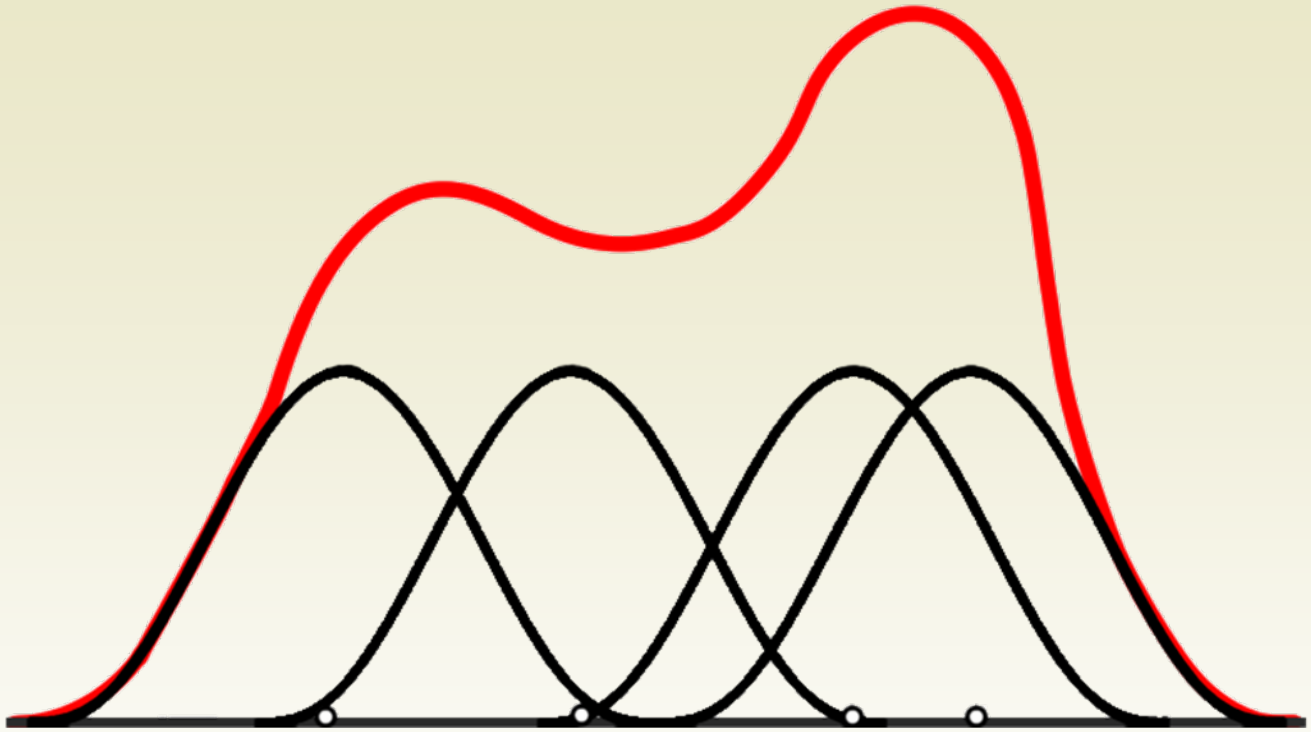
Smoothing Kernels



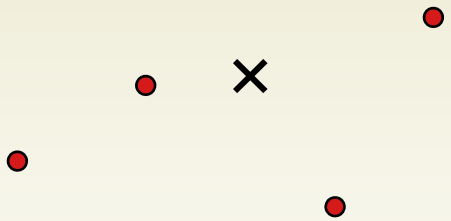
W(r)

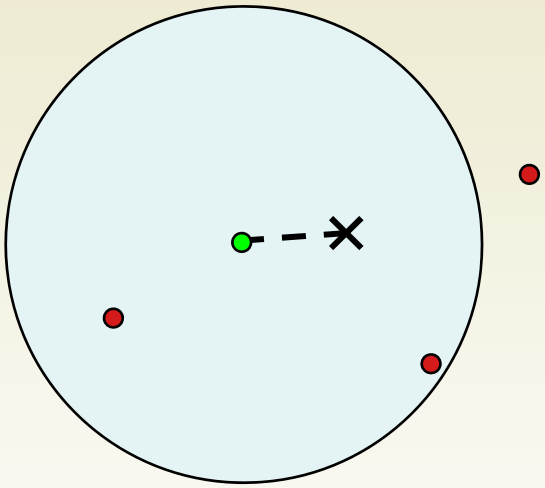


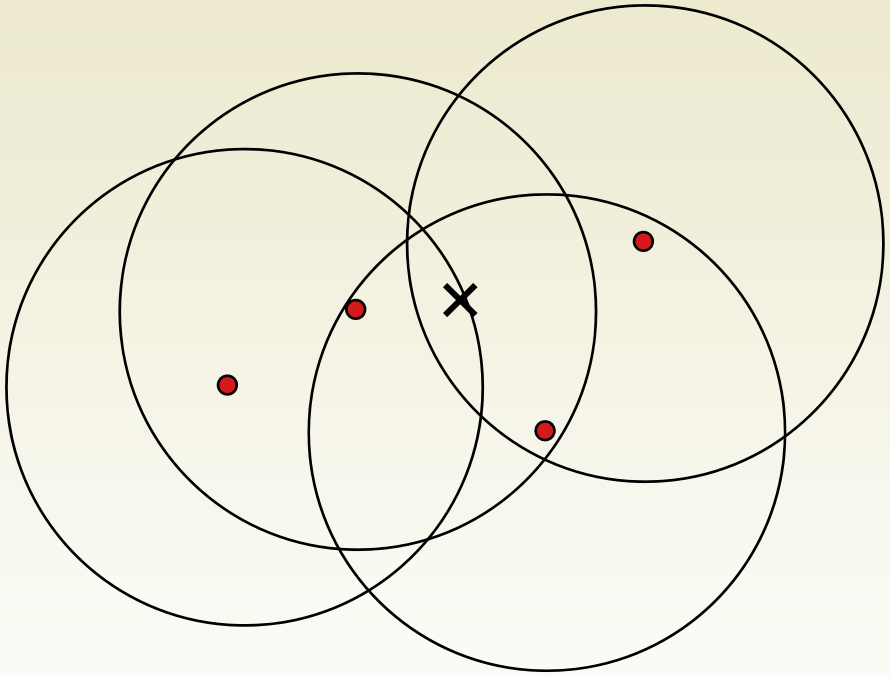
W(r,h)

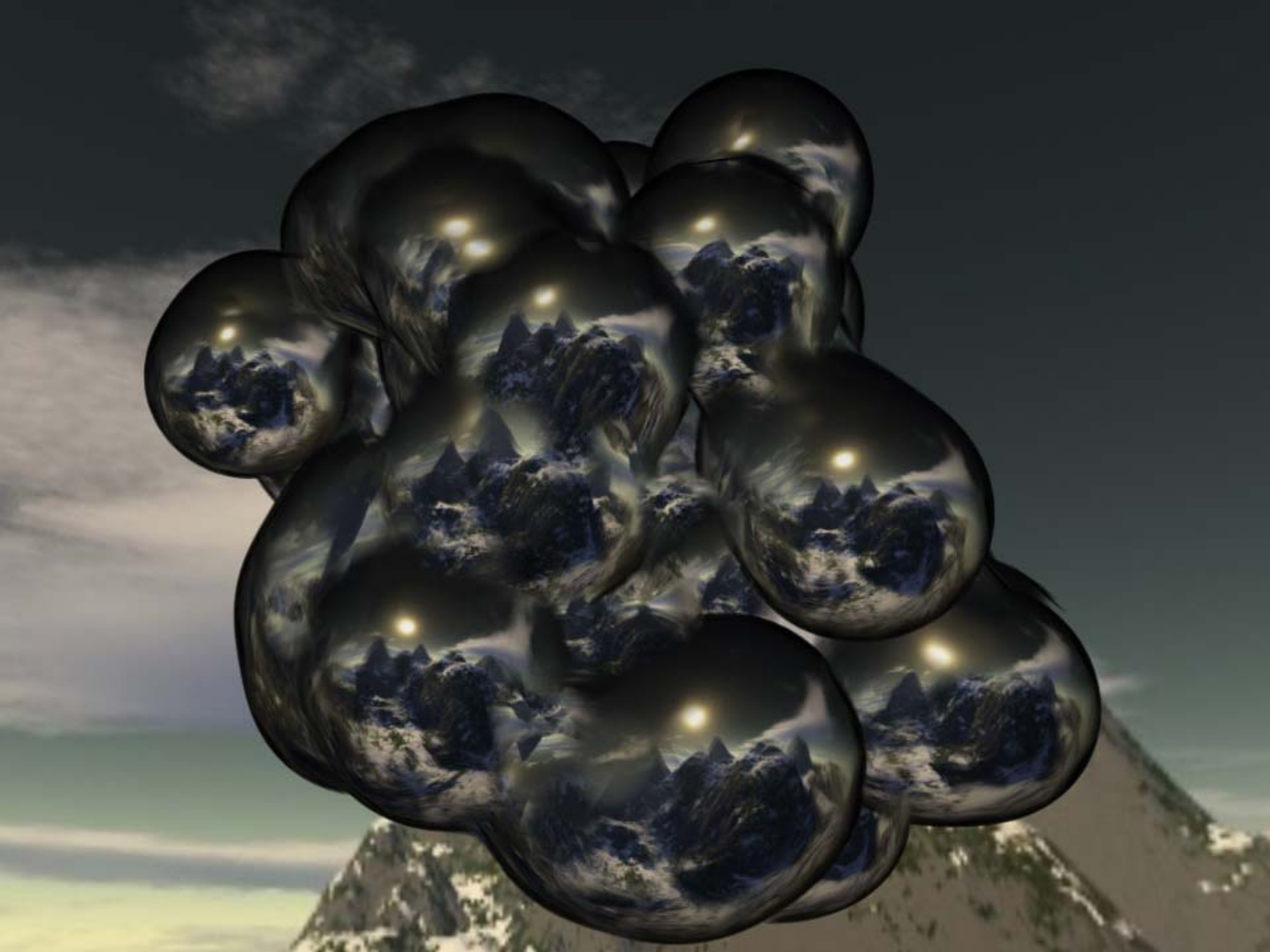


$$\sum W(r,h)$$

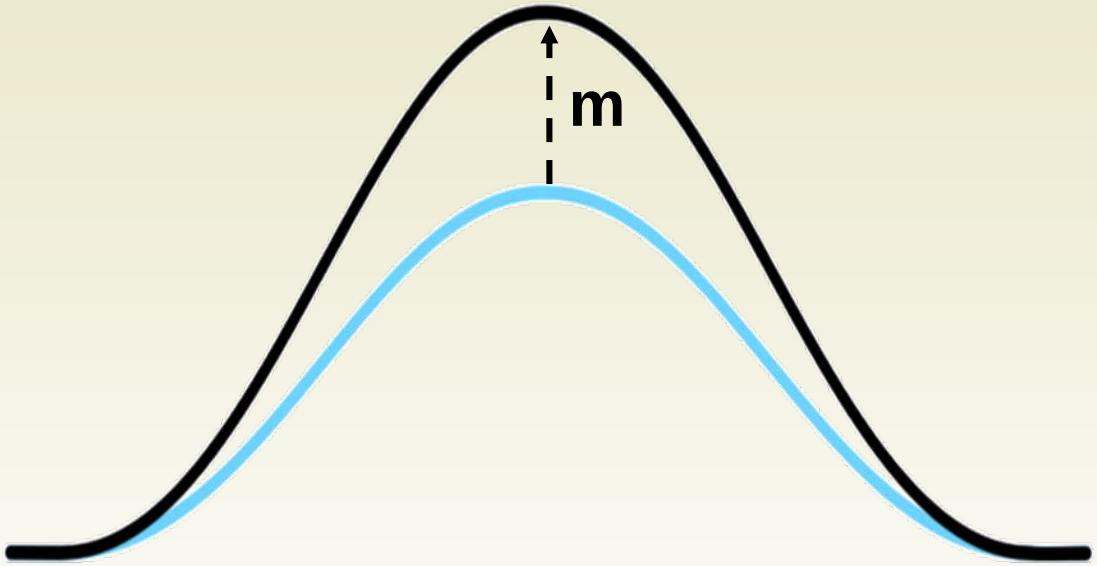




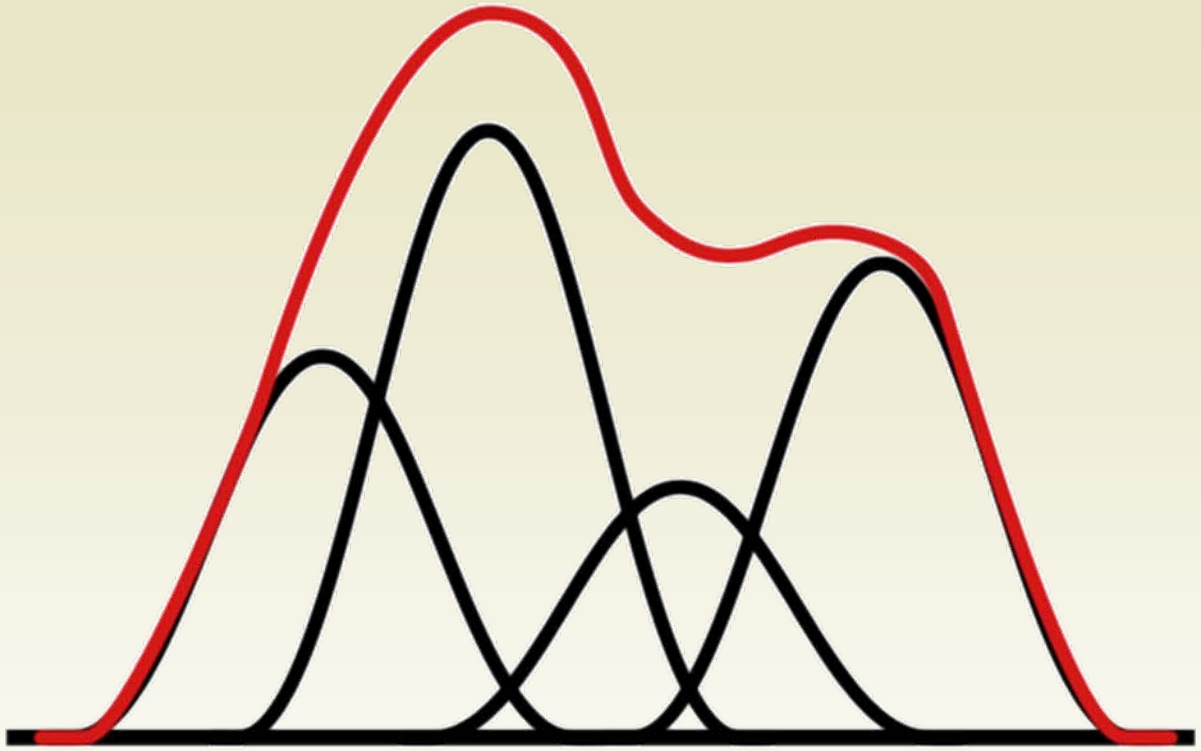




$$\mathbf{d}(x) = \mathbf{W}(r)$$



$$\mathbf{d}(x) = \mathbf{m}\,\mathbf{W}(r)$$

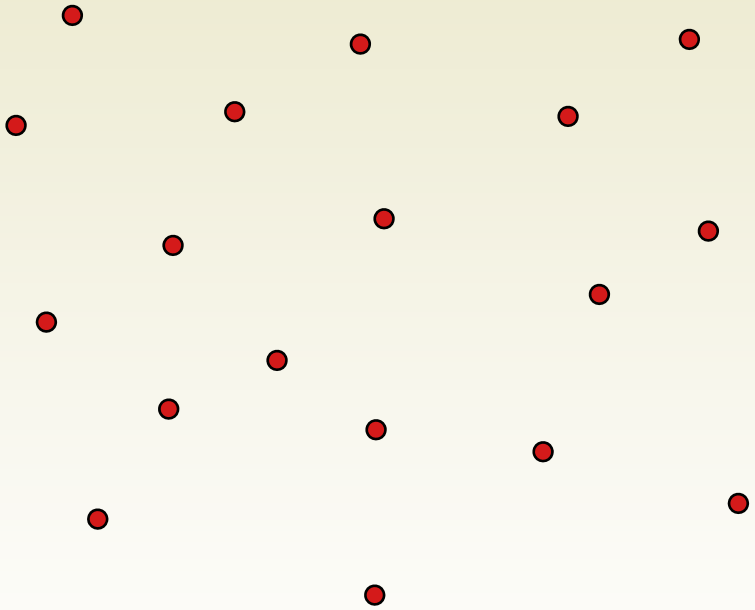


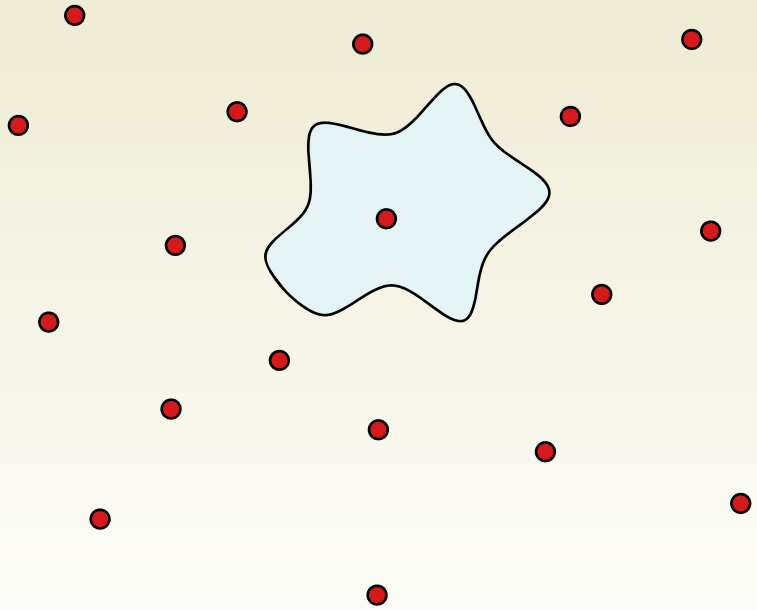
$$\mathbf{d}(x) = \sum_b \mathbf{m}_b \mathbf{W}(r_b)$$

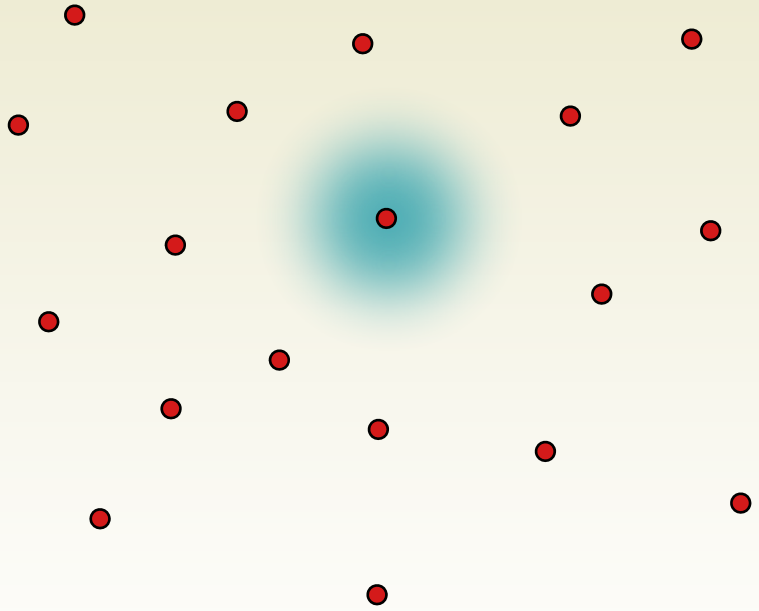
$$f_{internal}$$

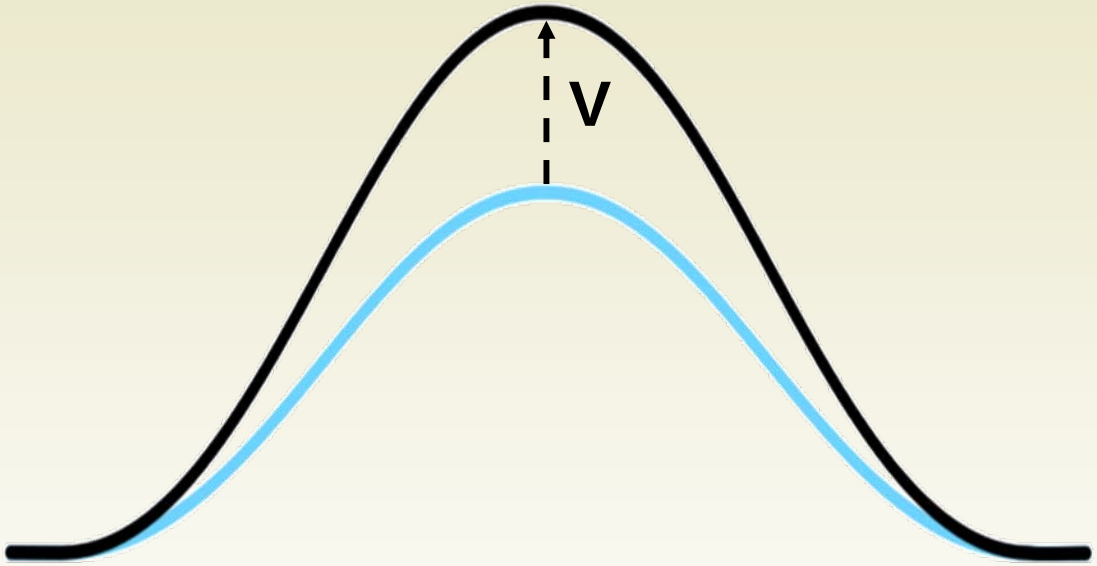
$$f_p \sim p$$

$$f_v \sim v$$

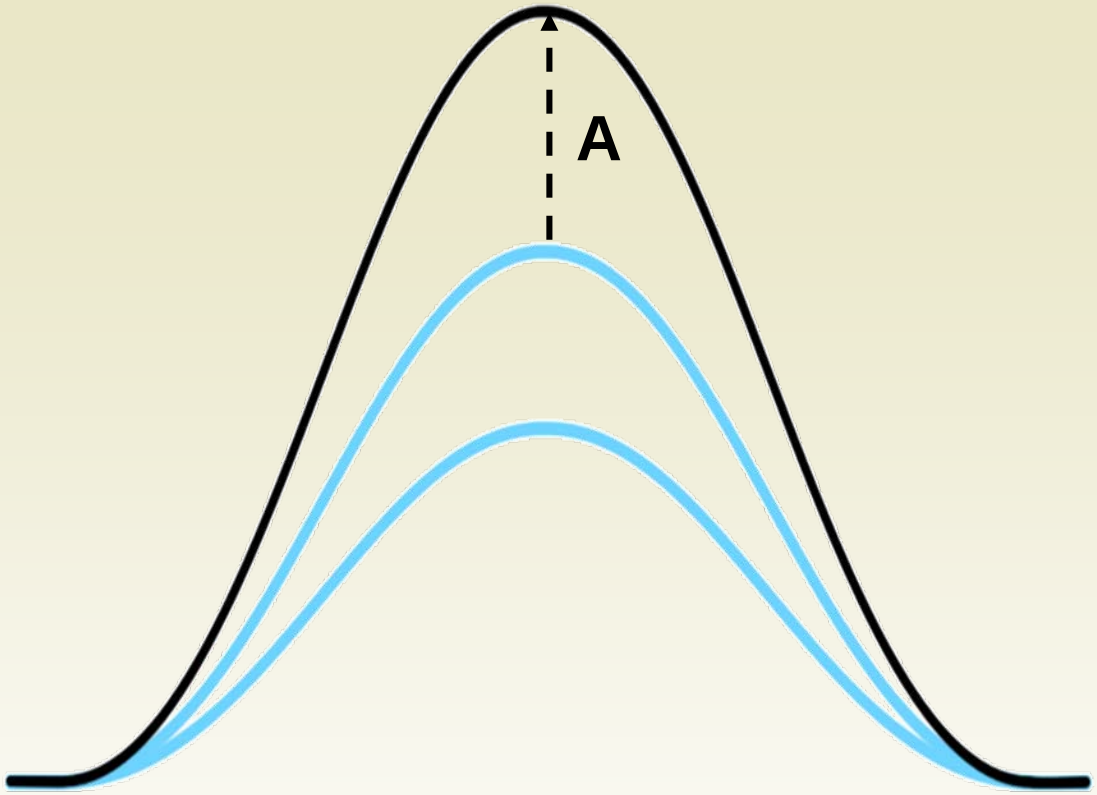




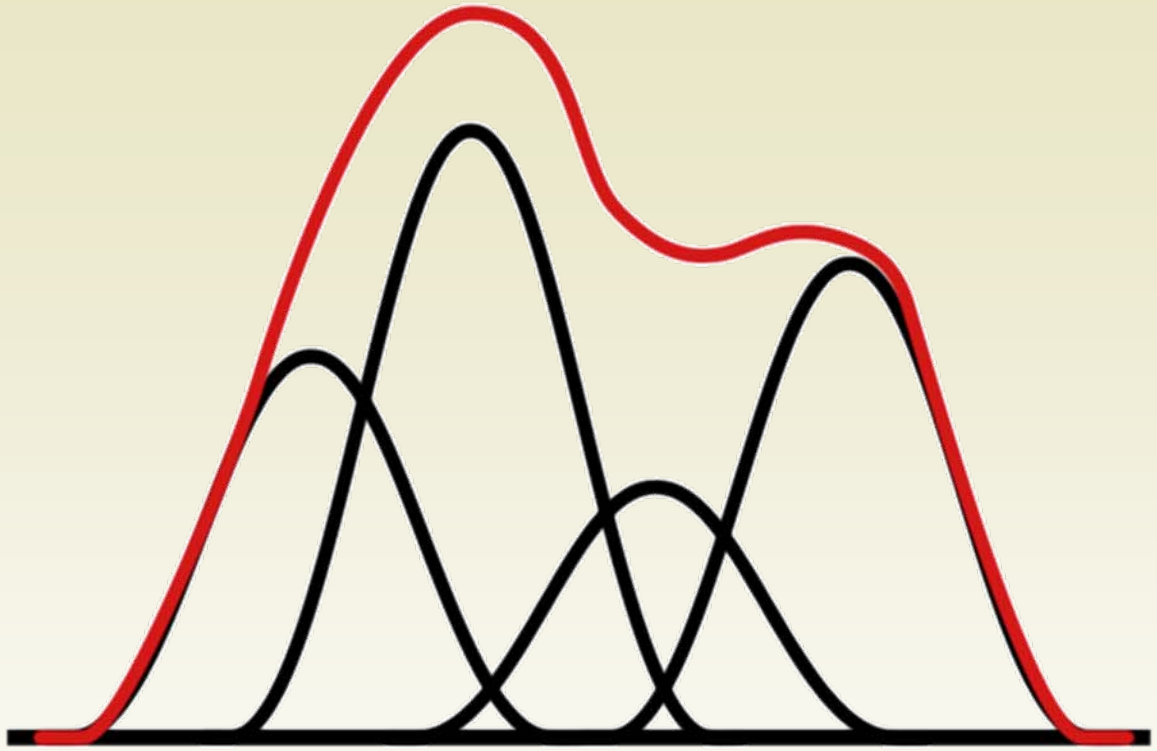




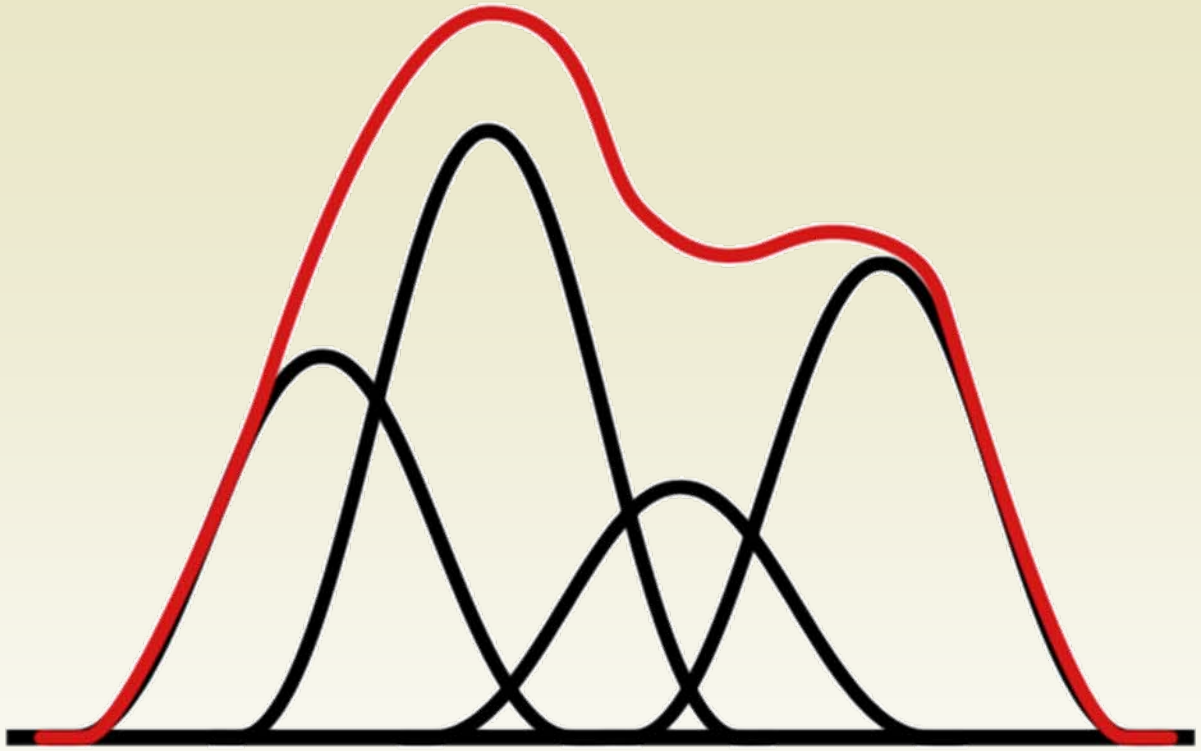
$$\mathbf{A}(x) = \mathbf{V}_b \ldots \mathbf{W}(r_b)$$



$$\mathbf{A}(x) = \mathbf{V}_b \, \mathbf{A}_b \, \mathbf{W}(r_b)$$



$$\mathbf{A}(x) = \sum_b \mathbf{V}_b \mathbf{A}_b \mathbf{W}(r_b)$$



$$\mathbf{A}(x) = \sum_b \frac{\mathbf{m}_b}{\mathbf{d}_b} \mathbf{A}_b \mathbf{W}(r_b)$$

d

$$\mathbf{A}(x) = \sum_b \frac{m_b}{d_b} \mathbf{A}_b \mathbf{W}(r_b)$$

$$\mathbf{d}(x) = \sum_b \frac{\mathbf{m}_b}{\mathbf{d}_b} \mathbf{d}_b \mathbf{W}(r_b)$$

$\mathbf{d}$

$$\mathbf{d}(x) = \sum_b \mathbf{m}_b \mathbf{W}(r_b, h)$$

p

$$\mathbf{A}(x) = \sum_b \mathbf{V}_b \, \mathbf{A}_b \, \mathbf{W}(r_b)$$

$$\mathbf{p}(x) = \sum_b \mathbf{V}_b \, \mathbf{p}_b \, \mathbf{W}(r_b)$$

p

$$p_b = d_b\, k$$

$$\mathbf{p}(x) = \sum_b \mathbf{V}_b\, \mathbf{p}_b\, \mathbf{W}(r_b)$$

V

$$\mathbf{A}(x) = \sum_b \mathbf{V}_b \, \mathbf{A}_b \, \mathbf{W}(r_b)$$

$$\mathbf{v}(x) = \sum_b \mathbf{V}_b \, \mathbf{v}_b \, \mathbf{W}(r_b)$$

$$f_{internal}$$

$$f_p \sim p$$

$$f_v \sim v$$

f_p

$\nabla p(x)$

$$\mathbf{f}_p$$

$$\nabla(\sum_b \mathbf{v}_b \mathbf{p}_b \mathbf{W}(\mathbf{r}_b))$$

$$\mathbf{f}_p$$

$$\sum_b \mathbf{v}_b \mathbf{p}_b \mathbf{W}'(r_b) \underline{r}_b$$

$$f_p$$

$$\sum_b V_b \frac{(p_a + p_b)}{2} W'(r_b) \underline{r}_b$$

f_v

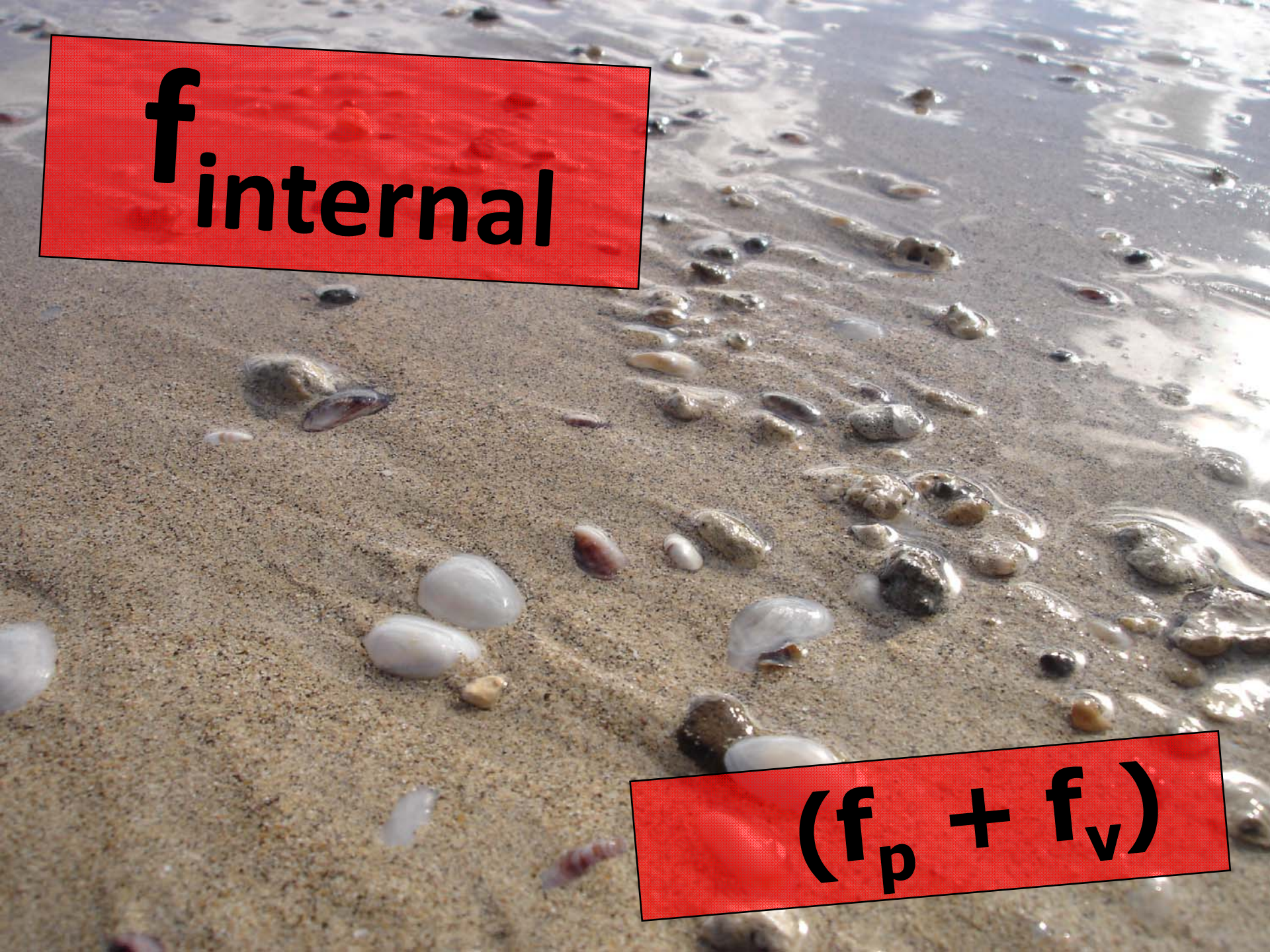
$\nabla^2 v(x)$

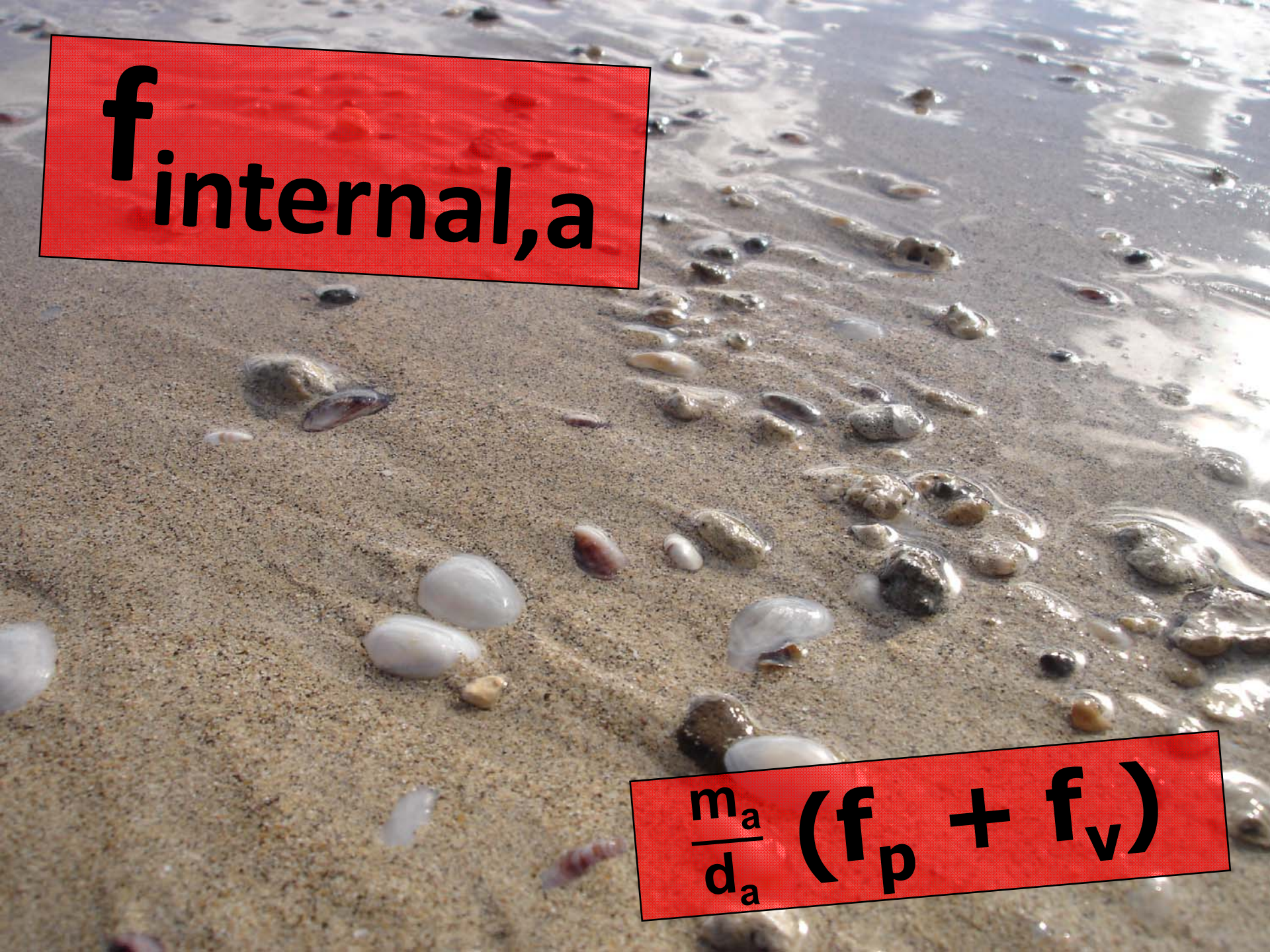
$$\mathbf{f}_{\mathbf{v}}$$

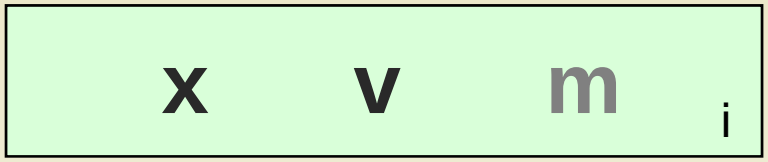
$$\sum_b \mathbf{V}_b \mathbf{v}_b \mathbf{W''}(r_b)$$

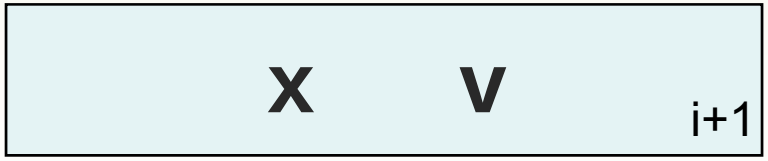
$$\mathbf{f}_{\mathbf{v}}$$

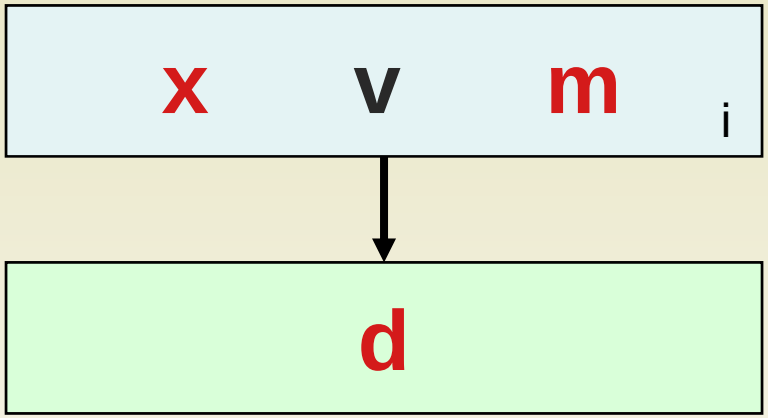
$$\sum_b \mathbf{V}_b \, (\mathbf{v}_b - \mathbf{v}_a) \, \mathbf{W''}(r_b)$$

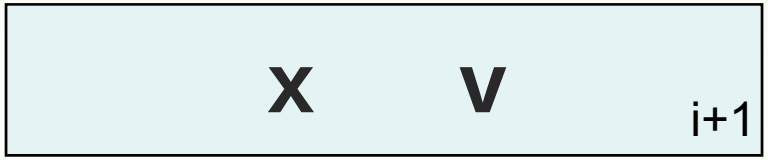

$f_{internal}$
$(f_p + f_v)$

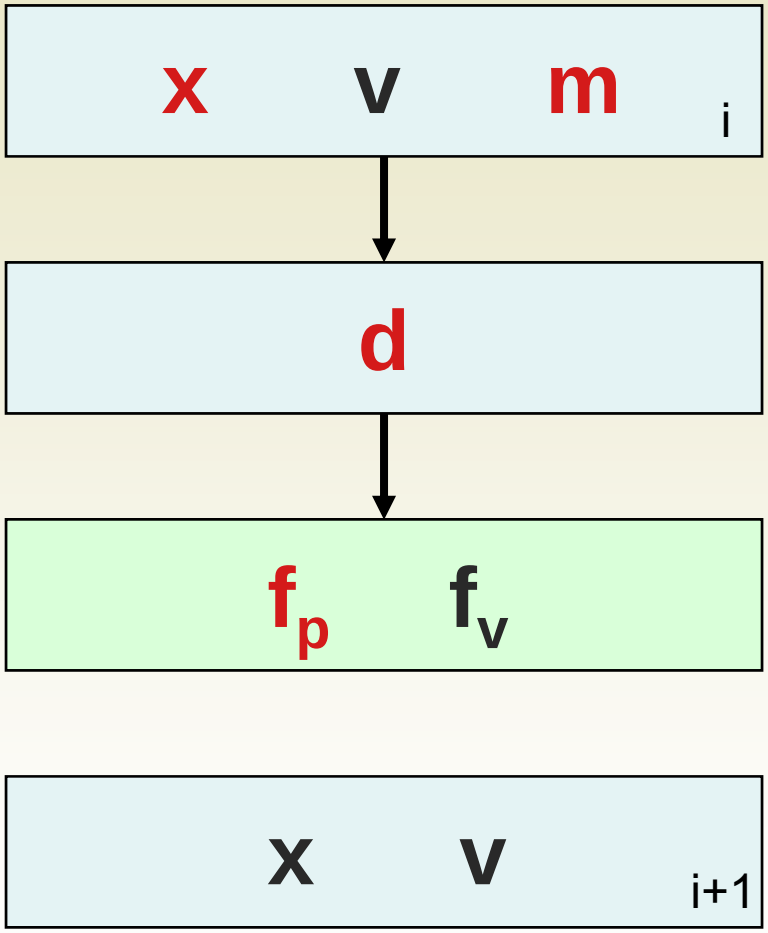

$f_{internal,a}$
$\frac{m_a}{d_a}(f_p + f_v)$

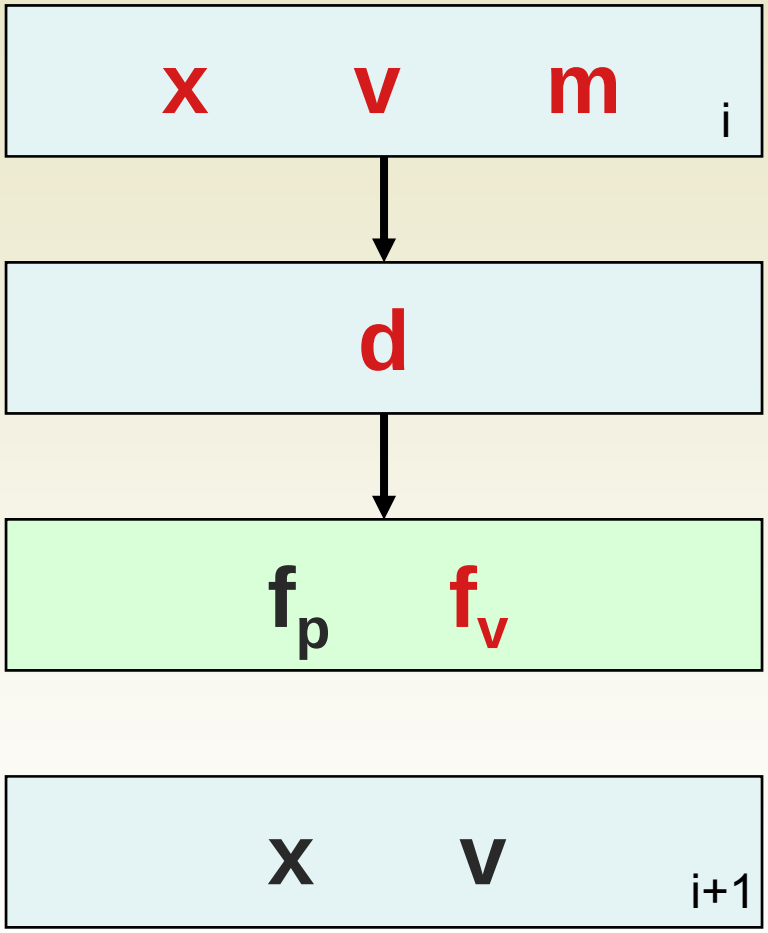

x
v
m
i

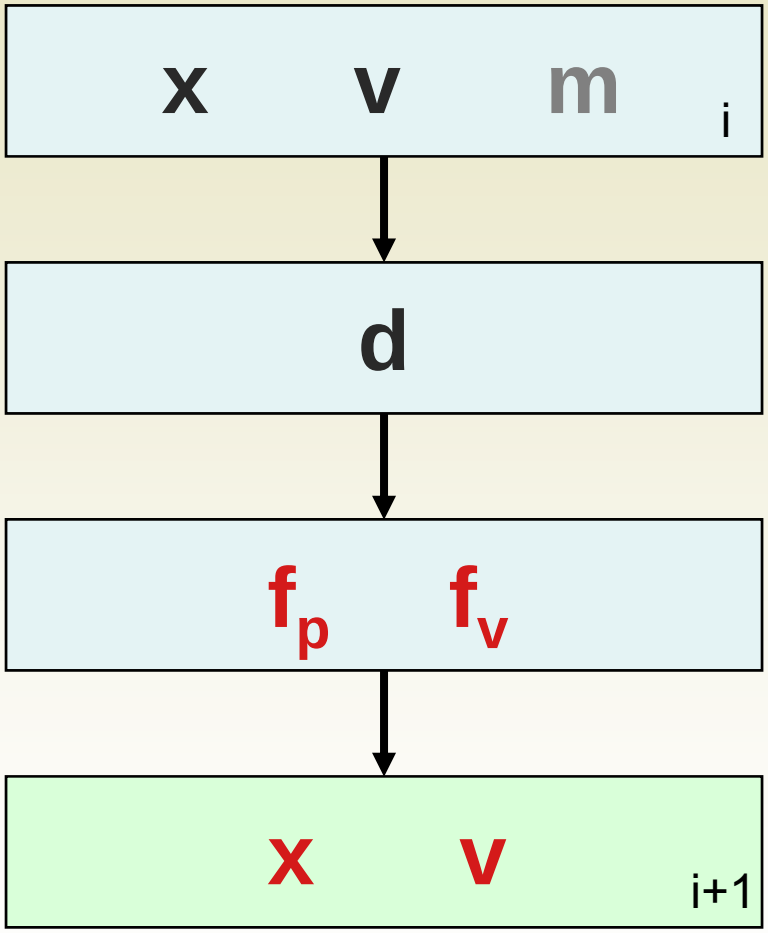

x
v
i+1


x
v
m
i
d


x
v
i+1


x v m
i
d
f_p f_v
x v
i+1

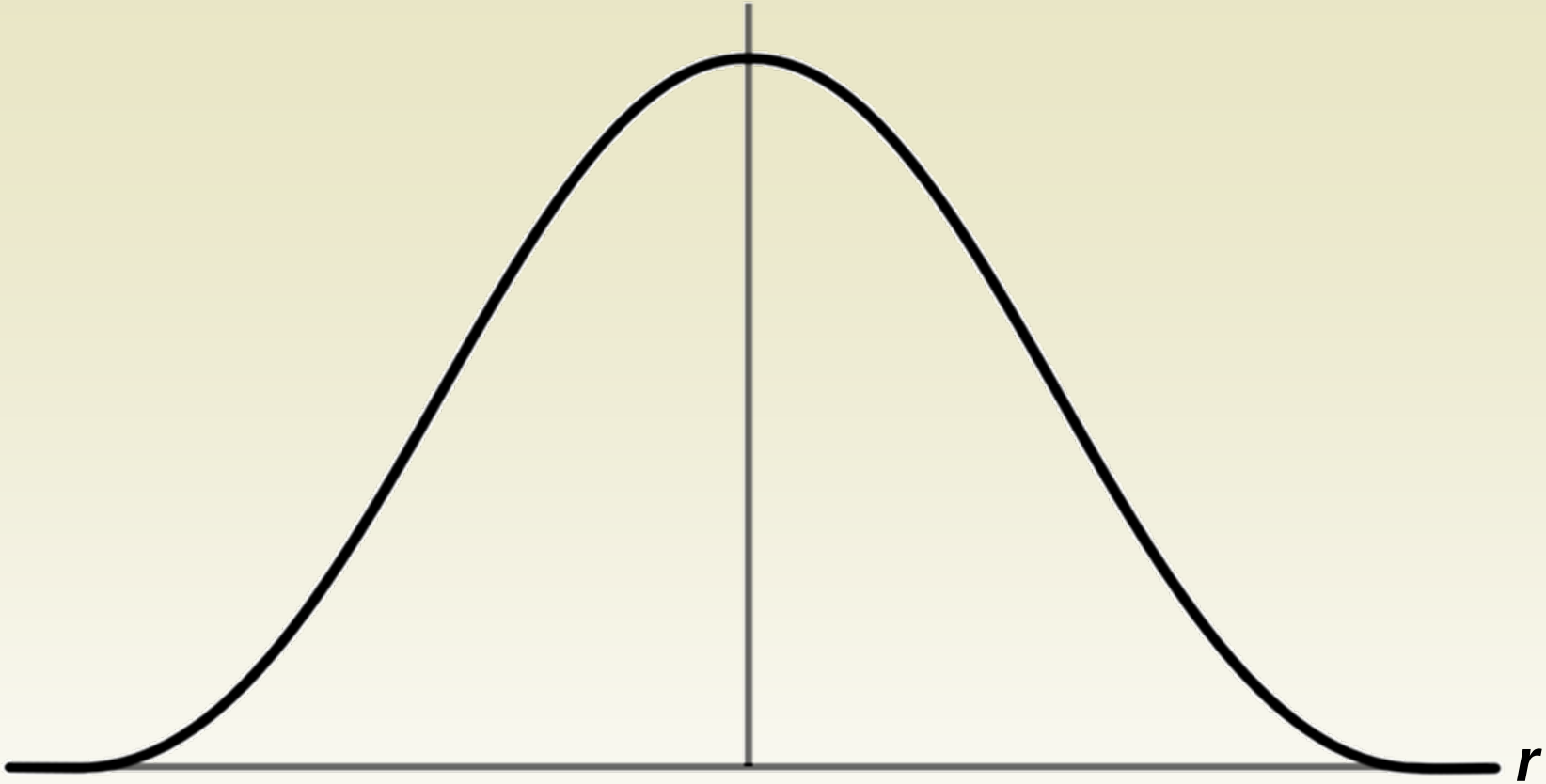

x v m
i
d
f_p f_v
x v
i+1


x v m
i
d
f_p f_v
x v
i+1

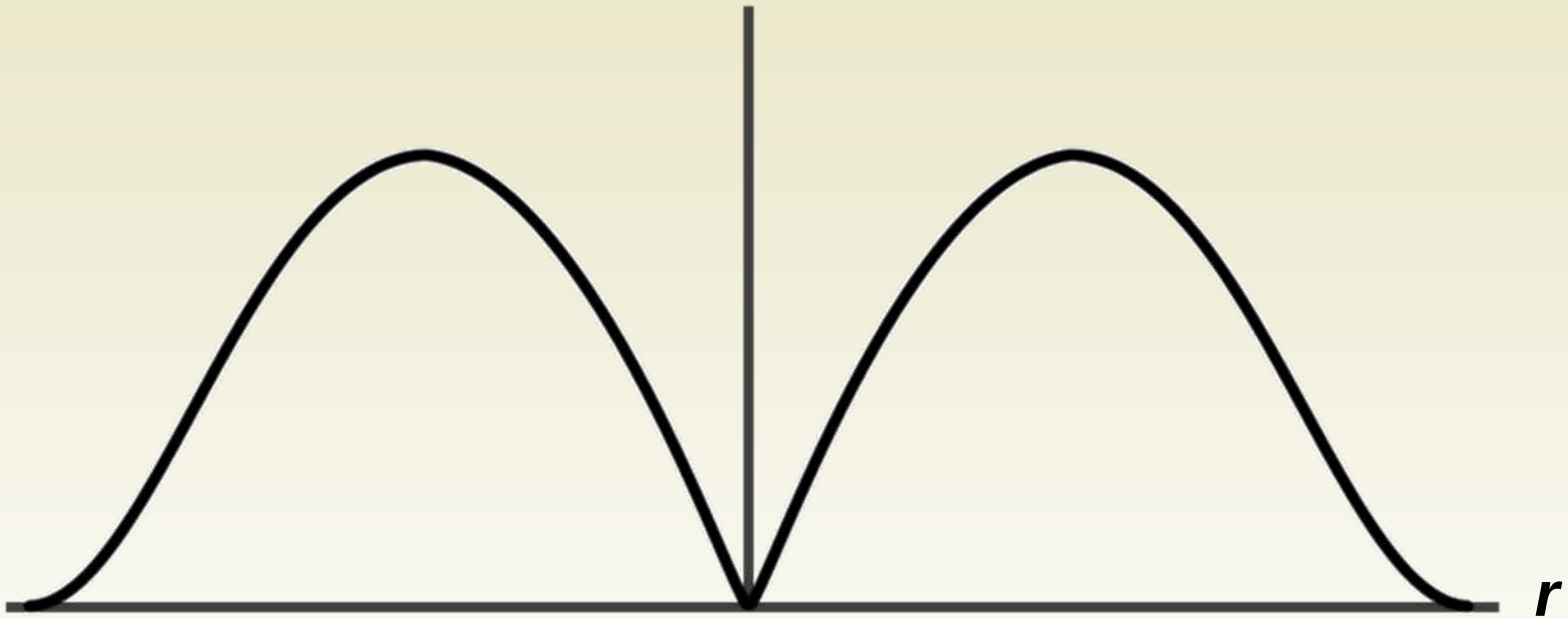
```
repeat
    forall particles a
      forall particles b
        accumulate d_a using b
```

```
repeat
    forall particles a
      forall particles b
        accumulate d_a using b

    forall particles a
      forall particles b
        accumulate f_internal,a using b
```

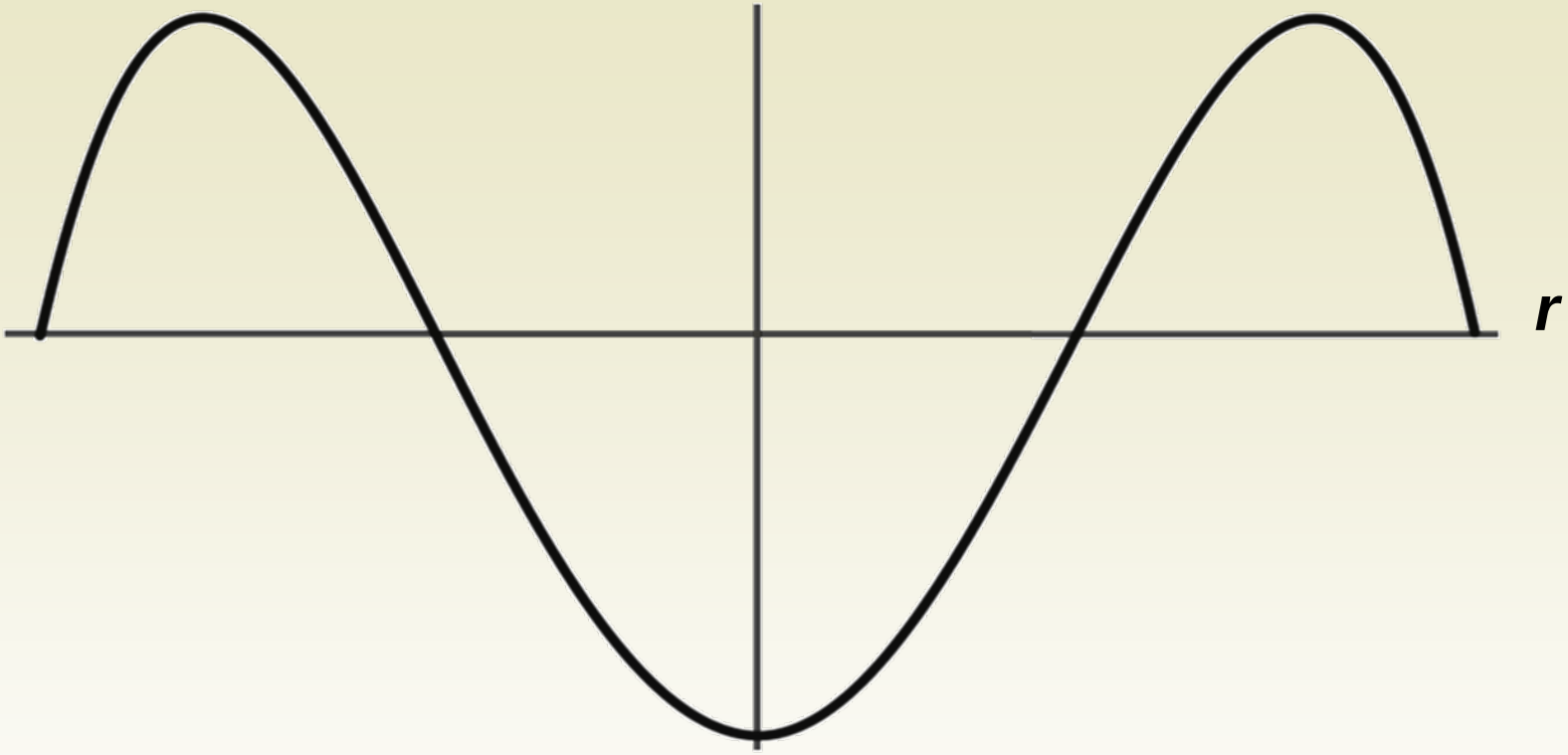
```
repeat
    forall particles a
      forall particles b
        accumulate d_a using b

    forall particles a
      forall particles b
        accumulate f_internal,a using b

    forall particles a
      integrate x_a and v_a
```



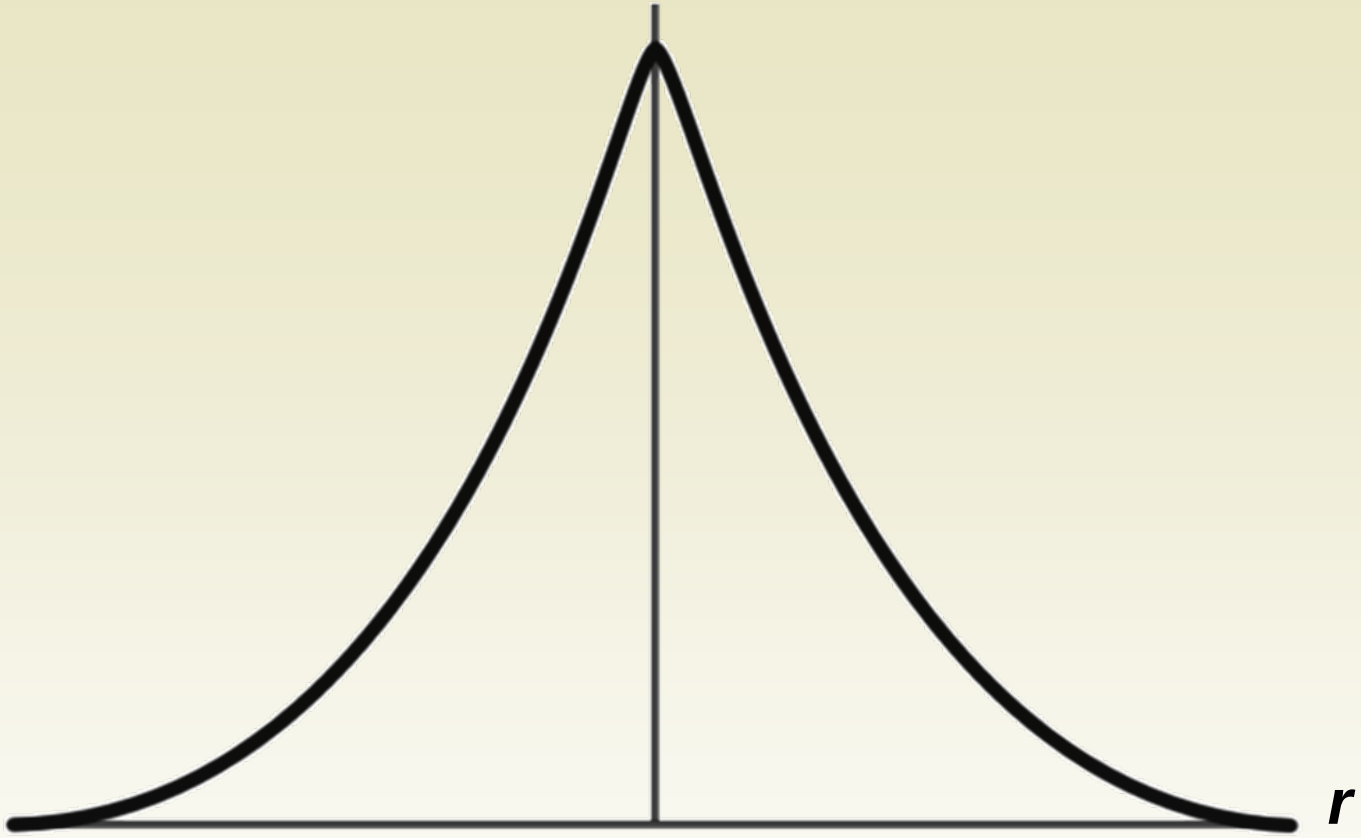
$$s\,(\,h^2 - r^2\,)^3$$



W’(r,h)



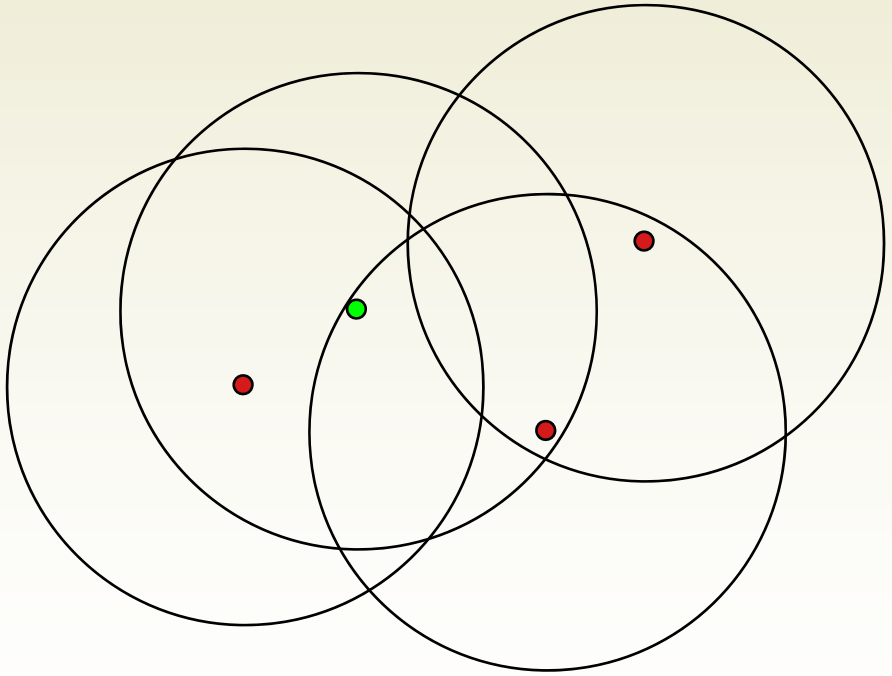
W”(r,h)



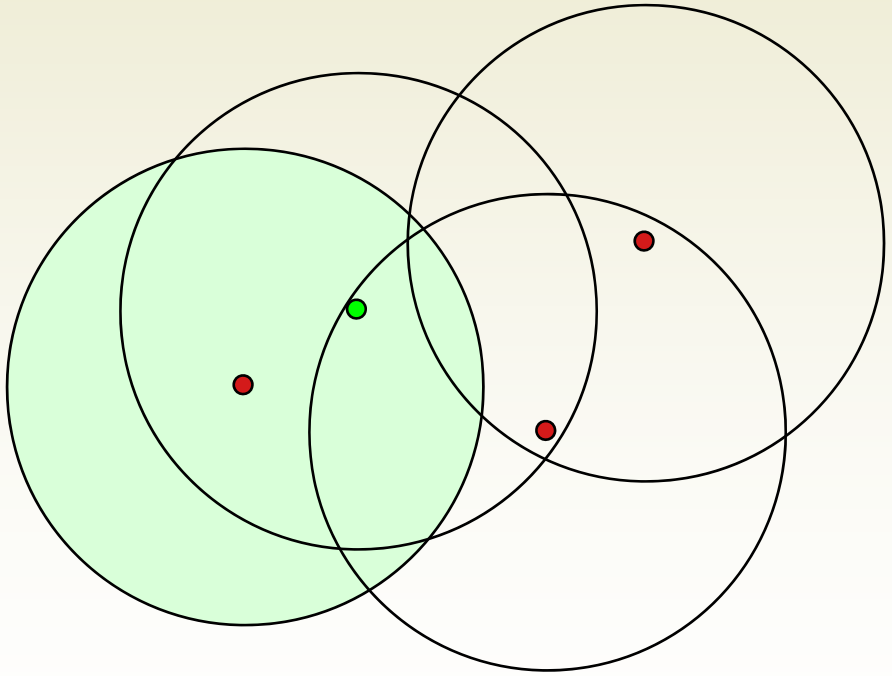
$$s\,(h - r)^3$$


Make it fly

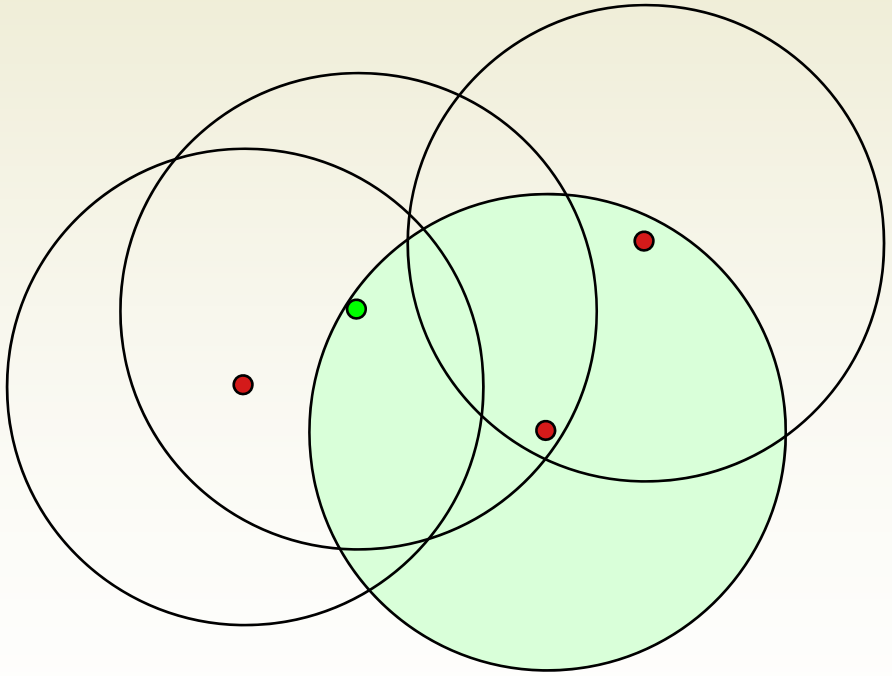
Broad phase



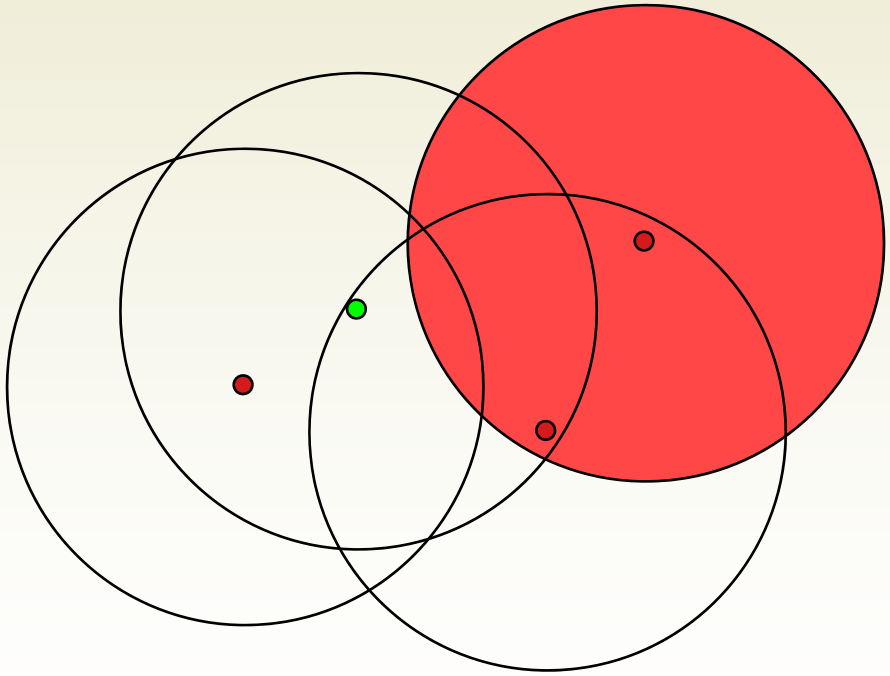
Broad phase



Broad phase



Broad phase



Collapse

```
repeat
    forall particles a
      forall particles b
        accumulate d_a using b

    forall particles a
      forall particles b
        accumulate f_internal,a using b

    forall particles a
      integrate x_a and v_a
```

```
repeat
    forall particles a
      forall particles b
        accumulate d_a
        accumulate f_internal,a
        integrate x_a and v_a
```

```
repeat
    forall particles a
      forall particles b
        accumulate d_a
        accumulate f_internal,a
        integrate x_a and v_a
```

*Neighbors are **mix** of particles at **time i and i+1***

Thanks!

kees.vankooten@gmail.com

Demos

NVIDIA fluid particle PhysX demo

www.nvidia.com/cuda

Real-Time Particle-Based Fluid Simulation

http://www.ss.iij4u.or.jp/~amada/fluid/

GPU-based Fast Ray Casting for a Large Number of Metaballs

http://nis-lab.is.s.u-tokyo.ac.jp/~pierrot/projects/metaball/

Point-Based Visualization of Metaballs on a GPU

http://developer.nvidia.com/object/gpu-gems-3.html - or mail me

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