



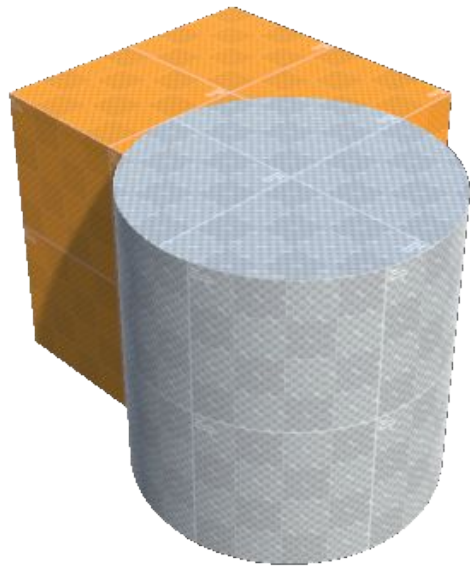
# Geometry in milliseconds: Real-time Constructive Solid Geometry

Sander van Rossen  
Unity Technologies

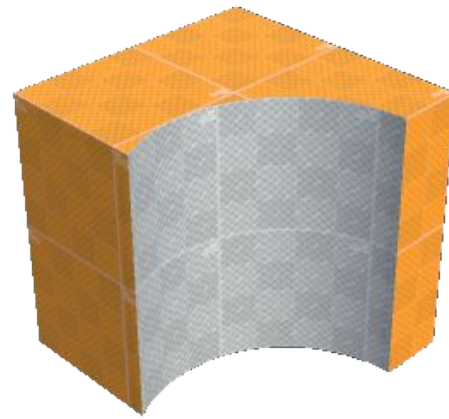
**GAME DEVELOPERS CONFERENCE**  
MARCH 16–20, 2020 | #GDC20

# So what *is* Constructive Solid Geometry?

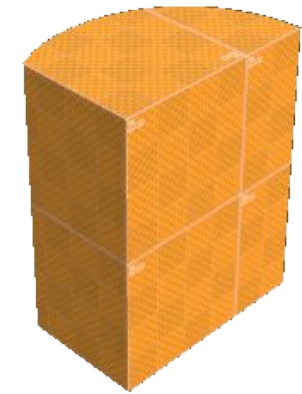
# Boolean Operations



Additive

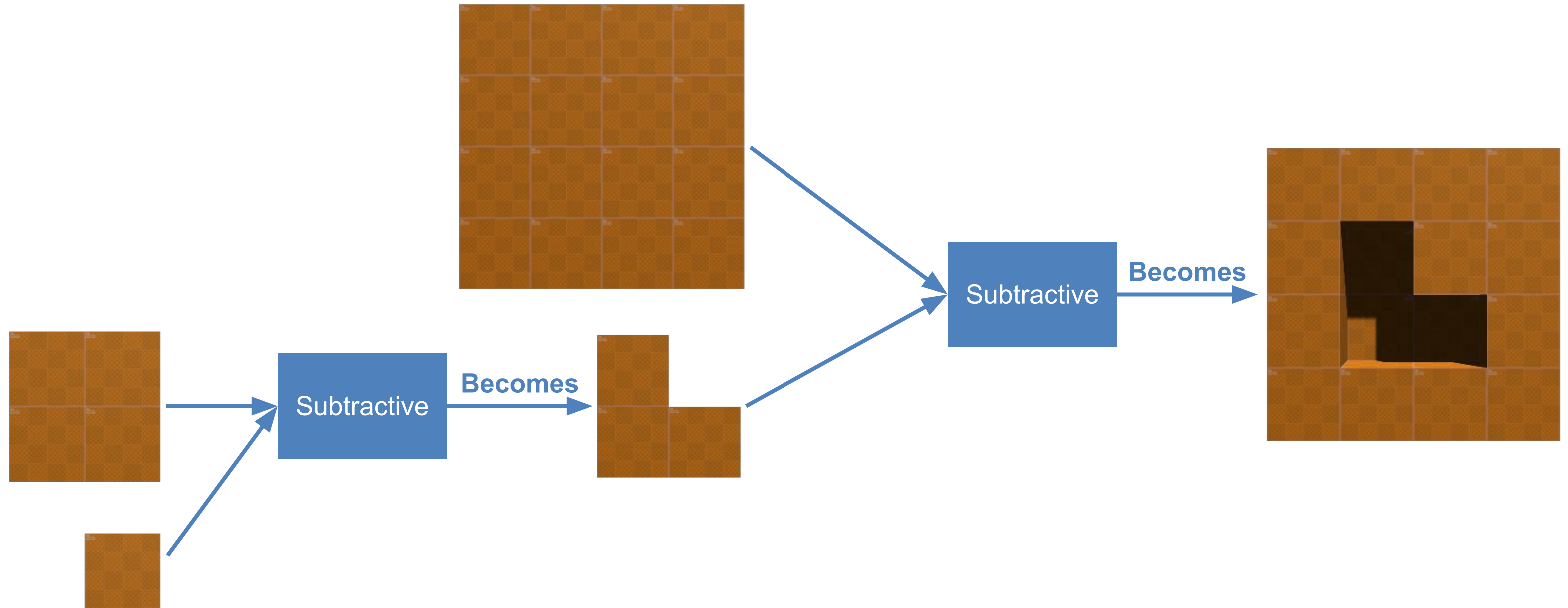


Subtractive

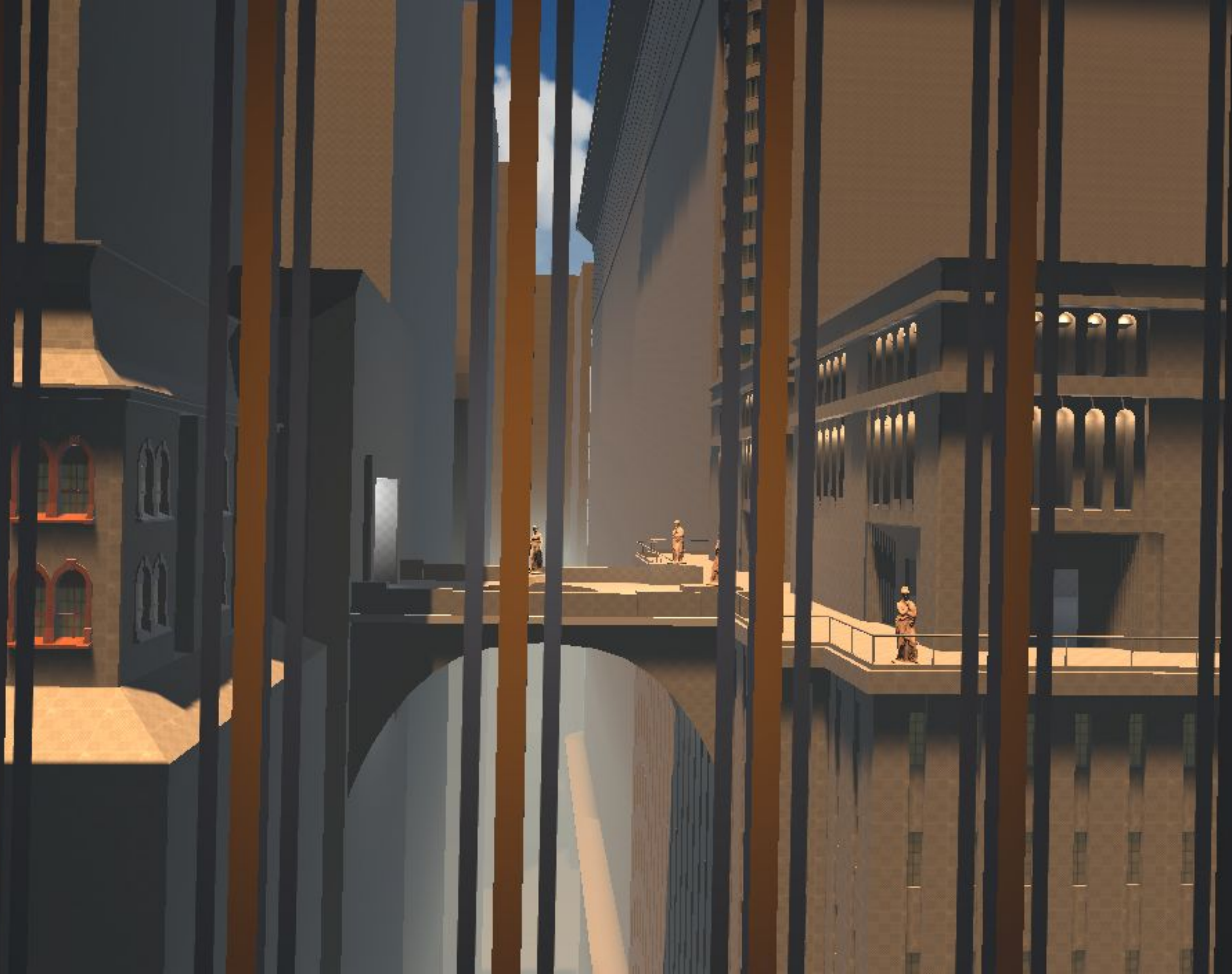


Intersection

# CSG Hierarchy







Thank you!

Well,  
there's a little more to it...

# Overview

## 1. History of CSG

### The algorithm

## 2. Iterative updates

## 3. Intersections

## 4. Mesh Generation

## 5. Polygon categories, Routing & Operation tables

## 6. Putting it all together



# First some history

# History

- Originated outside of the game industry
  - Used in the CAD industry
- Long history in the game industry
  - Quake/iD tech engines / iD Software
    - Many build on top are still in use today (mostly completely rewritten)
      - Source engine 1-2 / Valve
  - Unreal engine 1-4 / Epic Games
  - Torque, Roblox, and many more

# History

- Games with CSG level editors often spawned mod communities
- Some mods turned into full games
  - Counter strike
  - Team Fortress
  - Portal
  - Black Mesa
  - The Stanley Parable
- Many professional level designers started out as modders



# History





## History

- Early implementations used Binary Space Partitions (BSP)
  - Scales poorly with number of polygons
    - Unusable beyond a relatively small number
- The tooling build around CSG hasn't evolved much
  - Unreal, for example, still uses the BSP code Tim Sweeney wrote decades ago

But... why?



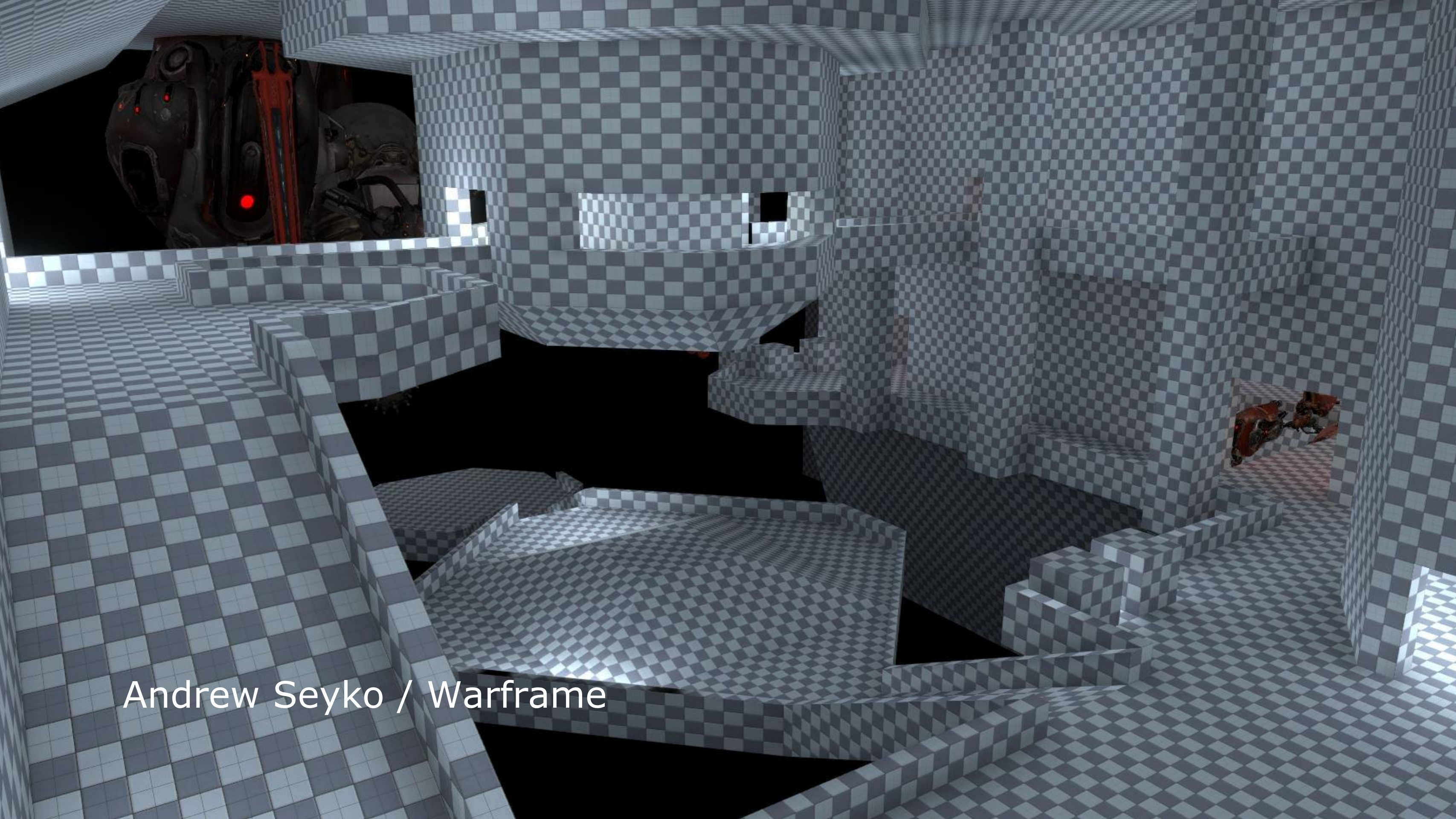
## Workflow

- Fast & non-destructive iteration
  - Brushes can easily be moved around, replaced, hidden/shown
    - Your level geometry will automatically get adjusted
  - Fast to quickly mock/block out levels, test gameplay
  - Easy to try out different game layouts quickly
- Easy to learn / very intuitive / Allows for playful exploration
- Mostly used to design larger outline and flow of levels
  - Complemented with modeled props
  - Sections replaced with pieces of modeled geometry

# #blocktober

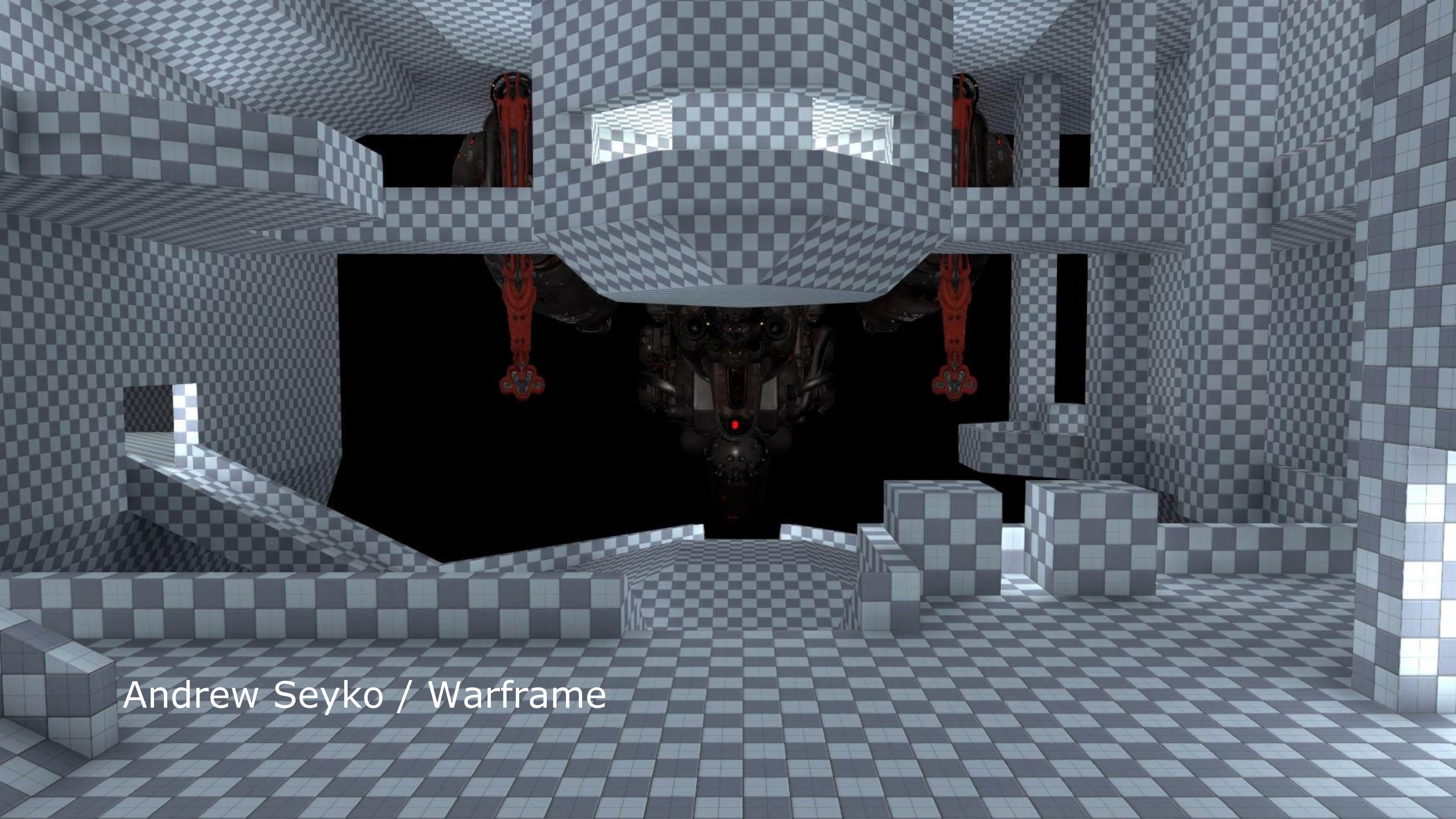






Andrew Seyko / Warframe





Andrew Seyko / Warframe





Andrew Seyko / Warframe



The image shows a high-angle view of a game map from Apex Legends, specifically the area known as 'The Gorge'. The map is a large, open arena with a floor and walls made of grey, square tiles. A large, curved structure on the right side of the map is illuminated with bright blue light. In the center of the map, there are several small, colorful objects (green, yellow, and orange) that appear to be loot or weapons. A large, dark, rectangular object lies on the floor near the center. In the background, there are some small, white, box-like structures. The overall lighting is dim, with the blue light from the curved structure providing a strong focal point. The text 'Alex Graner / Apex Legends' is overlaid in the bottom left corner.

Alex Graner / Apex Legends



name: unnamed  
pos: 27705.53 8130.62 3204.76  
ang: 29.29 -137.45 -0.04  
vel: 0.00

Alex Graner / Apex Legends





Alex Graner / Apex Legends





Alex Graner / Apex Legends



Alex Graner / Apex Legends



## Workflow

- CSG forces a focus on the large first, details later
  - You can fine tune your game with simpler geometry
    - Before you spend resources on making it pretty
- CSG creates *solid* geometry without gaps, ideal for physics
  - Easy to make invisible infinitely thin gaps in a 3D modeling tool
    - Unlikely for this to happen with CSG
  - Not something you want to worry about during design

## Workflow

- CSG is well suited for procedurally generated geometry
  - All geometry created by CSG is physically plausible
  - Can very easily layer geometry by addition & subtraction
    - Allows the user to mix procedural geometry with hand created geometry seamlessly



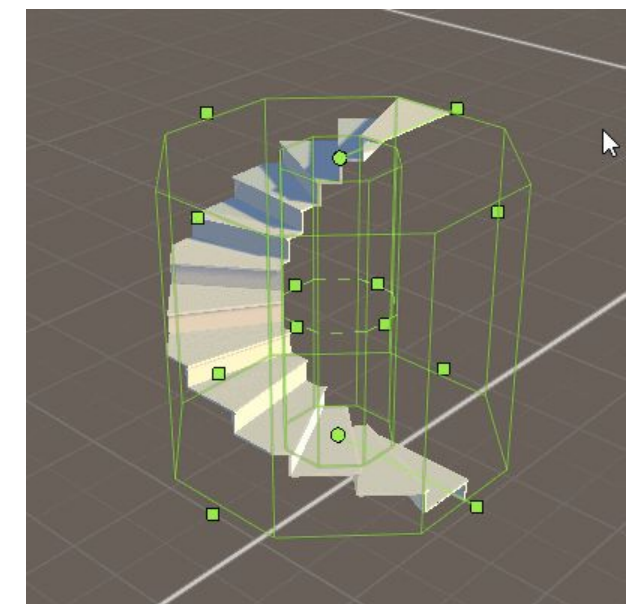
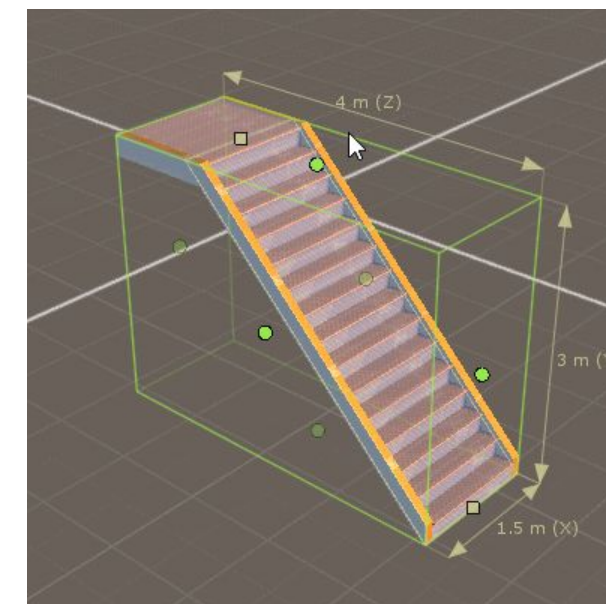
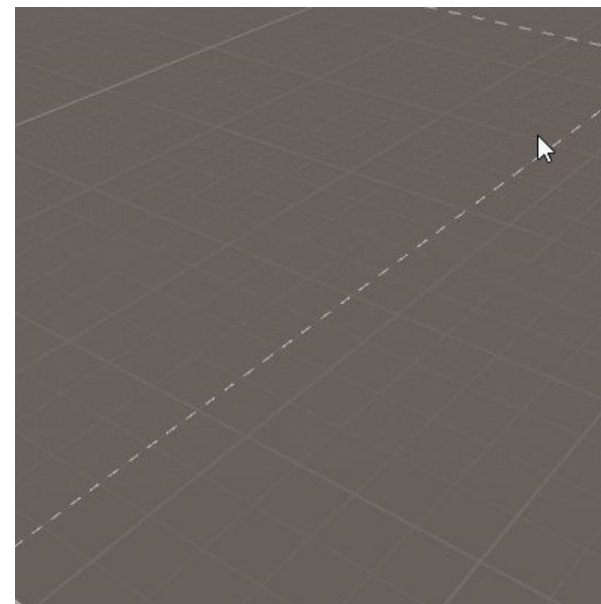
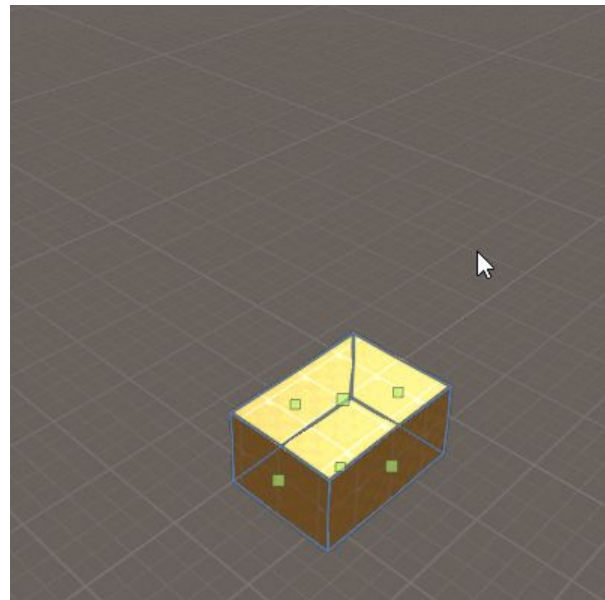
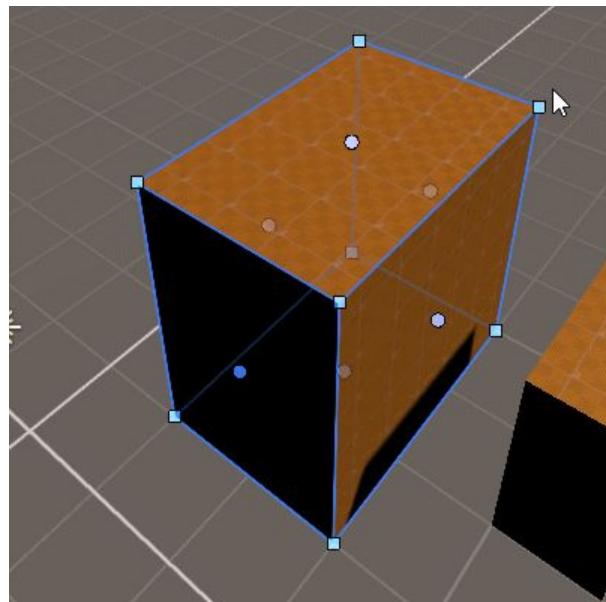
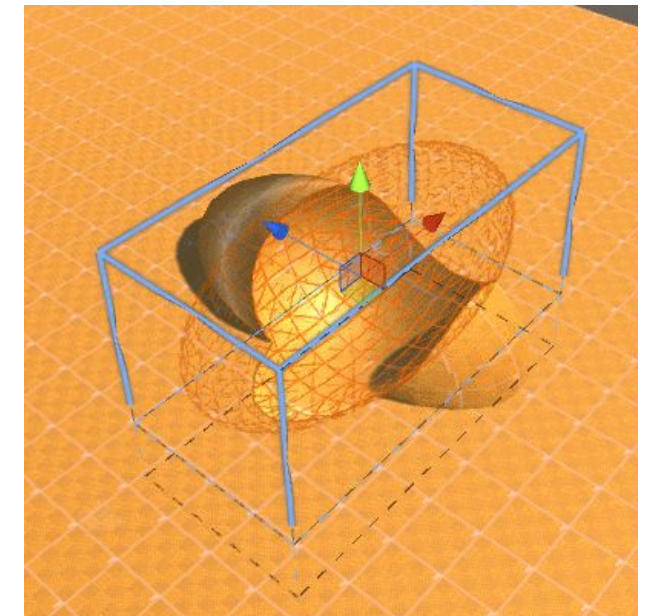
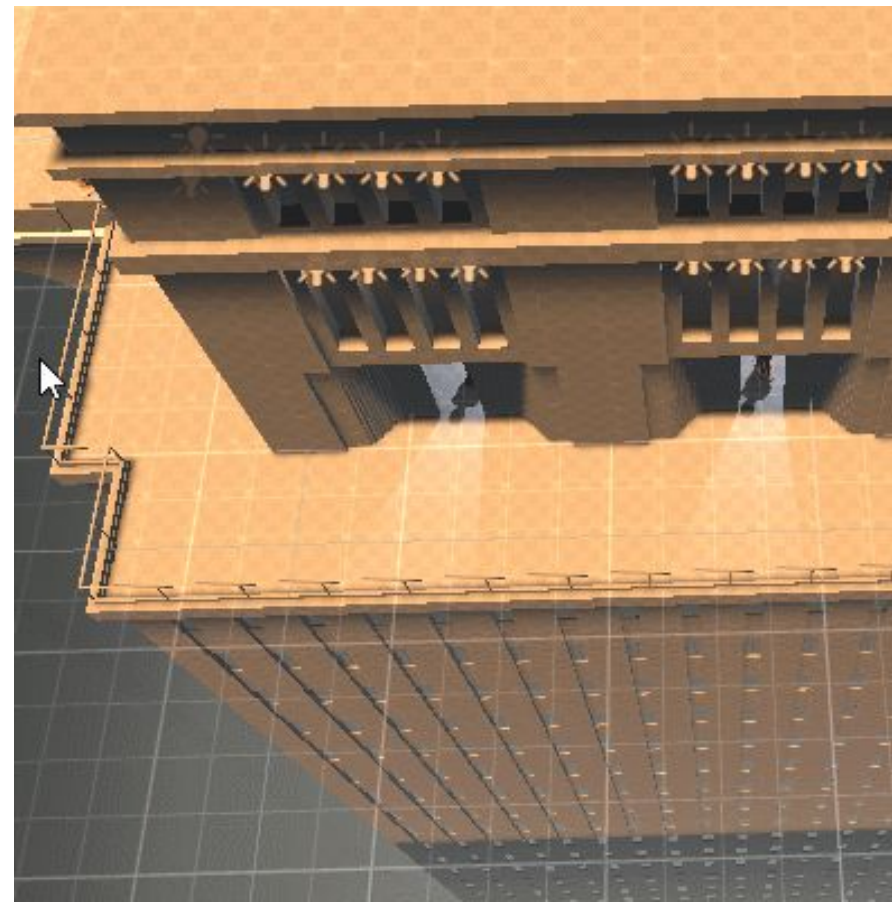
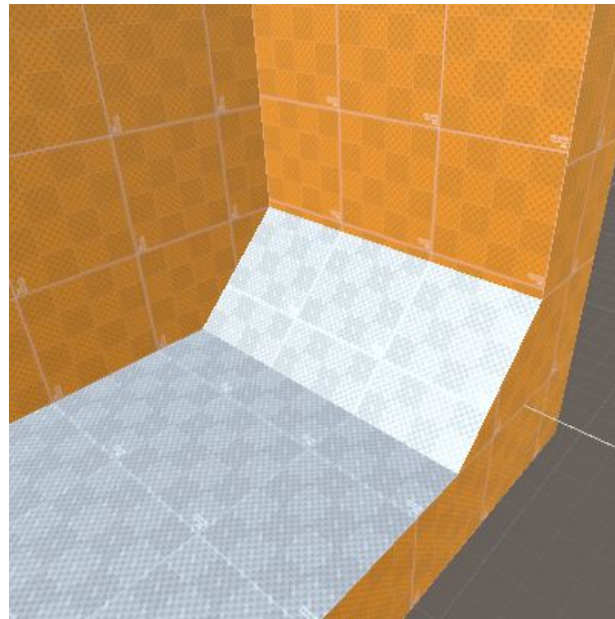
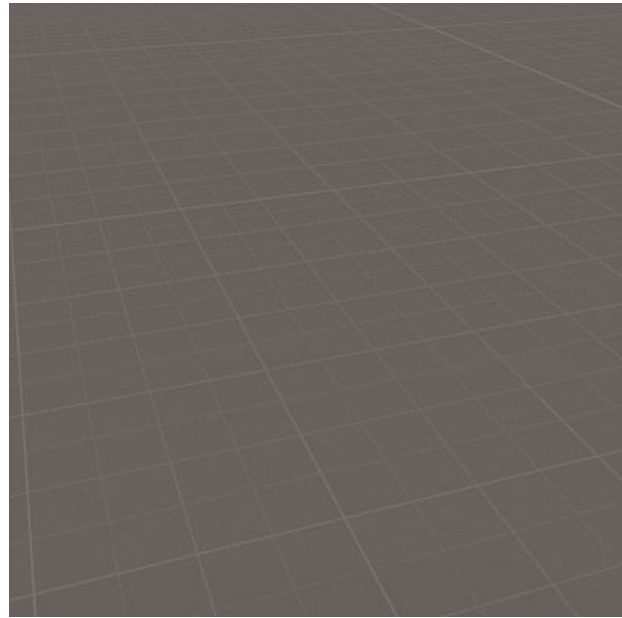
## Workflow

- Level design is not 3D modeling
  - Level designers and 3d artists are two different competencies
  - Level design is not just about what the geometry looks like
  - You always need the best tool for the job
    - You *can* mow your lawn with a scissor, *but why would you?*

## Perception

- Sadly, artists often equate CSG with BSP and old tools
  - Most common given reason not to use CSG is “it’s slow and blocky”
- Yet, it doesn’t have to be this way

## Modern CSG tools







Do I have your attention?

# How?

# Overview

## 1. History of CSG

The algorithm

## **2. Iterative updates**

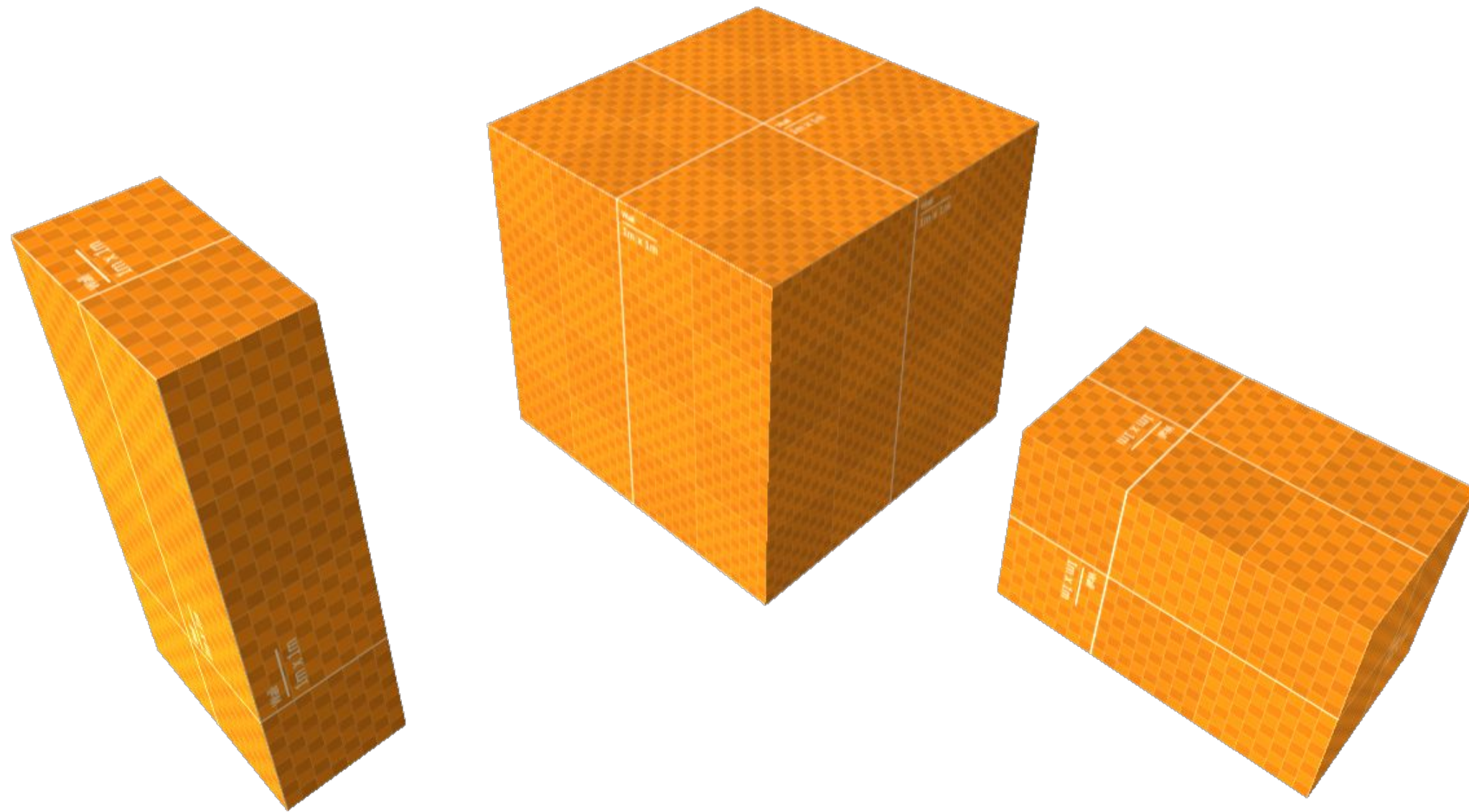
## 3. Intersections

## 4. Mesh Generation

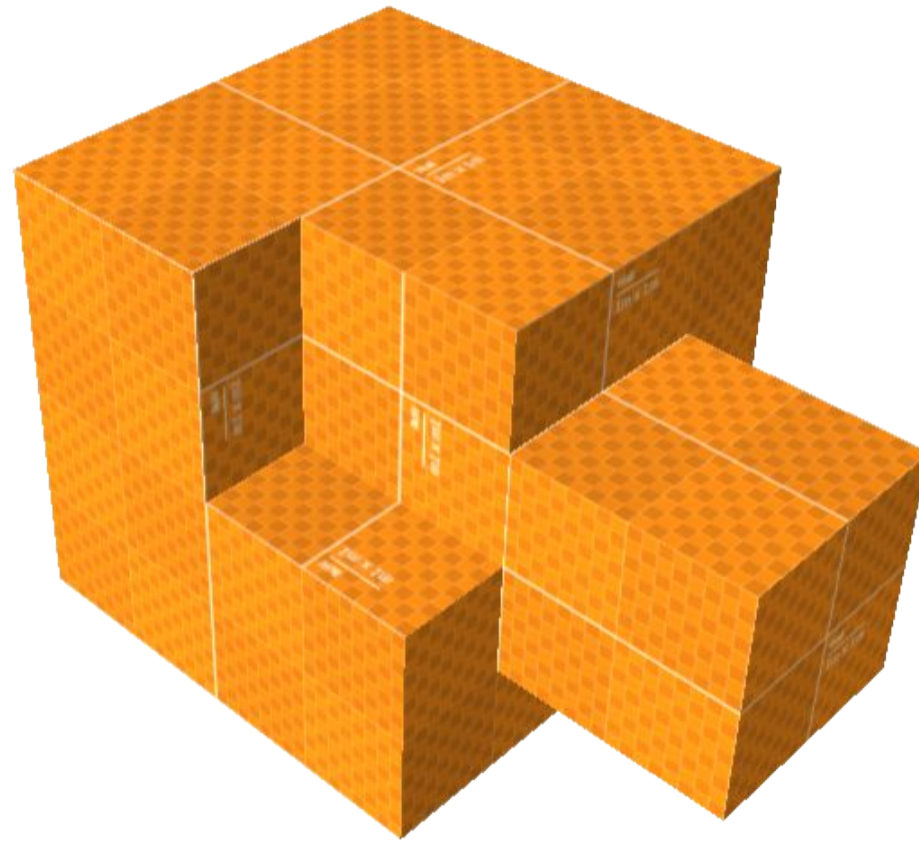
## 5. Polygon categories, Routing & Operation tables

## 6. Putting it all together

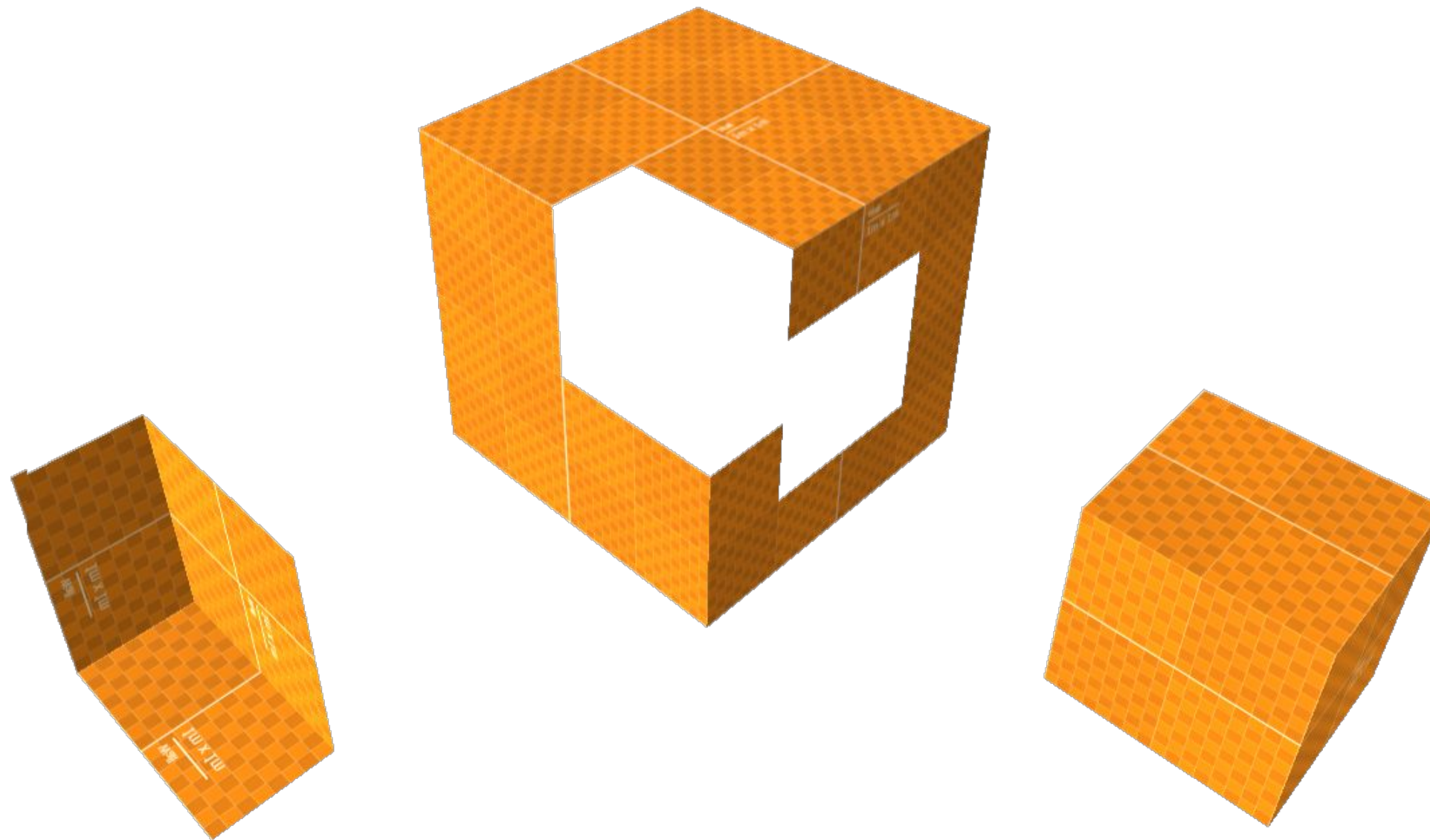




Suppose we perform CSG on some brushes

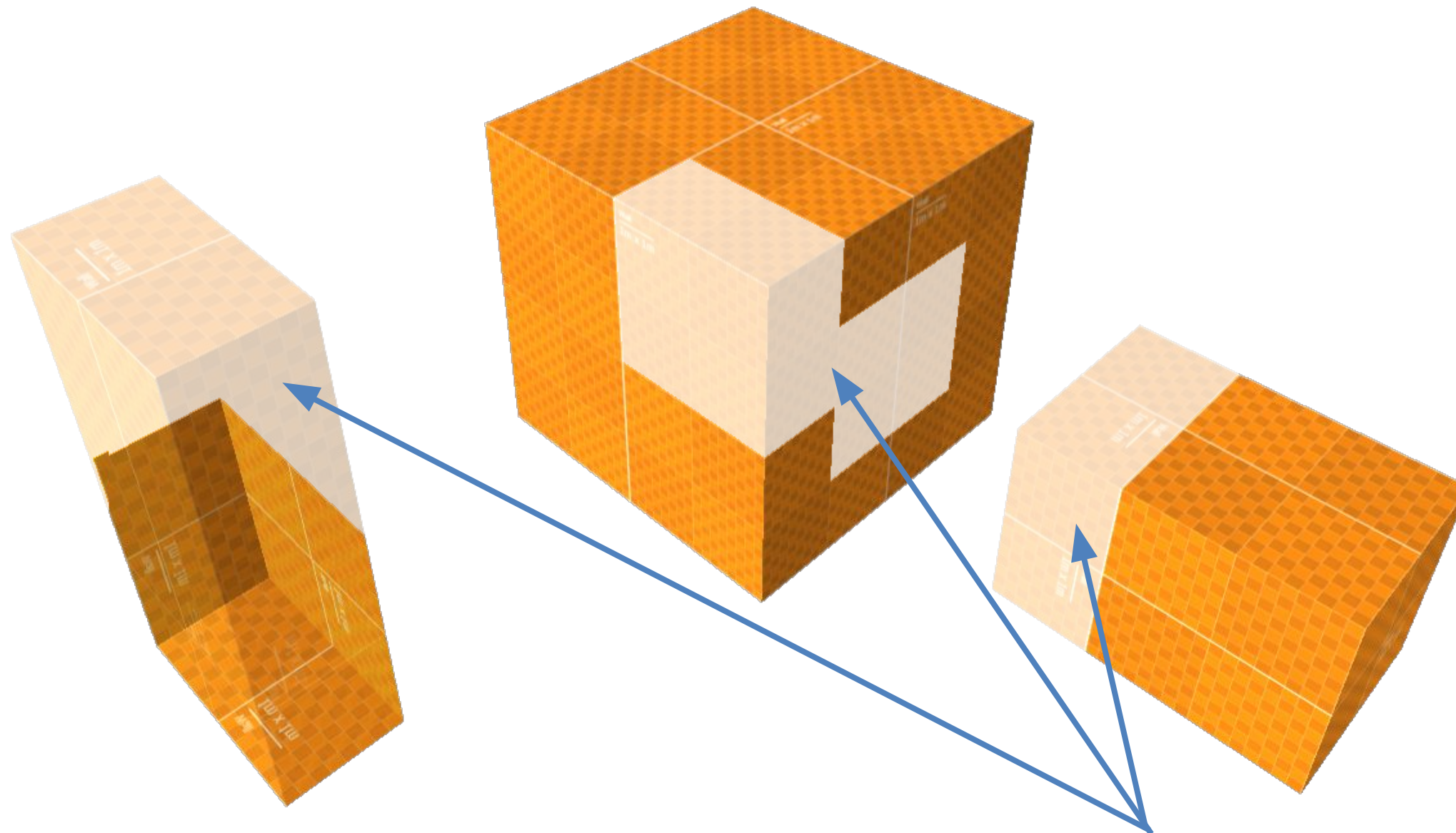


And we create a shape with those brushes,  
using a subtractive and an additive operation



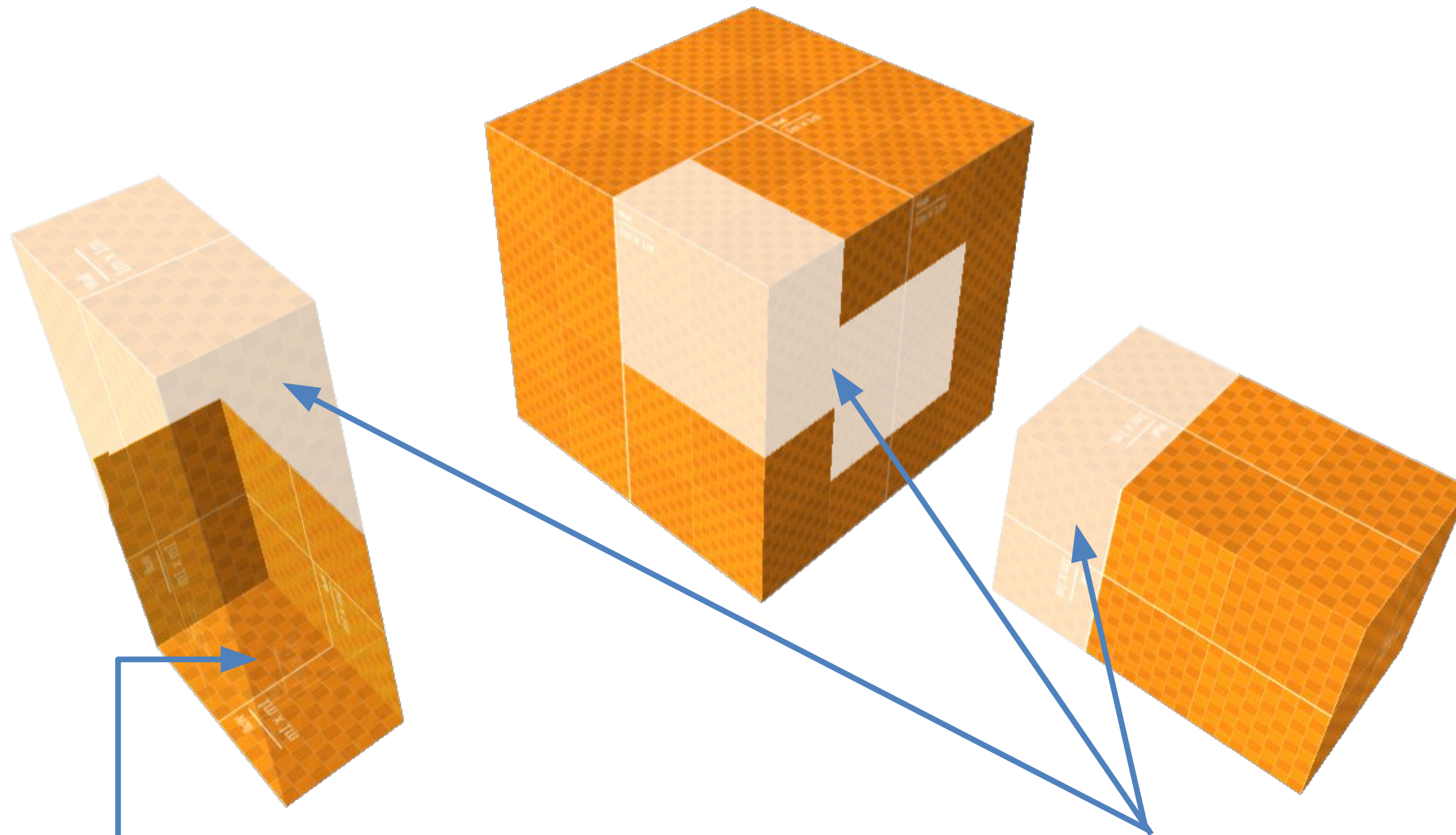
If we look at the contributions from  
each individual brush on the final shape



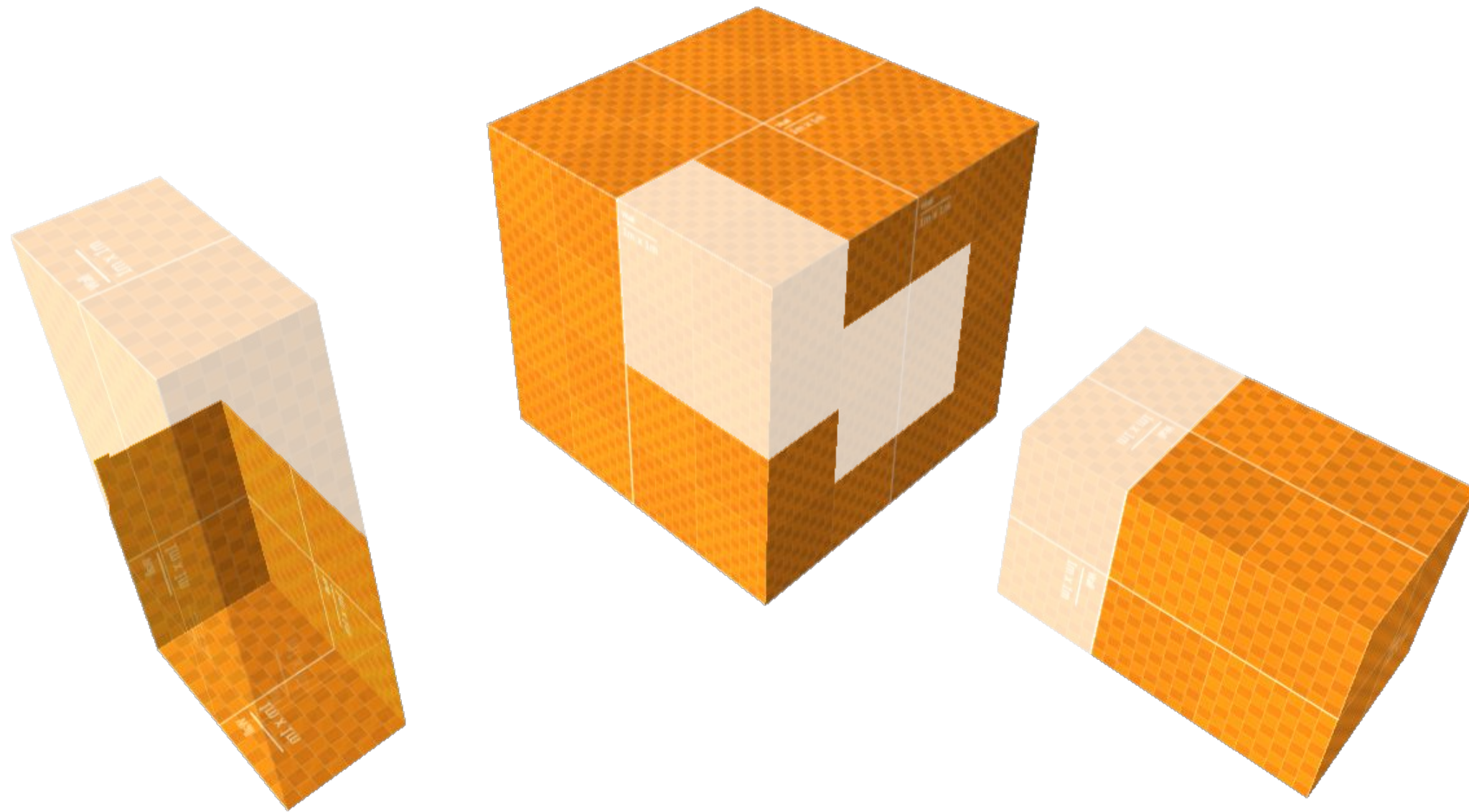


We can see that we only need to remove or **flip the orientation** of polygon pieces ..





We can see that we only need to remove or flip the orientation of polygon pieces ..

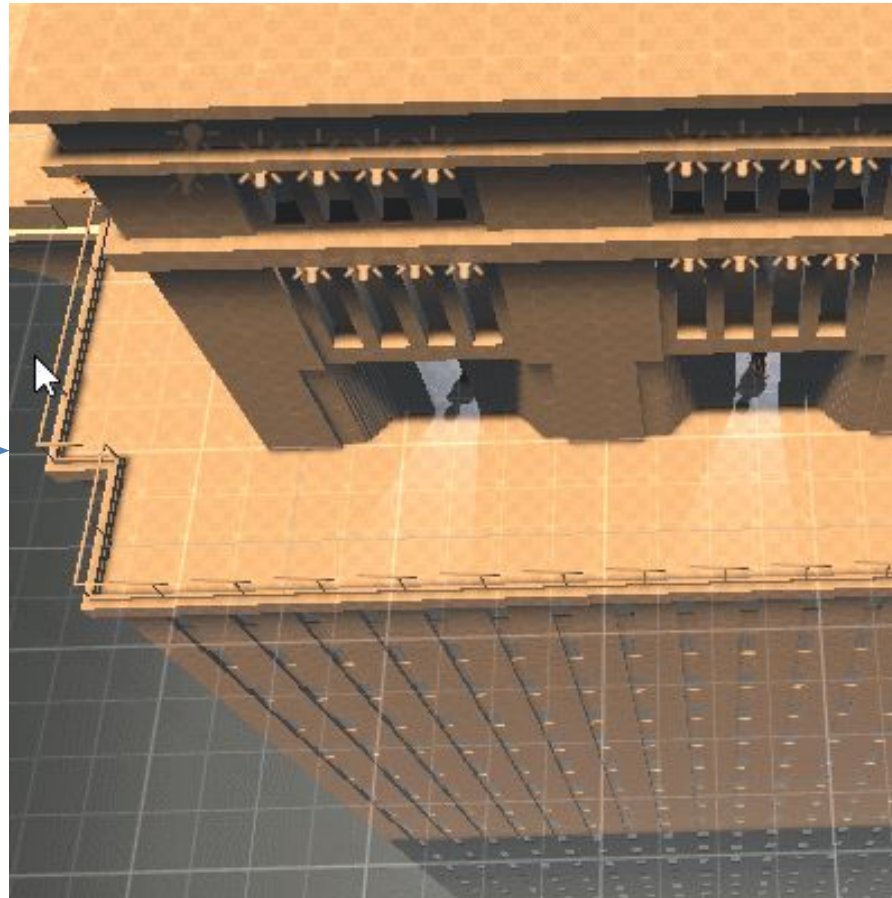


We can see that we only need to **remove**  
or **flip the orientation** of polygon pieces ..

**By finding those pieces, we can perform CSG per brush**

## Performing CSG per brush

- Allows for **iterative** updates
  - **Makes this possible**

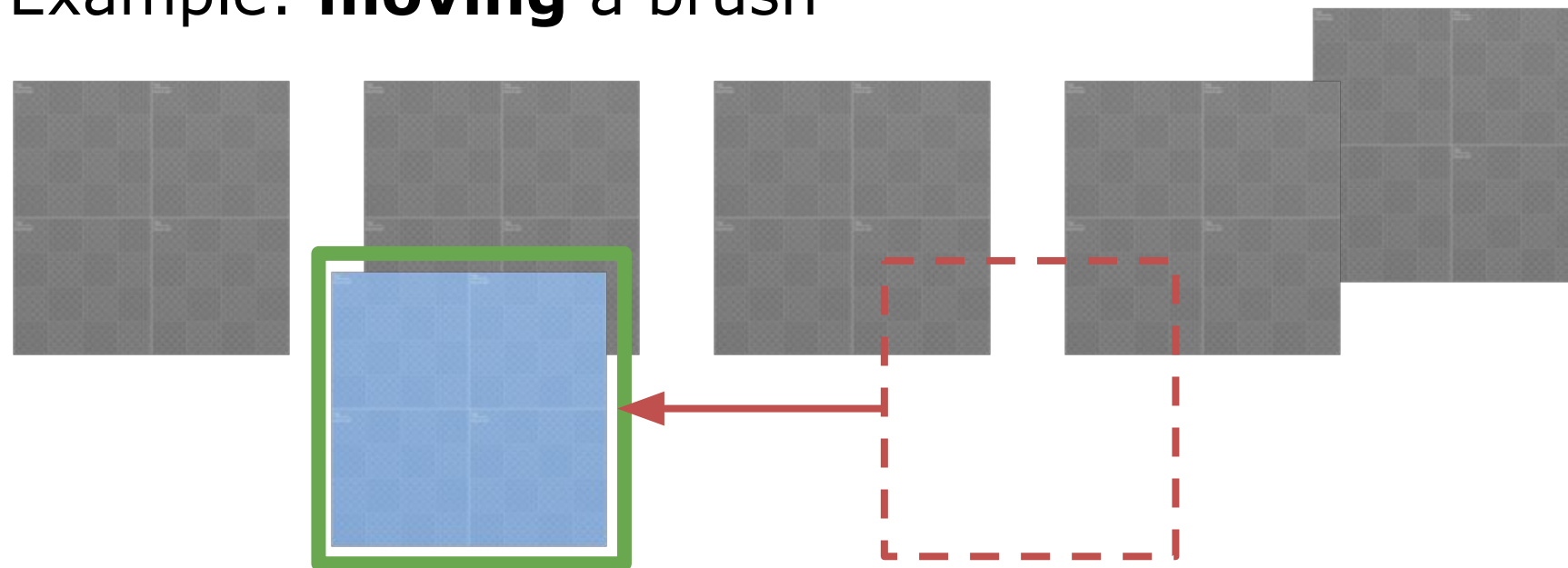




## Performing CSG per brush

- Allows for **iterative** updates
  - Only need to **update a brush when its modified**
    - And all brushes that touched/touch it

Example: **moving** a brush

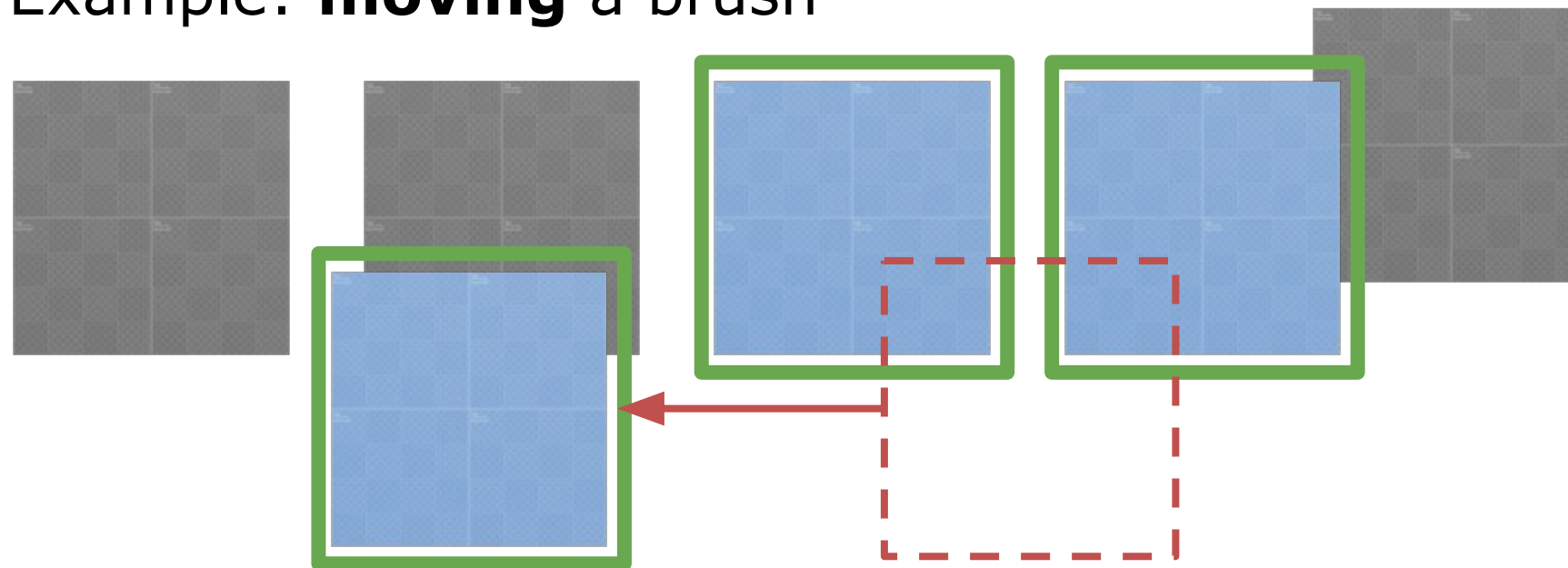


The brush itself is marked dirty

## Performing CSG per brush

- Allows for **iterative** updates
  - Only need to **update a brush when its modified**
    - And all brushes that **touched**/touch it

Example: **moving** a brush

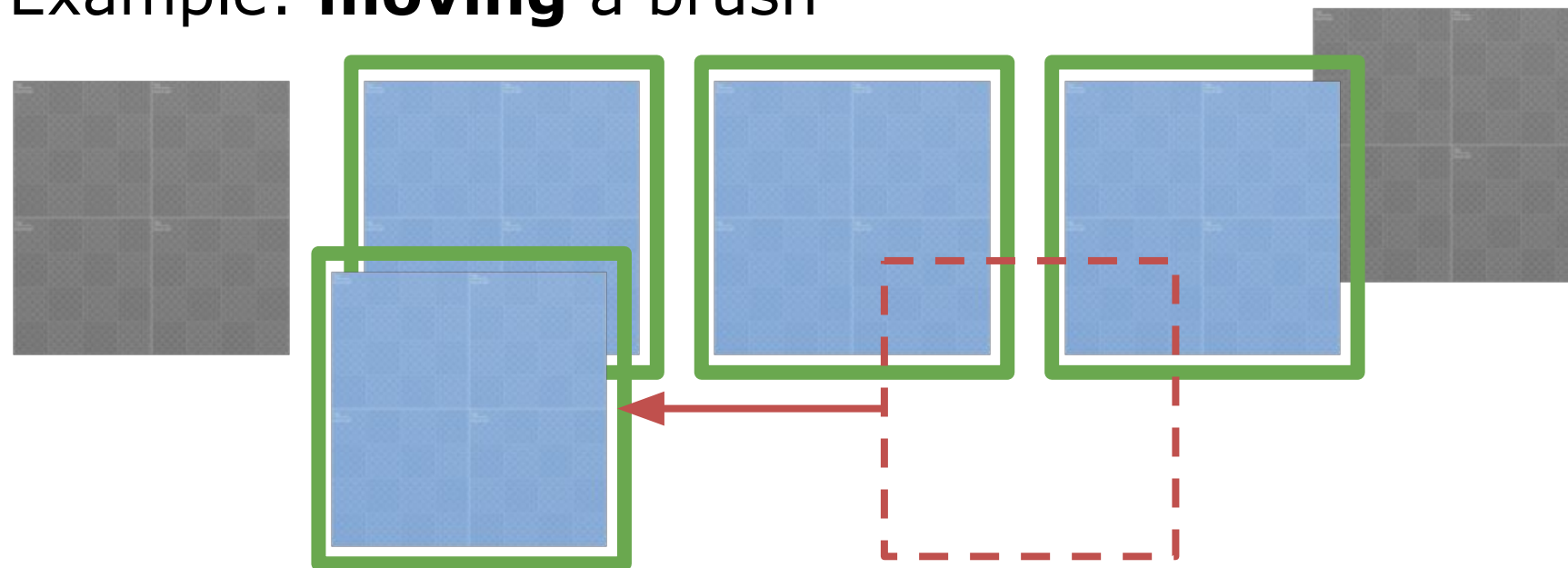


So are those that it touched before the move

## Performing CSG per brush

- Allows for **iterative** updates
  - Only need to **update a brush when its modified**
    - And all brushes that **touched/touch** it

Example: **moving** a brush



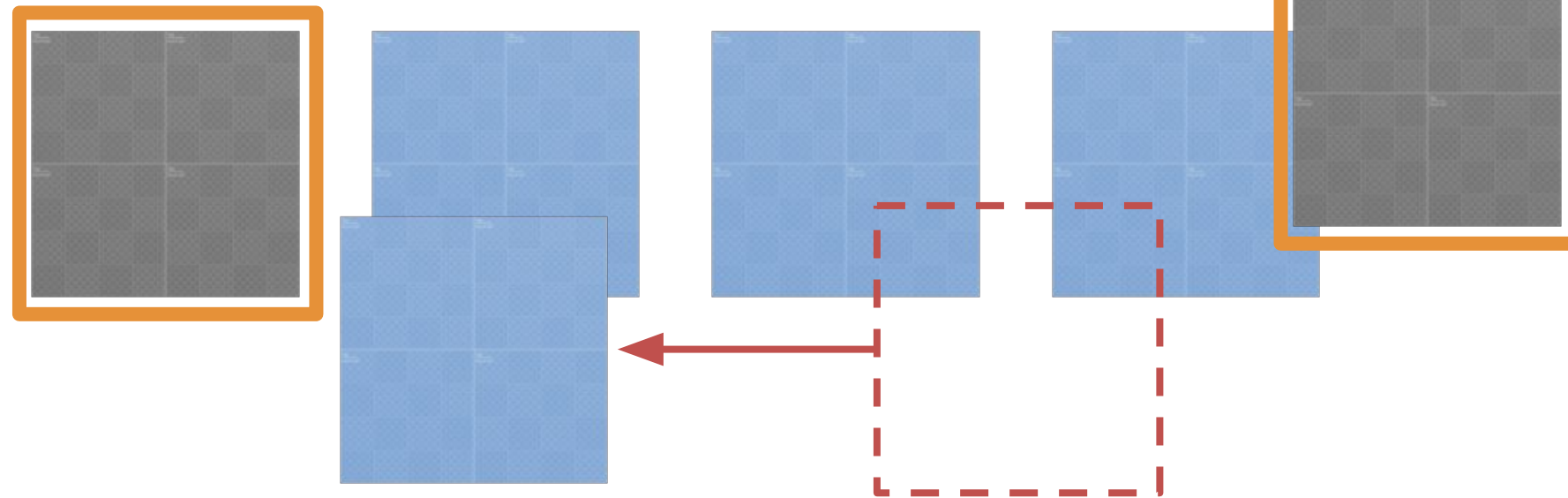
And those that it touches after the move



## Performing CSG per brush

- Allows for **iterative** updates
  - Only need to **update a brush when its modified**
    - And all brushes that **touched/touch** it
    - But not those that it didn't touch (**can be cached**)

Example: **moving** a brush

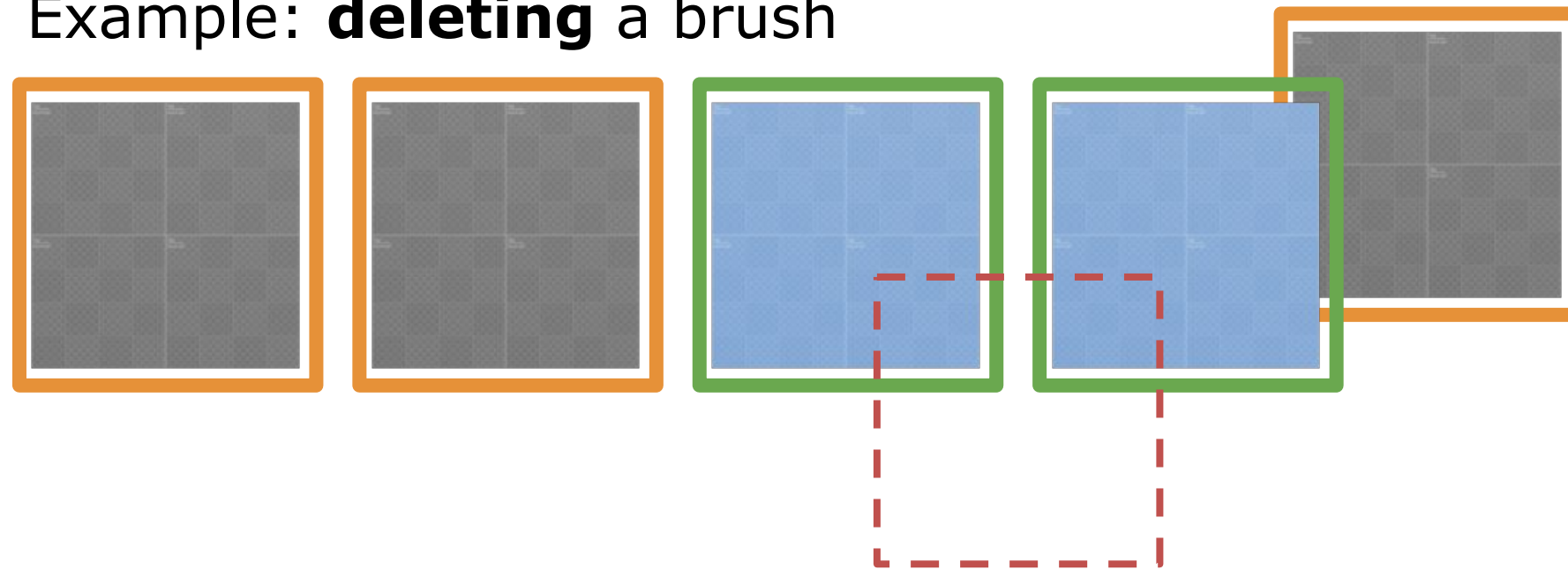


All the other brushes are left unmodified

## Performing CSG per brush

- Allows for **iterative** updates
  - Only need to **update a brush when its modified**
    - And all brushes that **touched/touch** it
    - But not those that it didn't touch (**can be cached**)

Example: **deleting** a brush



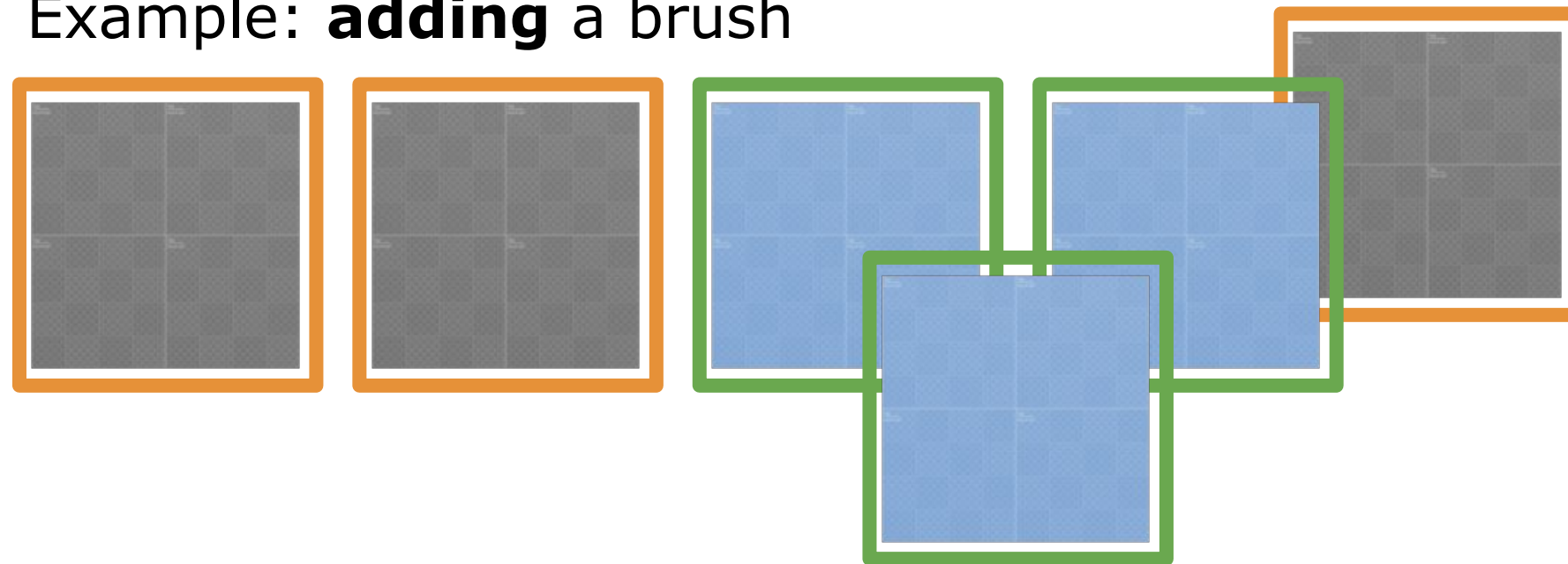
Here we update the brushes it touched before it was deleted



## Performing CSG per brush

- Allows for **iterative** updates
  - Only need to **update a brush when its modified**
    - And all brushes that **touched/touch** it
    - But not those that it didn't touch (**can be cached**)

Example: **adding** a brush

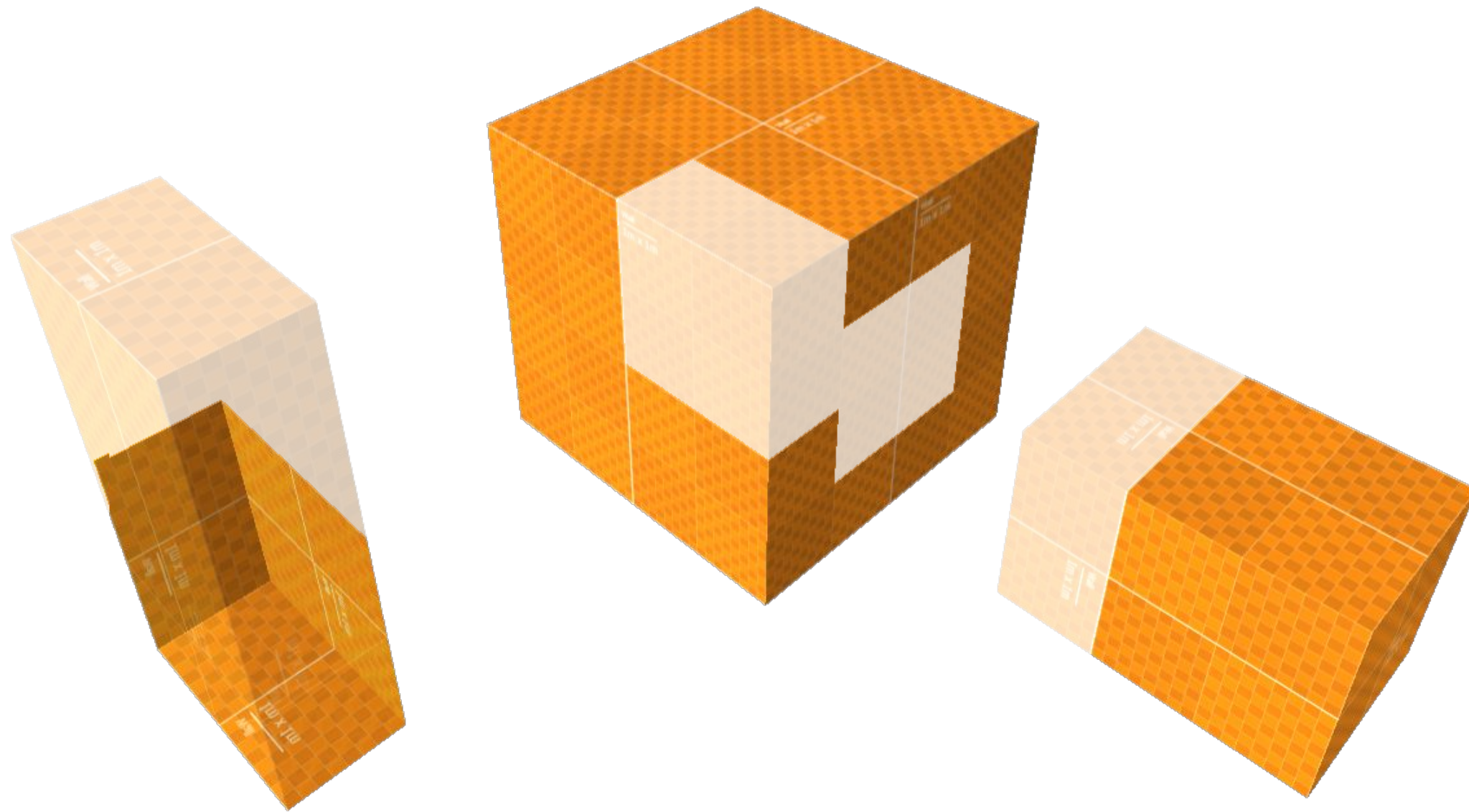


And here we update the brushes it touches after creation

## Performing CSG per brush

- Allows for **iterative** updates
  - Work can easily be split across **multiple cpu cores**
  - Work per brush doesn't get too expensive
    - **Scales well** with number of brushes





**Remember those polygon pieces?  
How do we find them?**

# Overview

## 1. History of CSG

The algorithm

## 2. Iterative updates

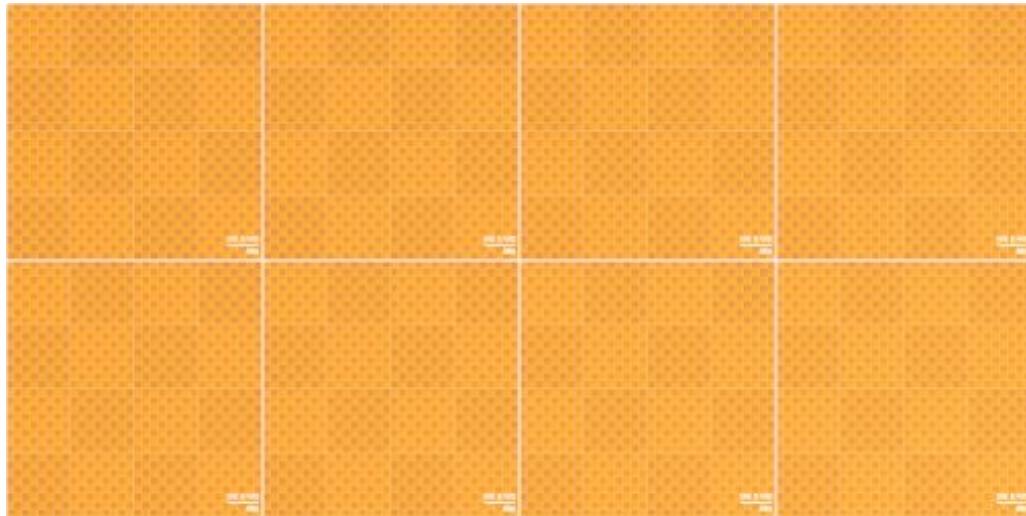
## **3. Intersections**

## 4. Mesh Generation

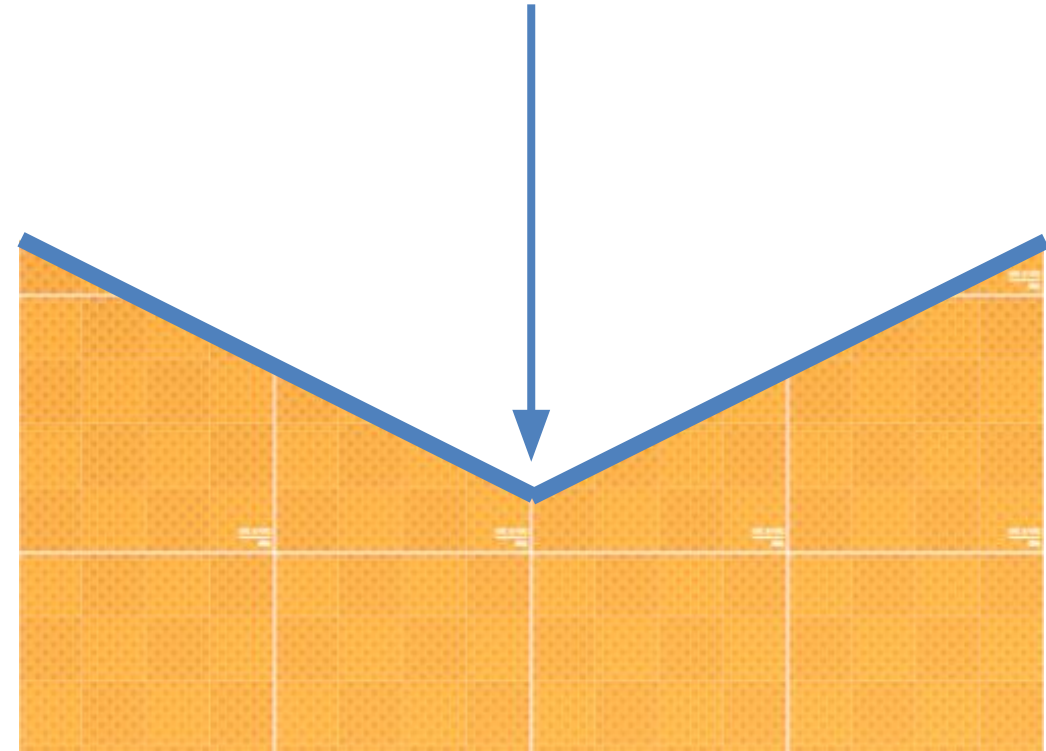
## 5. Polygon categories, Routing & Operation tables

## 6. Putting it all together

# Convexity



convex

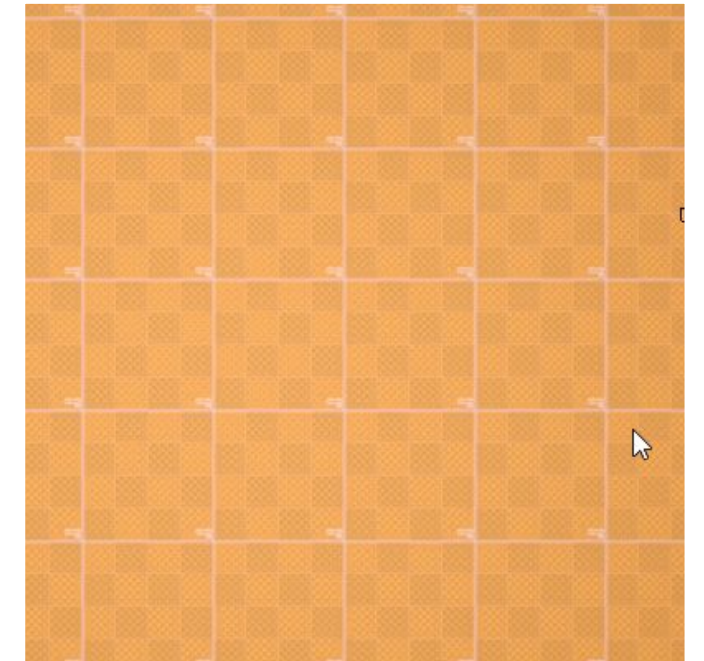


concave



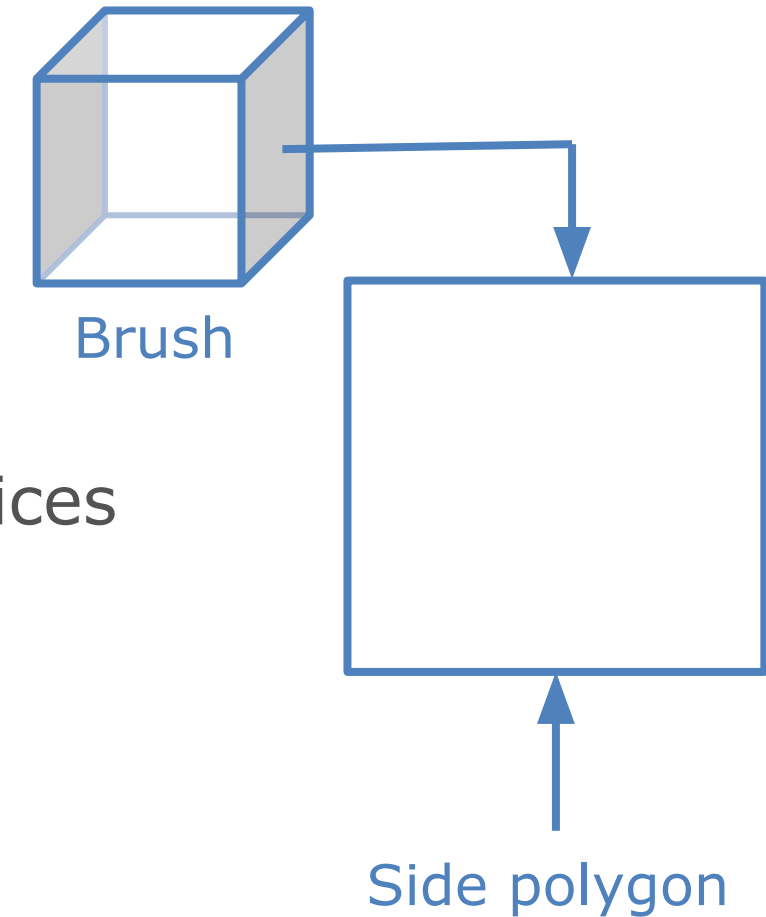
# Convex Brushes

- Can be thought of as an infinite cube sliced multiple times, leaving behind a convex shape
- These “slices” are infinite **planes**
  - Each plane has a facing direction
  - We essentially “remove” everything in front of the planes
- Conceptually convex brushes are “a list of planes”
- Convexity is not necessarily a requirement
  - But it makes everything *a lot* simpler & faster
  - You can still build any concave shape from multiple convex shapes

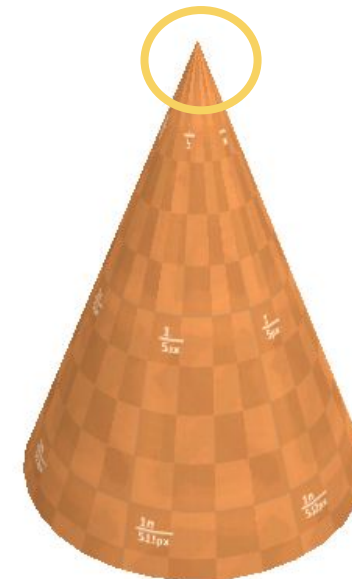
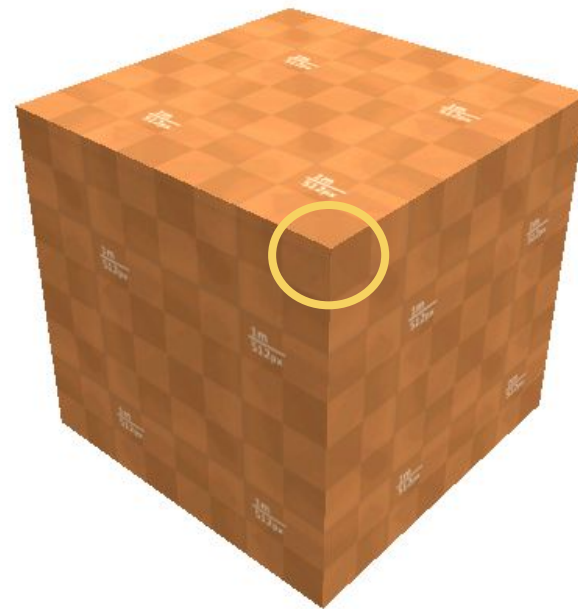


## Convex Brushes

- Edges are where **exactly** 2 planes intersect
- Vertices are where **at least** 3 planes intersect
- Side polygons are formed between these edges and vertices
  - Each polygon has a **single** plane going through it



A cube would have just 3 planes intersect at a corner



A cone could have an unbounded number of planes intersect at its peak

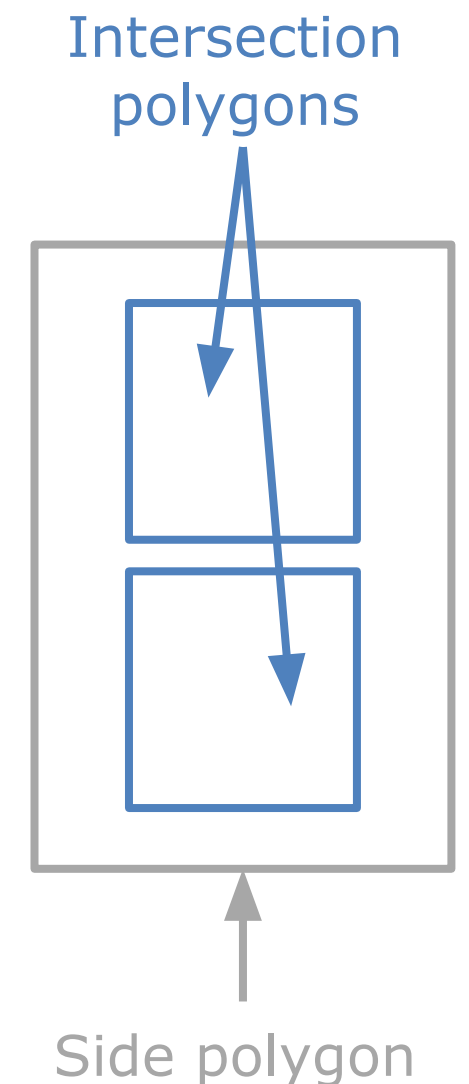
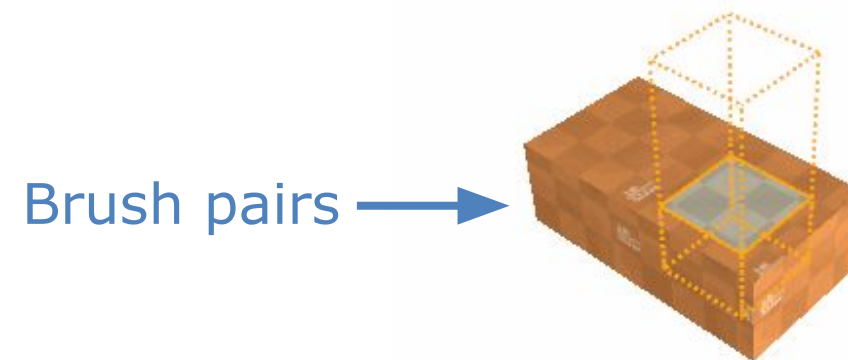
## Finding intersecting brushes

- Find intersecting brushes **at insertion time** or **after moving them**
  - Keep in mind that **intersection results are bi-directional**, so you only need to do this once for a pair of brushes.
    - **This ensures identical results**
  - Can use something like hierarchical hashed grids
  - Then, for each potential intersection
    - AABB intersection test
    - Check if vertices of a brush are outside the other brush
- Lots of ways of doing this, **this is *not* a bottleneck** however



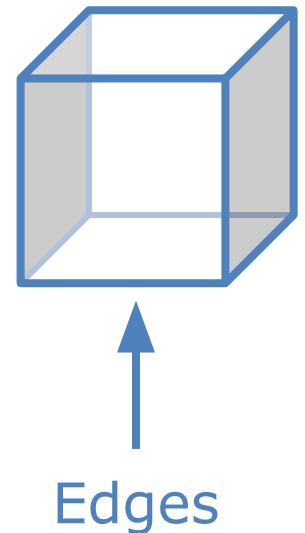
## Creating **intersection polygons**

- Process brush **pairs** together
  - Lots of shared information
  - Only consider polygons that intersect with the
    - Use space partition data structures to speed this up
      - Create per brush-shape, can be cached/shared
  - Find polygons that are formed at the intersection between pair
  - Polygons will always be convex if both brushes are convex
  - This ***is*** a bottleneck



## Creating **intersection polygons**

- Find all **vertices** of brush
  - Are inside other brush (inside all its other planes)
  - On a plane of the other brush (but inside all its other planes)
- Calculate intersections between brush **edges** with the other brush
  - Find intersection of edge with plane of other brush
    - Intersection vertex must be “inside” all *other* brush planes
  - We can only have 0-2 intersections per edge



## Creating **intersection polygons**

- Find all vertices that lie on the same plane on one brush
  - Do not calculate: store plane indices when finding vertices, use those
  - Remember: Our polygon is convex since our brushes are convex
- Allows us to find **edges** by finding vertex pairs that **share 2 planes**
- **Connect pairs** by finding **common vertices** between pairs
- Ensure ordering is correct
  - Calculate normal of vertices (newell's algorithm) and compare with plane normal
    - If dot product between both normals is negative, reverse order of vertices

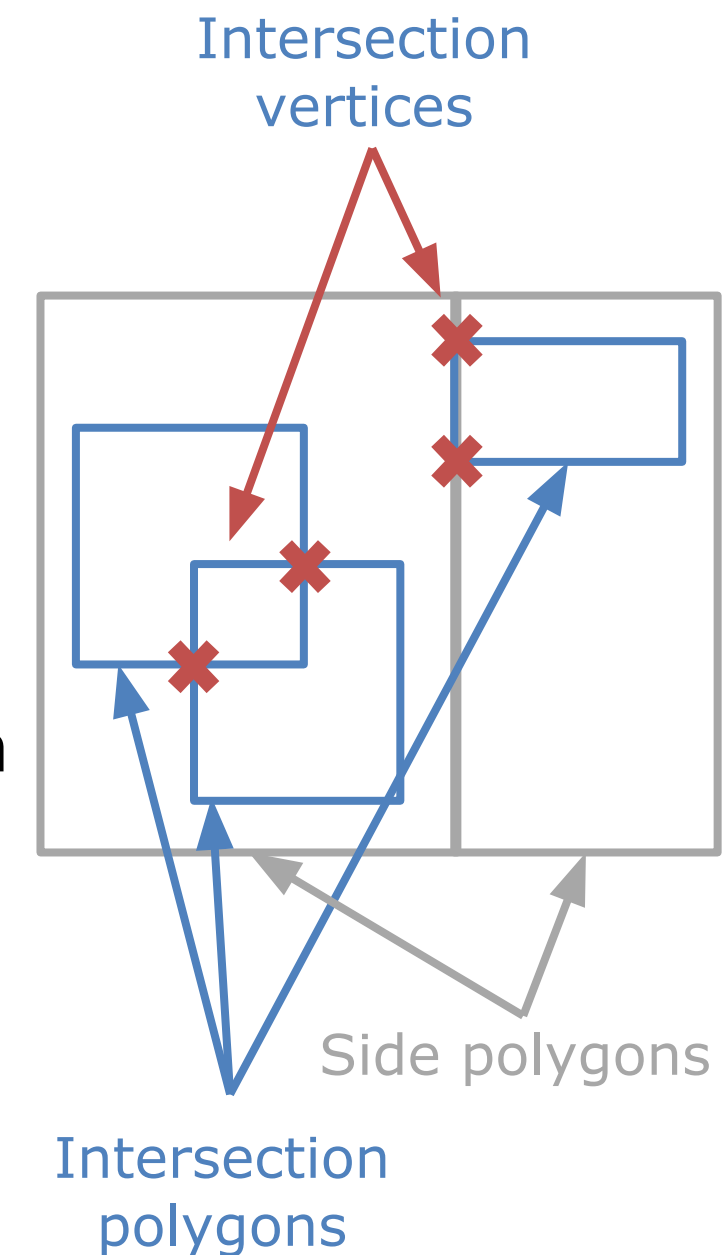


## Creating **intersection polygons**

- Store each intersection polygon together with the plane/brush polygon it's on
- For each intersection polygon
  - Store ***which brush we intersected with***
  - Store an ***interior category*** with this intersection polygon
    - If all the vertices lie on the surface of the other brush, our category is ***Aligned or Reverse Aligned*** (depending on the orientation of intersecting plane vs side polygon)
    - Otherwise, the polygon is ***Inside***
    - ***Can never be outside***, since this is an intersection
    - We will use this later on in the categorization part

## Creating **intersection polygons**

- Find all intersection polygons that **overlap**
  - Add intersection vertex to **both** polygons
    - These polygons are created by intersections with brushes
    - Ensure these vertices are also added to those brushes
      - This avoids gaps
- Do the same with the side polygon the intersection polygons lie on
  - Each edge brush is shared between 2 side polygons on a brush
  - Make sure this vertex exists on both polygons that share edge



## Precision

- Make sure that the found vertices are *copied* to the other brush, not recalculated.
  - When the vertices are identical between brushes, there won't be any gaps
    - It *ensures* that the vertices will be 100% identical on all edges
- Note: Snap vertices of intersecting brushes to each other as well, before you do any intersection calculations, for this exact same reason
  - Makes sure vertices are consistent between brushes
- We now have all the vertices we need, we don't need to create any more vertices



# Overview

## 1. History of CSG

### The algorithm

## 2. Iterative updates

## 3. Intersections

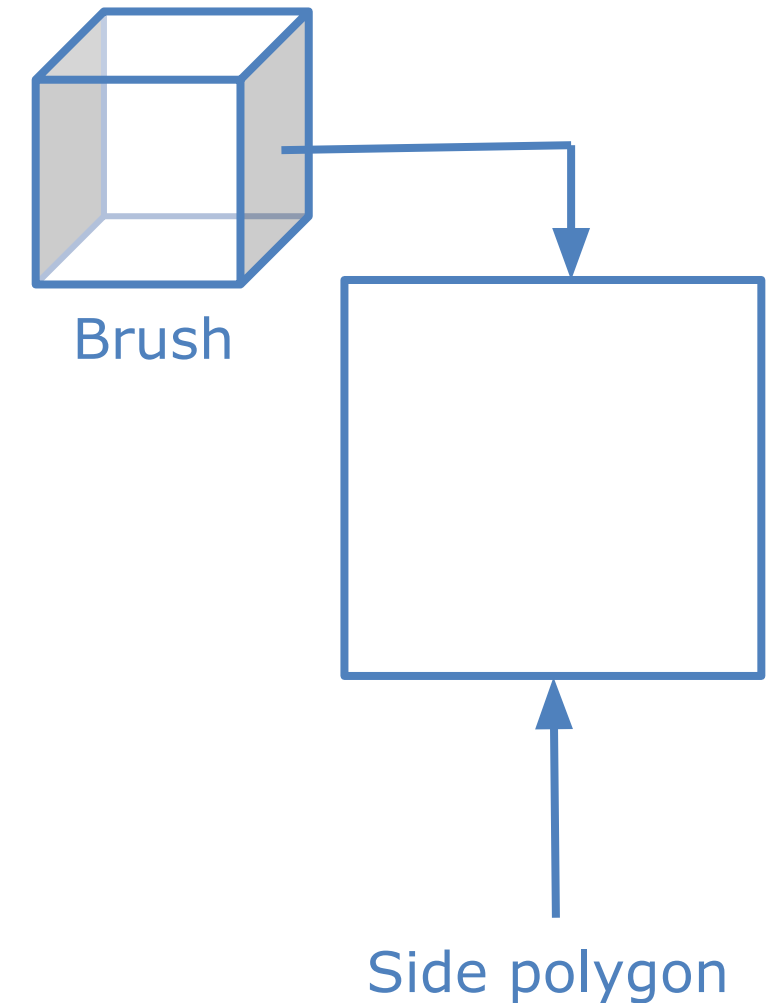
## **4. Mesh Generation**

## 5. Polygon categories, Routing & Operation tables

## 6. Putting it all together

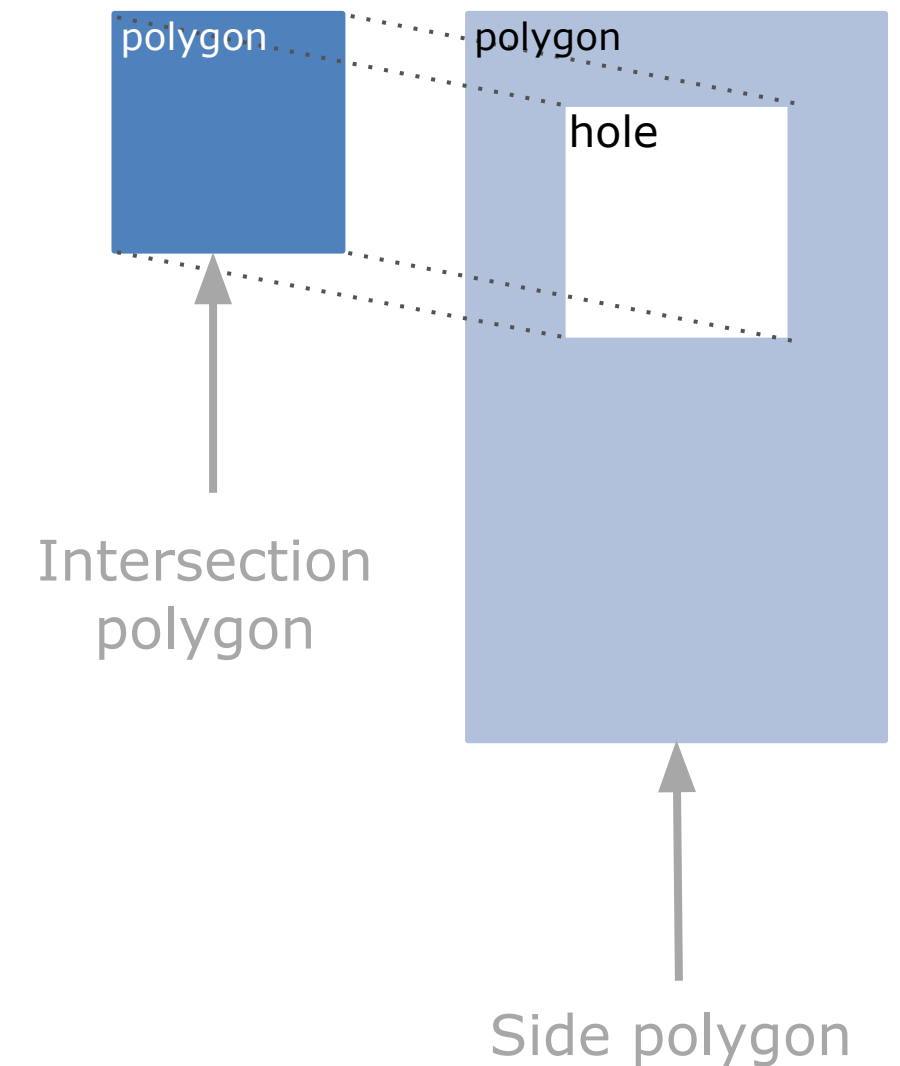
## Generating meshes

- We process each brush side separately
  - Here we apply each intersection polygon in order to split our brush side polygon into the pieces that we need



## Combining intersection polygons

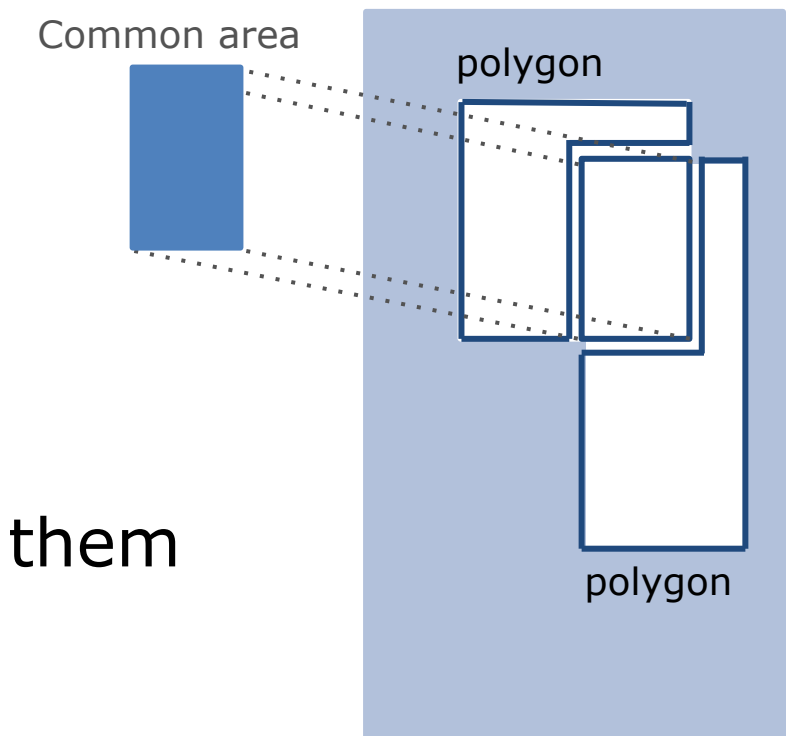
- Stored as **both** a **hole** on the current polygon and as a completely **new polygon**
  - Polygons are triangulated together with its holes





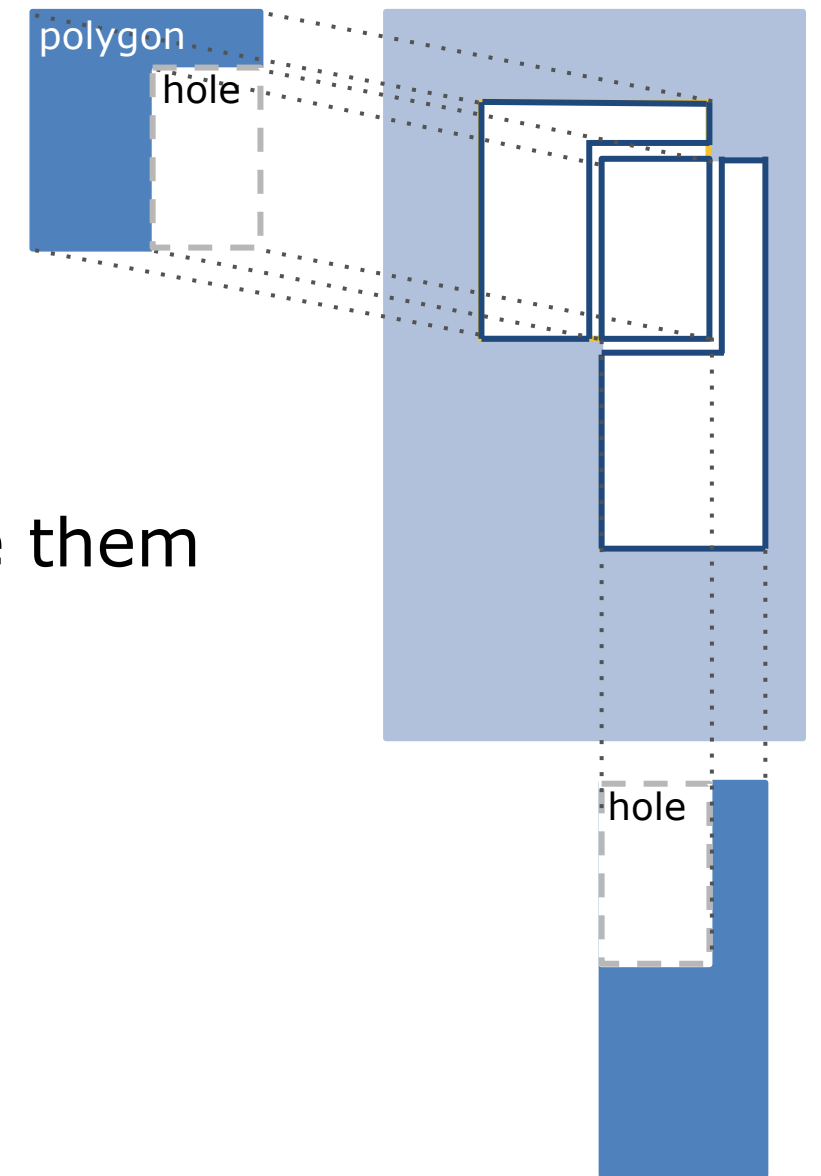
## Combining intersection polygons

- Stored as **both** a **hole** on the current polygon and as a completely **new polygon**
  - Polygons are triangulated together with its holes
  - Also need to handle **overlapping polygons**
    - Find **common area** between them
      - Find all edges that are inside/on both, combine them
      - Always works if both both polygons are convex



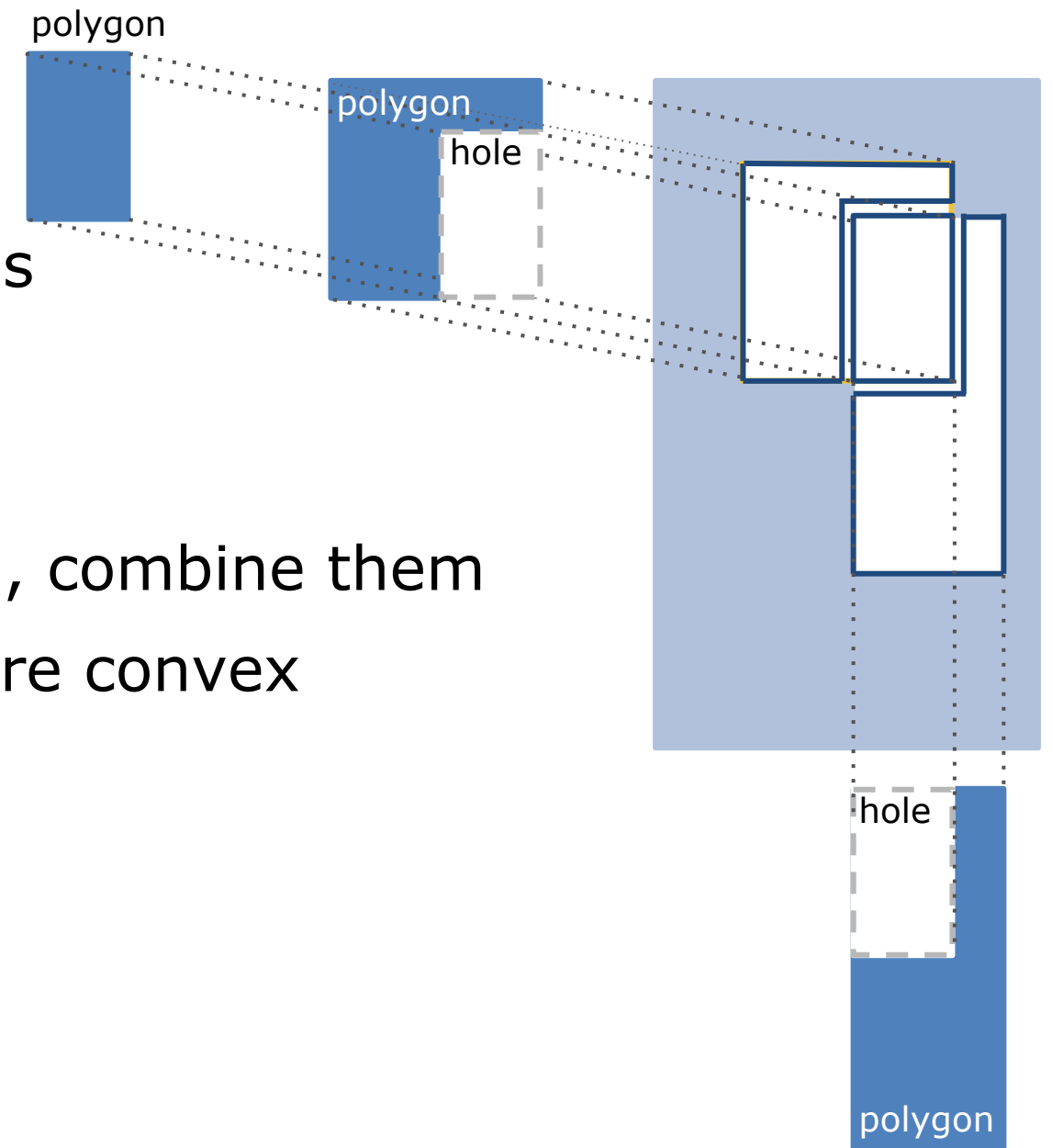
## Combining intersection polygons

- Stored as **both** a **hole** on the current polygon and as a completely **new polygon**
  - Polygons are triangulated together with its holes
  - Also need to handle **overlapping polygons**
    - Find **common area** between them
      - Find all edges that are inside/on both, combine them
      - Always works if both both polygons are convex
    - Becomes a **hole on both polygons**



## Combining intersection polygons

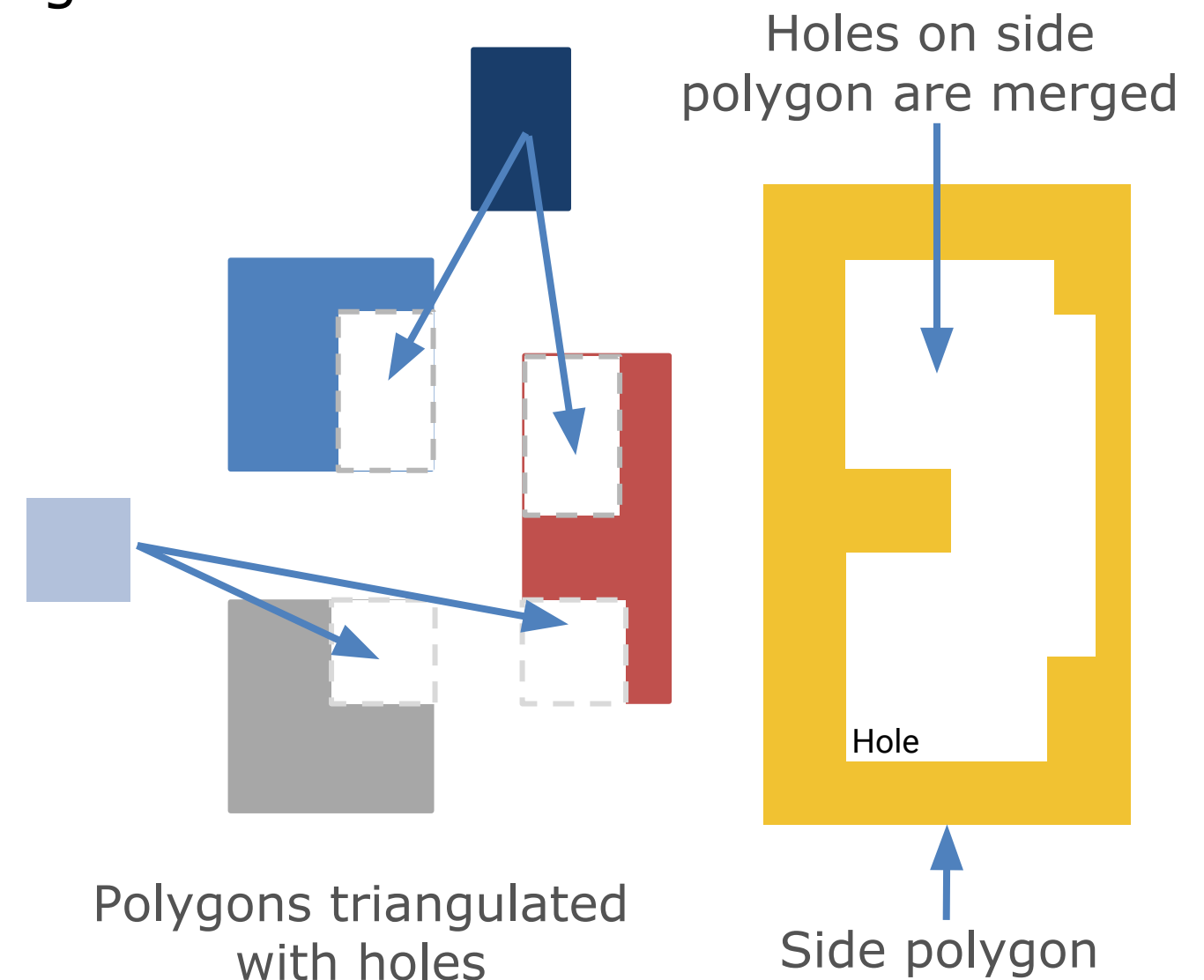
- Stored as **both** a **hole** on the current polygon and as a completely **new polygon**
  - Polygons are triangulated together with its holes
  - Also need to handle **overlapping polygons**
    - Find **common area** between them
      - Find all edges that are inside/on both, combine them
      - Always works if both both polygons are convex
    - Becomes a **hole on both polygons**
    - and a **new polygon**





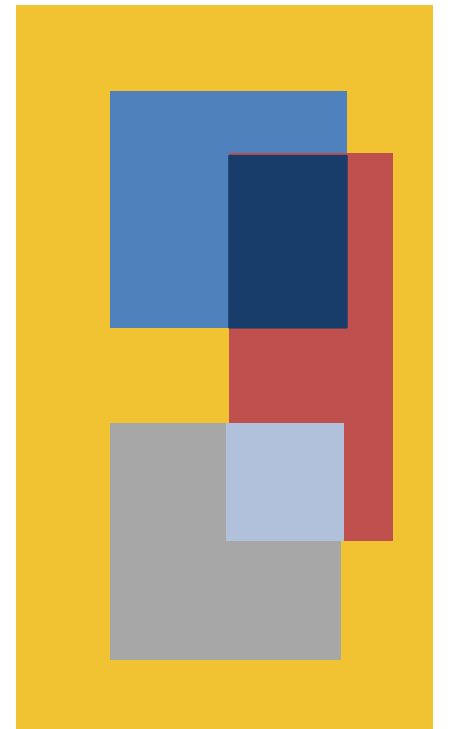
## Creating brush meshes

- We triangulate each polygon separately along with its holes
  - Merge the holes by removing overlapping edges and combining all the remaining edges



## Creating brush meshes

- We triangulate each polygon separately along with its holes
  - Merge the holes by removing overlapping edges and combining all the remaining edges
  - Each polygon is triangulated using vertex indices
    - Already found all vertices at the beginning



# Overview

## 1. History of CSG

### The algorithm

## 2. Iterative updates

## 3. Intersections

## 4. Mesh Generation

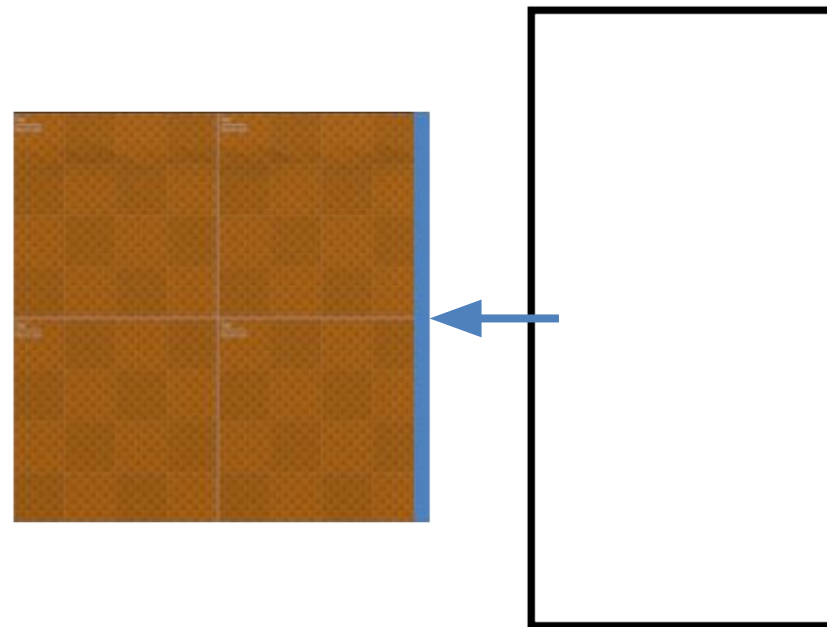
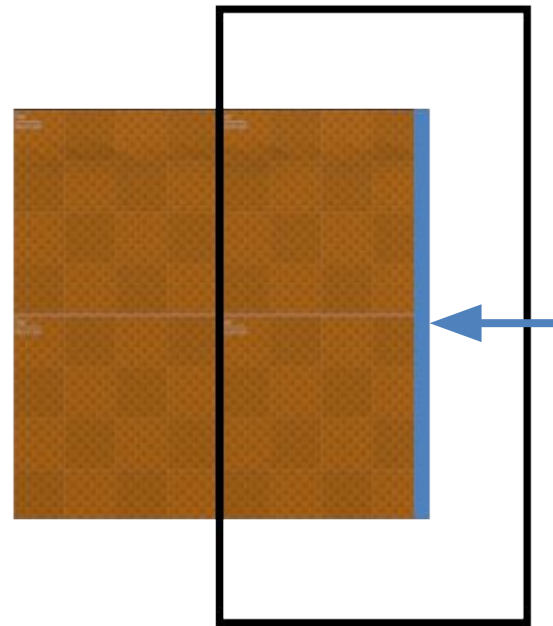
## **5. Polygon categories, Routing & Operation tables**

## 6. Putting it all together



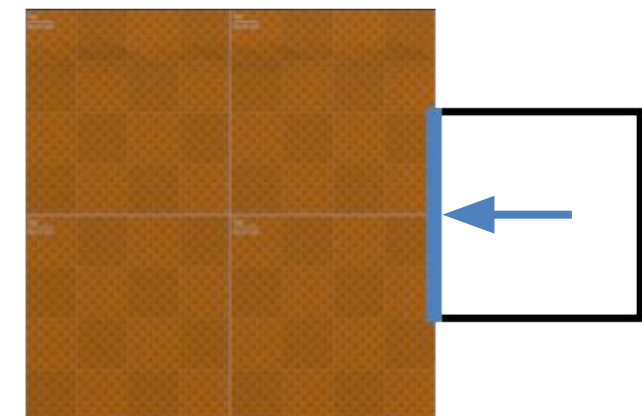
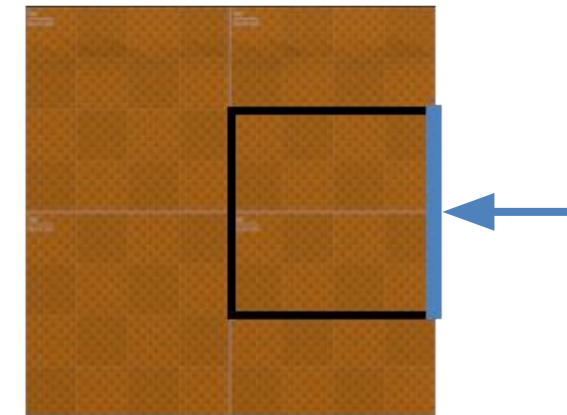
## Categories

### Inside



### Outside

### Aligned



### Reverse Aligned

## Brush categorization

- How to categorize a vertex against a single brush:
  - Calculate distance of a vertex against *each* plane
  - Positive value, compared to *any* plane: it's **outside** (early out)
  - *Near* zero value: it's **aligned**
  - Neither outside or aligned to *any* plane: it's **inside**

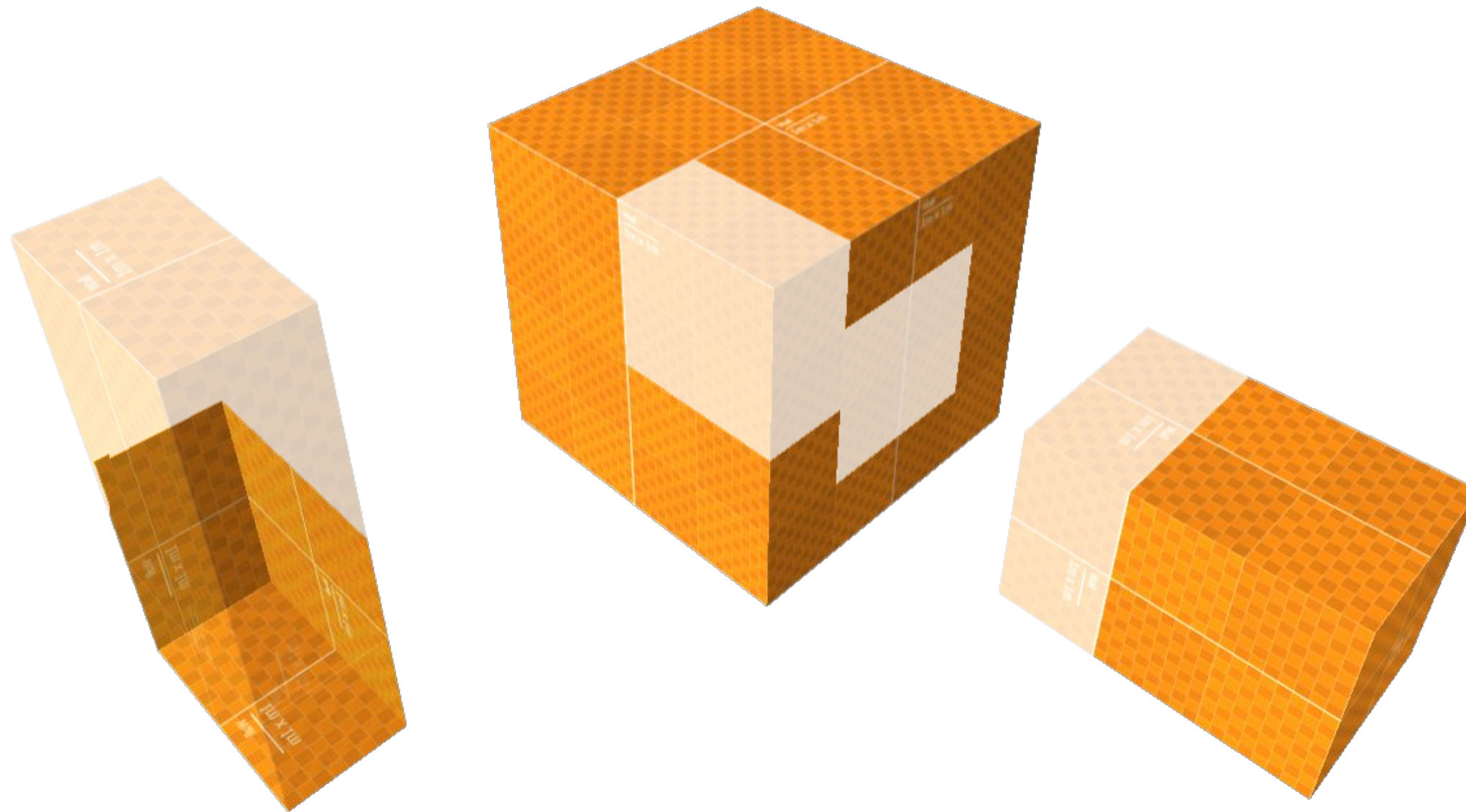
## Brush categorization

- How to categorize a polygon against a single brush:
  - Otherwise
    - If **all** vertices of a polygon are **(reverse) aligned**, then that's the polygons' category.
    - If **any** vertex is inside/outside, it's **inside/outside**
      - Some vertices might be aligned with/touch another brush
      - If one vertex of an edge is inside and the other is outside, then it's intersecting the brush
        - We already found all intersections, so this won't happen



## Brush categorization

- How to categorize a polygon against a single brush:
  - If it's aligned
    - Compare normal of polygon to normal of plane
      - Opposite direction: **reverse-aligned**
      - Same direction: **aligned**

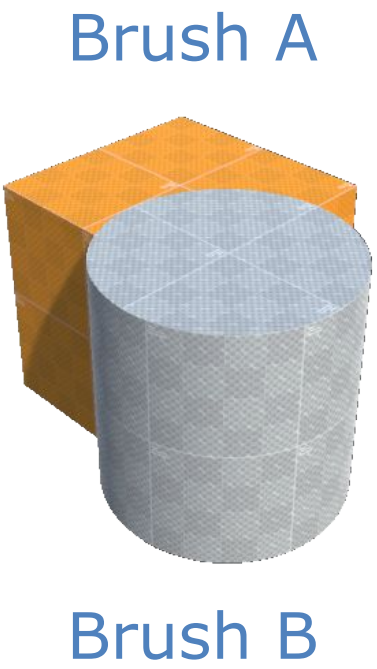


Which polygon piece is what,  
to the ***entire generated mesh?***

Use a lookup table to combine categories among multiple brushes

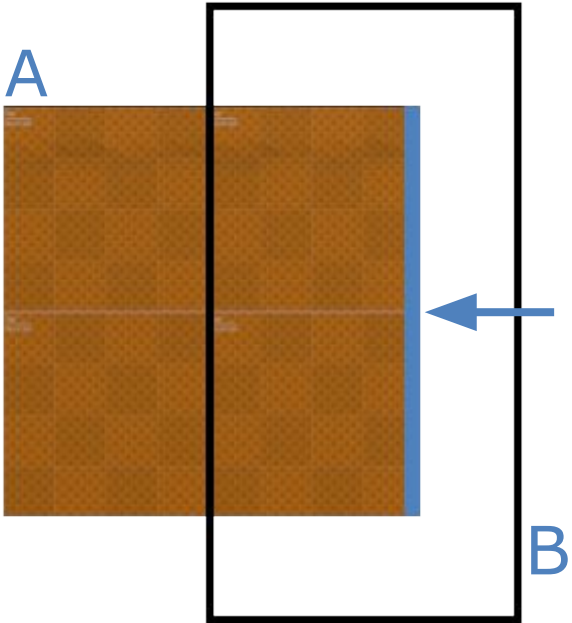
- Find the polygon category for each brush individually
- Combine categories using an operation table
- Note: Polygon does not need to be part of either brush

| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |



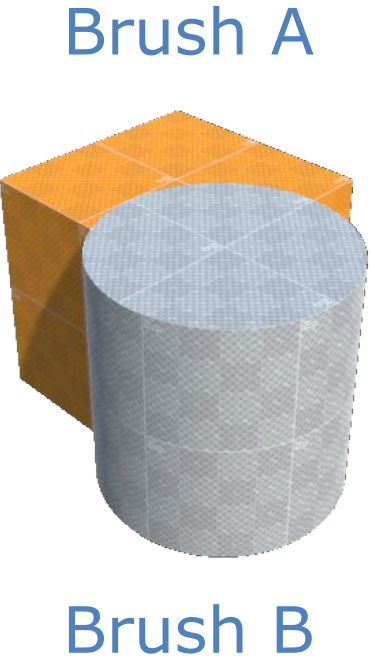


Use a lookup table to combine categories among multiple brushes

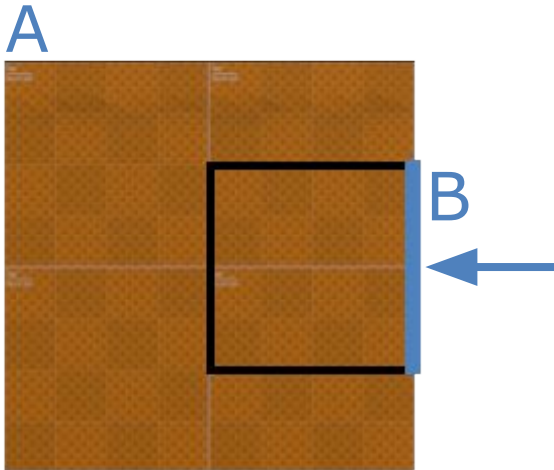


If polygon has the **inside** category for **either** brush, it's inside **both** brushes

| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |

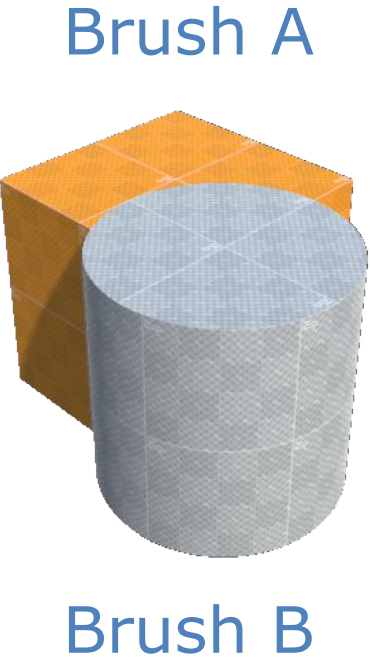


Use a lookup table to combine categories among multiple brushes

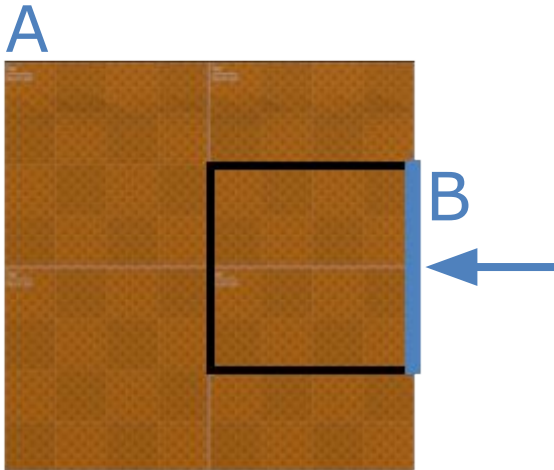


If polygon has the **aligned** category for **both** brushes, it's aligned

|         |             | Brush A |         |             |             |
|---------|-------------|---------|---------|-------------|-------------|
|         |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B | Inside      | Inside  | Inside  | Inside      | Inside      |
|         | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|         | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|         | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |

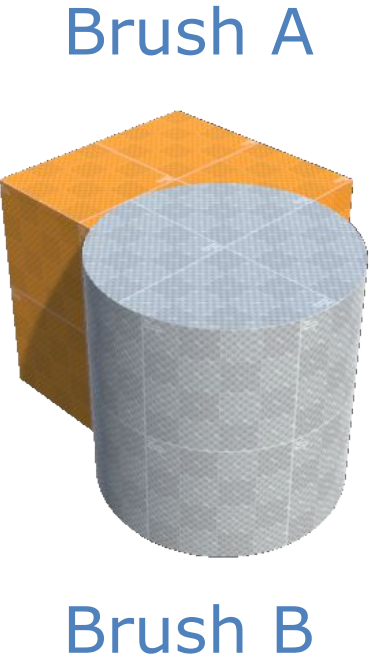


Use a lookup table to combine categories among multiple brushes



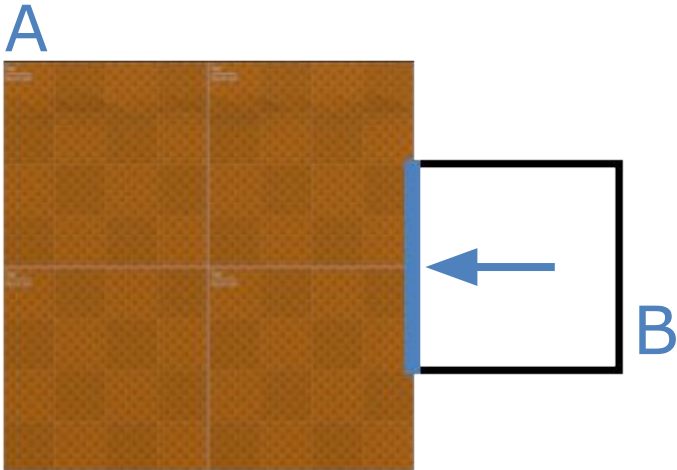
If polygon has the **reverse-aligned** category for **both** brushes, it's reverse-aligned

| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |



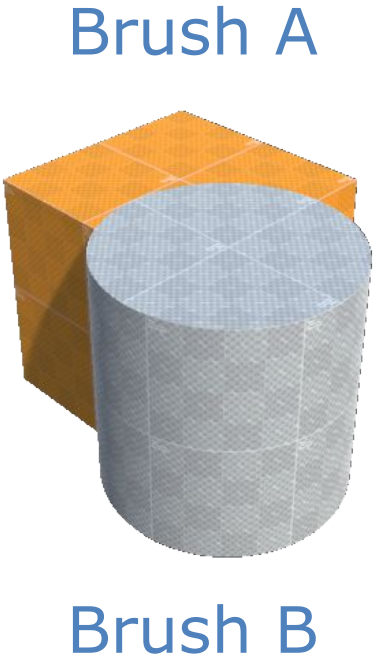


Use a lookup table to combine categories among multiple brushes

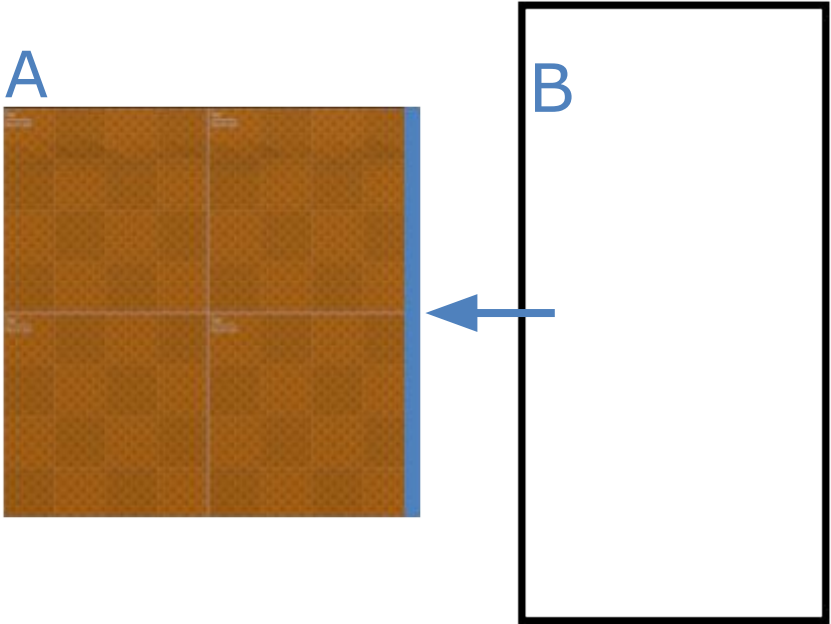


If categories are **reverse-aligned** and **aligned**, the final category is **inside** (surfaces cancel each other out)

| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |

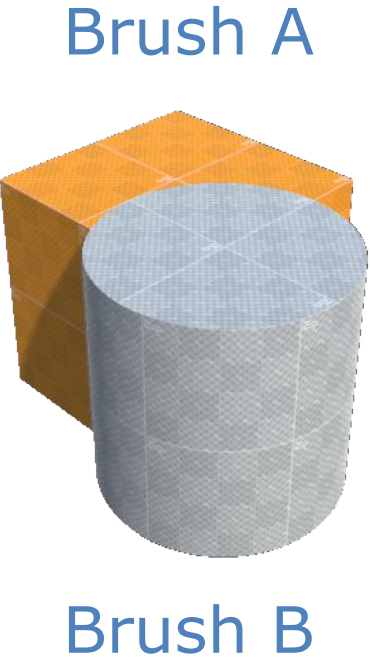


Use a lookup table to combine categories among multiple brushes

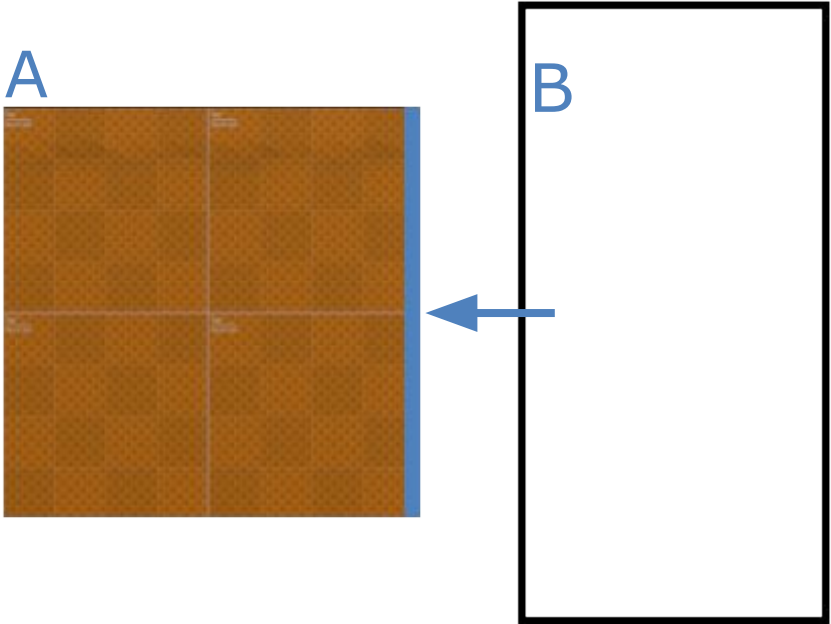


If polygon has the **outside** category for **either** brush, it's the **category of the other brush**

| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |

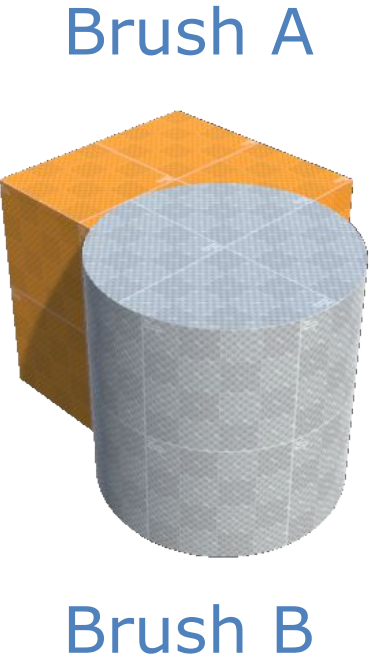


Use a lookup table to combine categories among multiple brushes



If polygon has the **outside** category for **both** brushes, it's **outside**

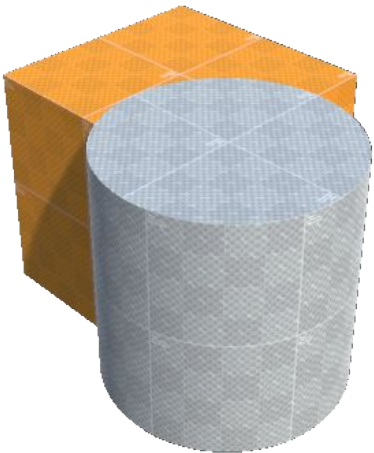
| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |





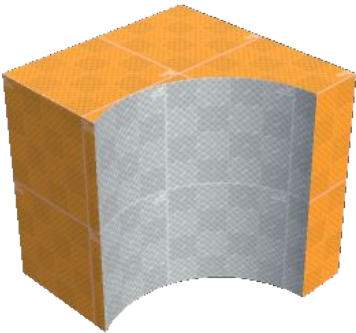
Use a lookup table to combine categories among multiple brushes

| Additive<br>Operation Table |             | Brush A |         |             |             |
|-----------------------------|-------------|---------|---------|-------------|-------------|
|                             |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                     | Inside      | Inside  | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside  | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside  | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside  | Aligned | Rev-Aligned | Outside     |



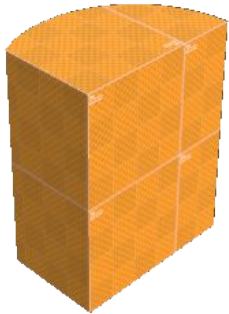
Use a lookup table to combine categories among multiple brushes

| Subtractive<br>Operation Table |             | Brush A |             |             |             |
|--------------------------------|-------------|---------|-------------|-------------|-------------|
|                                |             | Inside  | Aligned     | Rev-Aligned | Outside     |
| Brush B                        | Inside      | Outside | Rev-Aligned | Aligned     | Inside      |
|                                | Aligned     | Outside | Outside     | Aligned     | Aligned     |
|                                | Rev-Aligned | Outside | Rev-Aligned | Outside     | Rev-Aligned |
|                                | Outside     | Outside | Outside     | Outside     | Outside     |



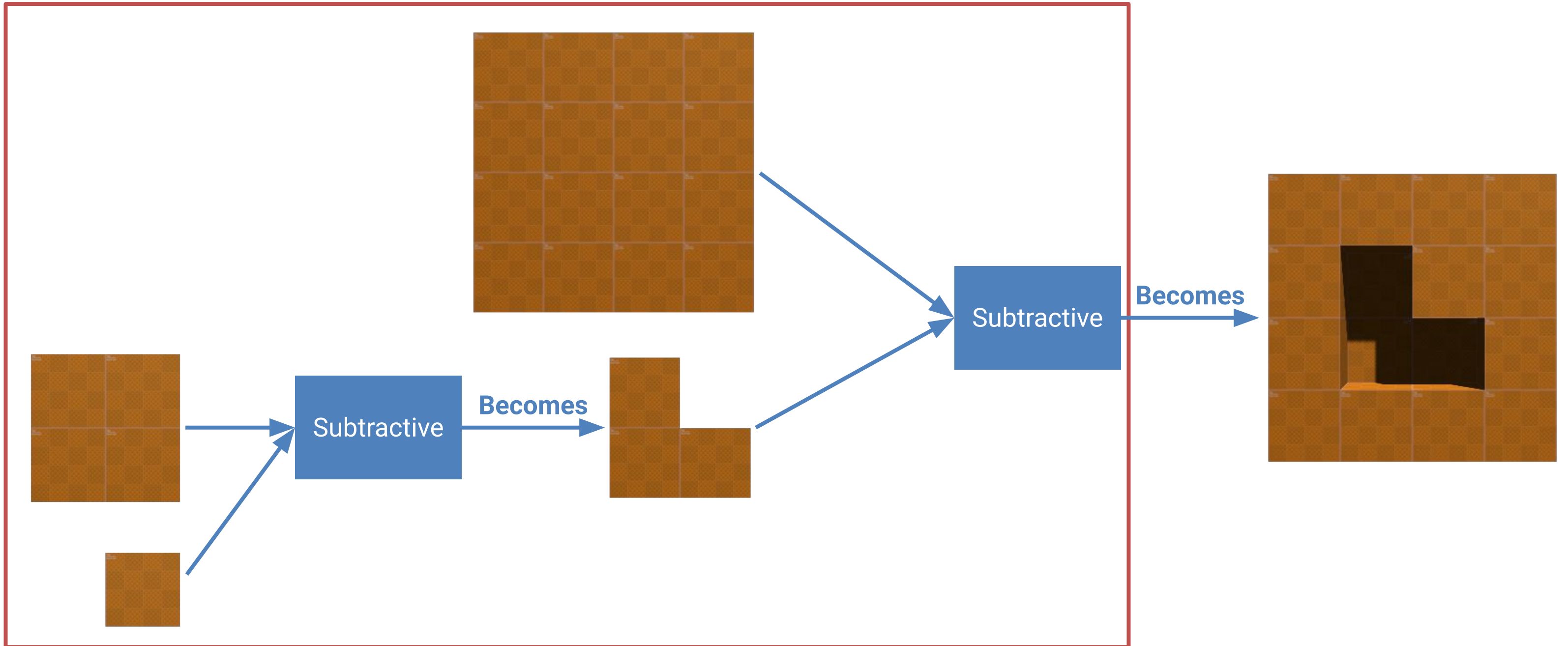
Use a lookup table to combine categories among multiple brushes

| Intersecting<br>Operation Table |             | Brush A     |         |             |         |
|---------------------------------|-------------|-------------|---------|-------------|---------|
|                                 |             | Inside      | Aligned | Rev-Aligned | Outside |
| Brush B                         | Inside      | Inside      | Aligned | Rev-Aligned | Outside |
|                                 | Aligned     | Aligned     | Aligned | Outside     | Outside |
|                                 | Rev-Aligned | Rev-Aligned | Outside | Rev-Aligned | Outside |
|                                 | Outside     | Outside     | Outside | Outside     | Outside |



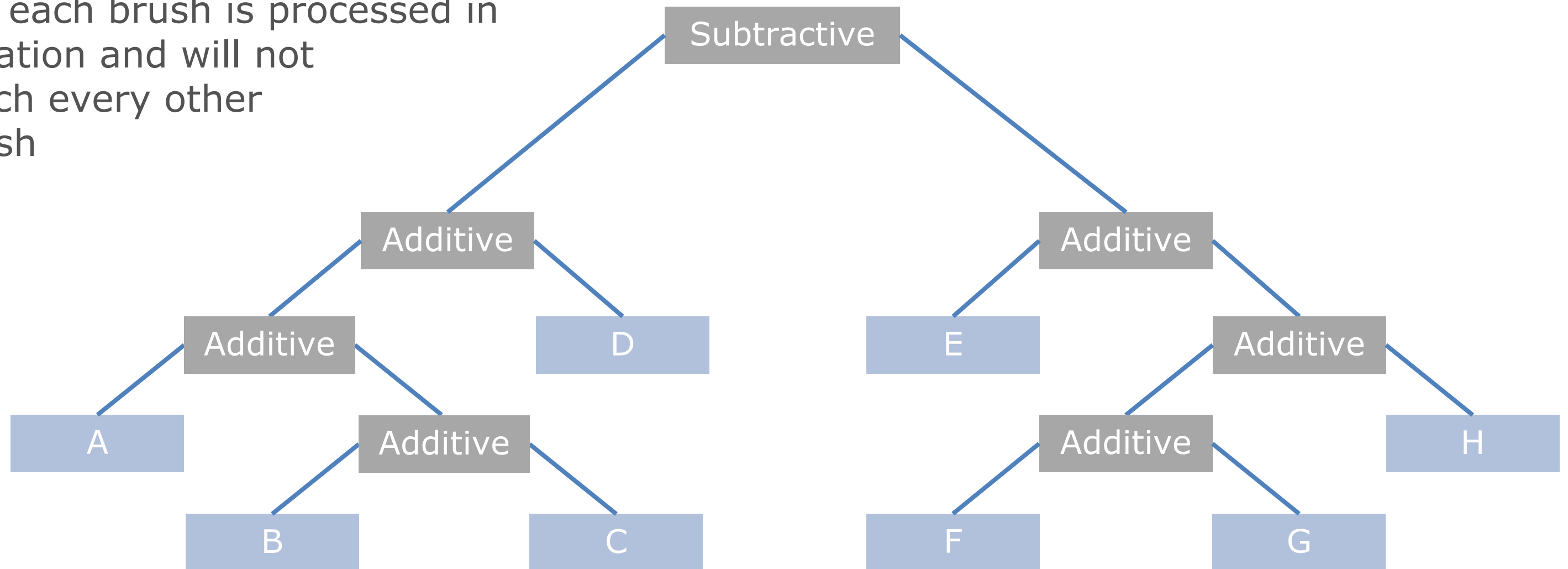


## CSG Tree



# CSG Tree

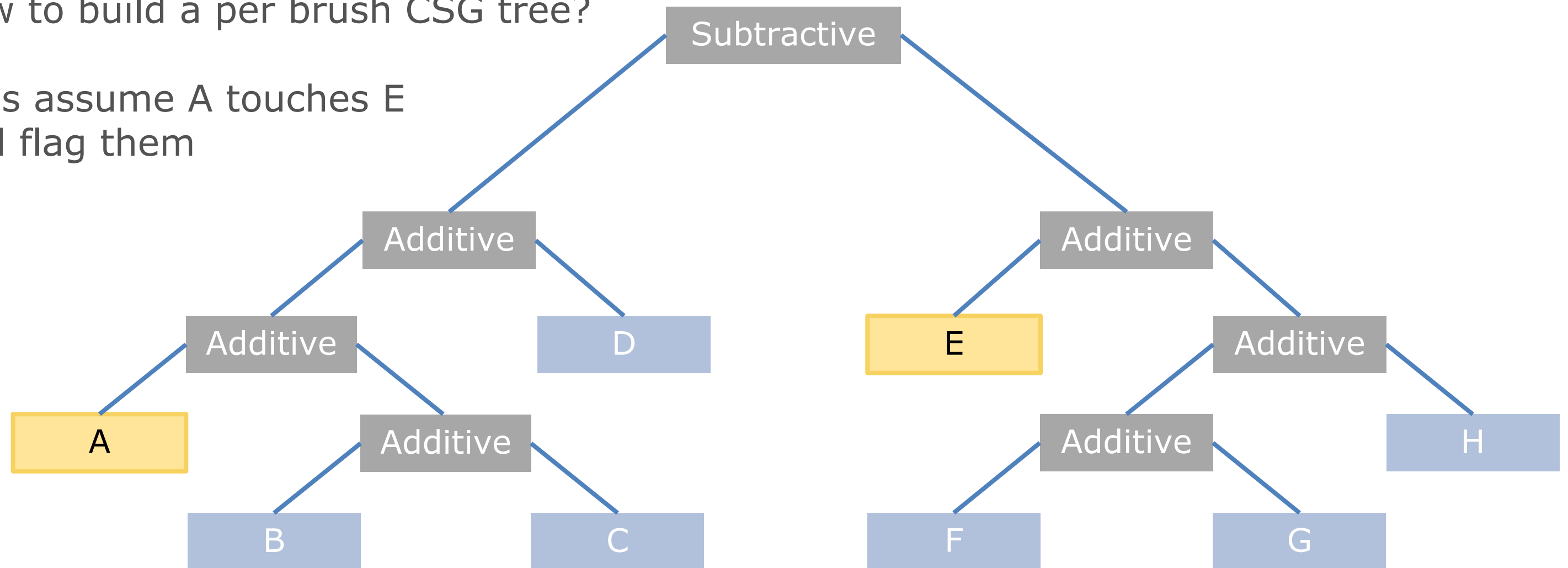
But each brush is processed in isolation and will not touch every other brush



## Per brush CSG Tree

How to build a per brush CSG tree?

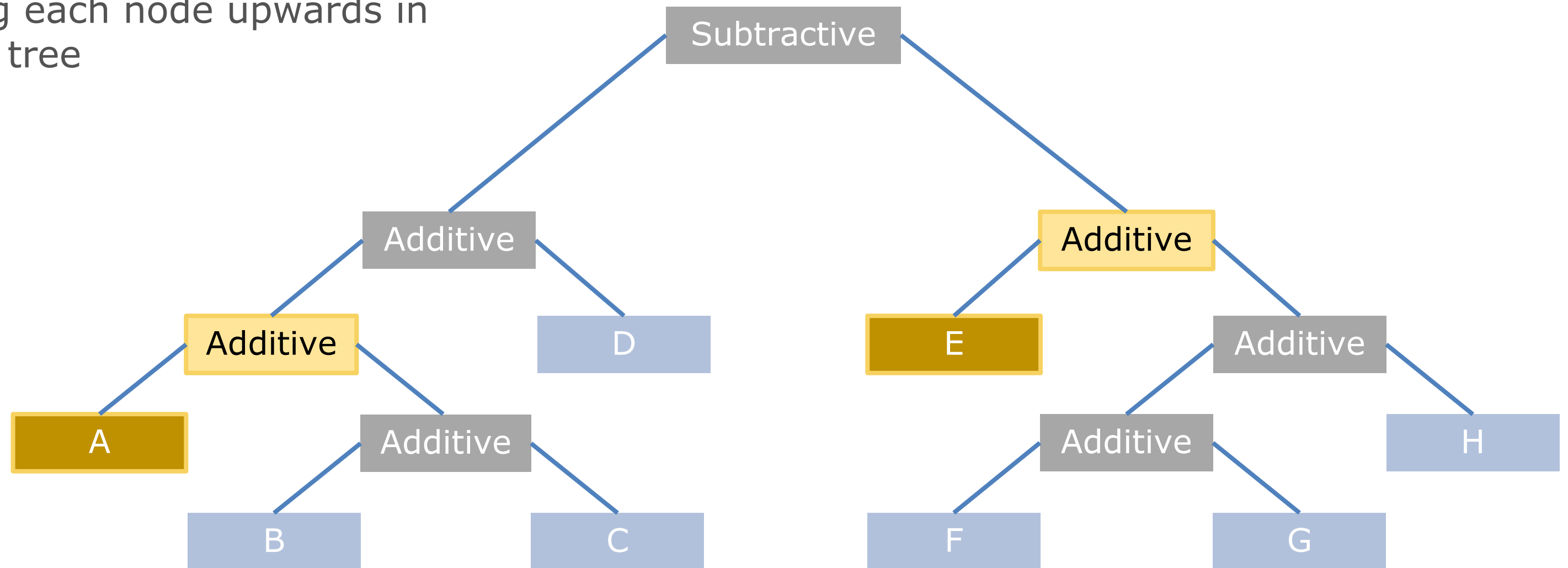
Let's assume A touches E  
And flag them





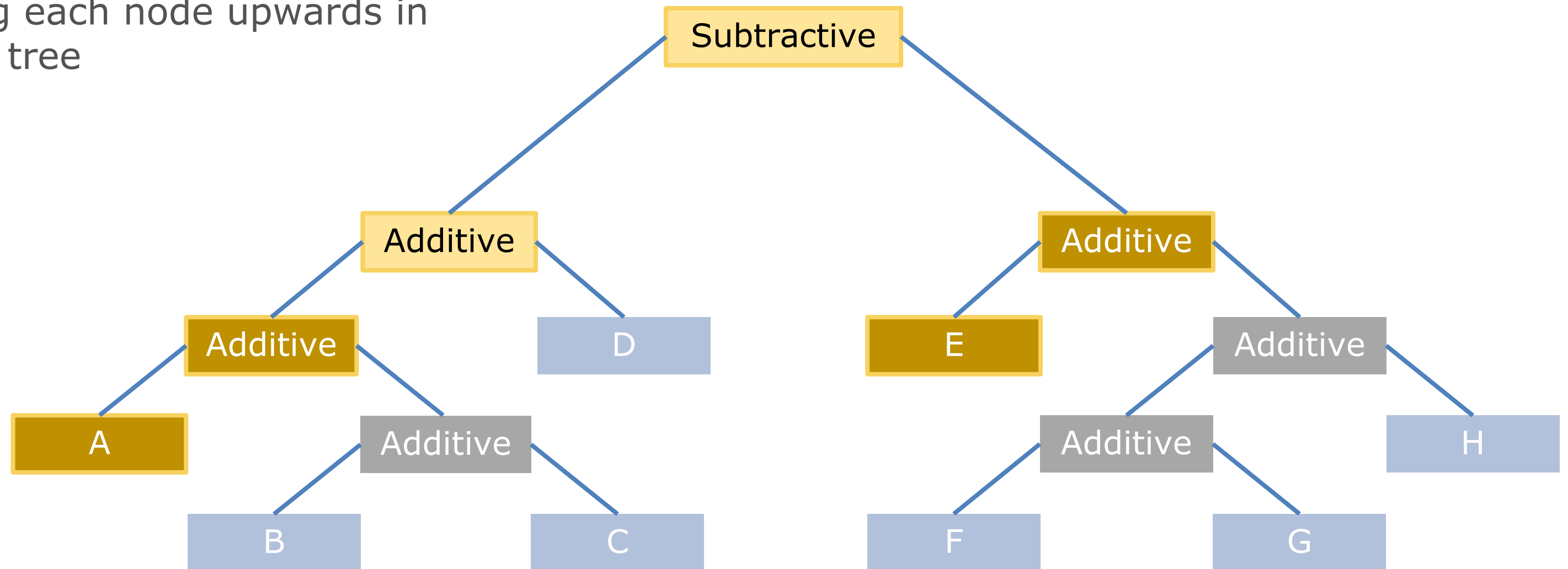
## Per brush CSG Tree

Flag each node upwards in the tree



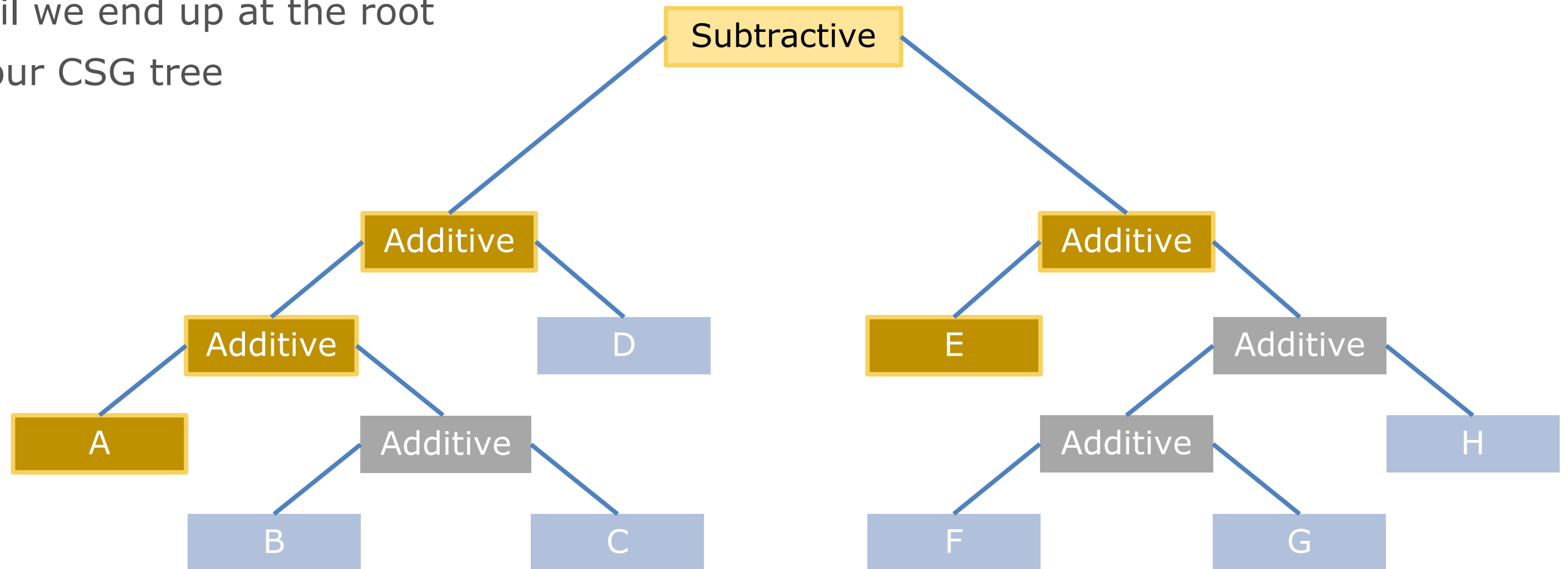
## Per brush CSG Tree

Flag each node upwards in the tree



## Per brush CSG Tree

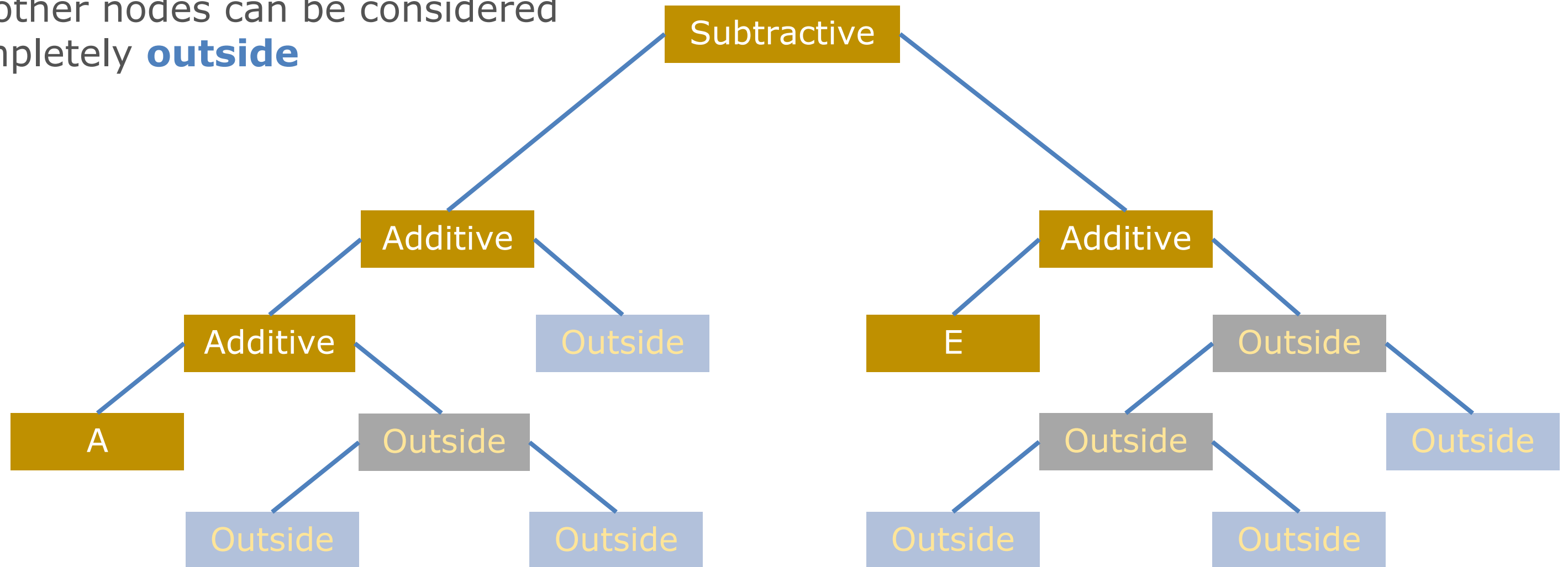
Until we end up at the root  
of our CSG tree





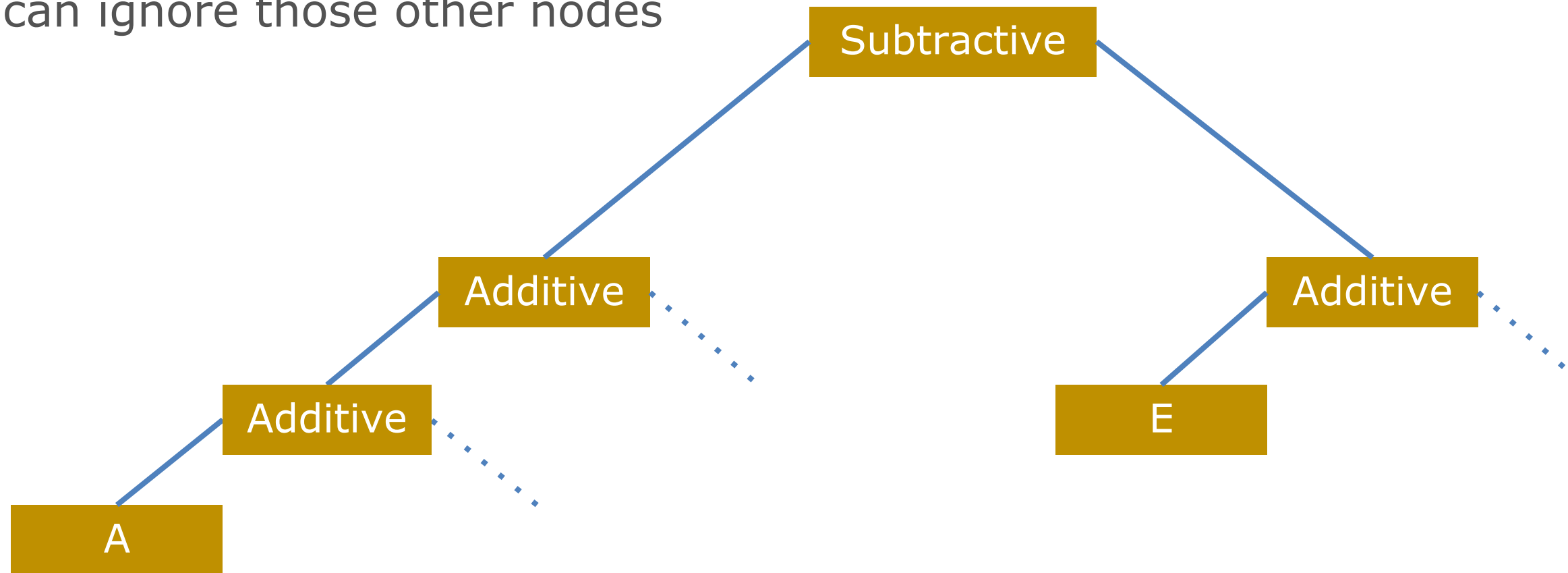
## Per brush CSG Tree

All other nodes can be considered completely **outside**

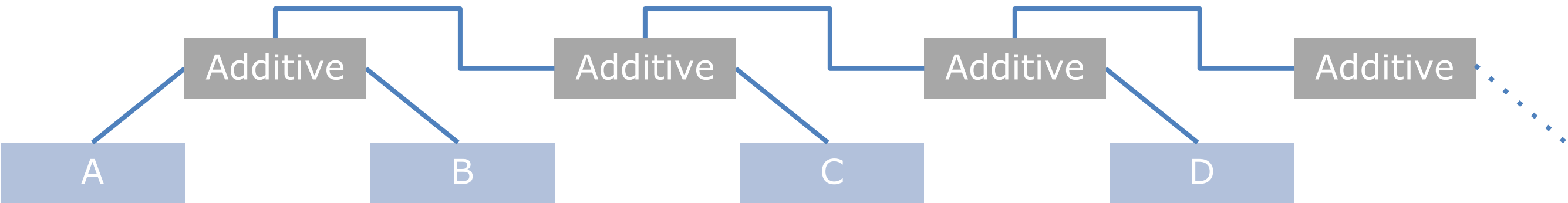


## Per brush CSG Tree

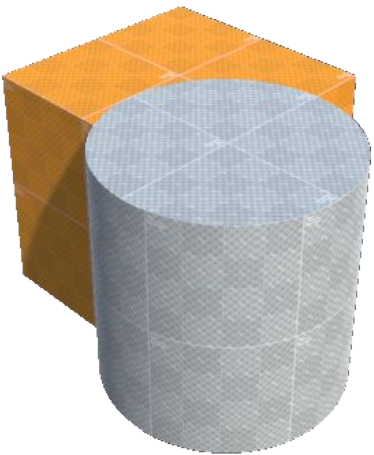
We can ignore those other nodes



Use a lookup table to combine categories among multiple brushes



| Additive<br>Operation Table |             | Left   |         |             |             |
|-----------------------------|-------------|--------|---------|-------------|-------------|
|                             |             | Inside | Aligned | Rev-Aligned | Outside     |
| Right                       | Inside      | Inside | Inside  | Inside      | Inside      |
|                             | Aligned     | Inside | Aligned | Inside      | Aligned     |
|                             | Rev-Aligned | Inside | Inside  | Rev-Aligned | Rev-Aligned |
|                             | Outside     | Inside | Aligned | Rev-Aligned | Outside     |





## Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside     |
|---------|---------------|--------|---------|-------------|-------------|
| Brush A | -             | Inside | Aligned | Rev-Aligned | Outside     |
| Brush B | Inside        | Inside | Inside  | Inside      | Inside      |
|         | Aligned       | Inside | Aligned | Inside      | Aligned     |
|         | Rev-Aligned   | Inside | Inside  | Rev-Aligned | Rev-Aligned |
|         | Outside       | Inside | Aligned | Rev-Aligned | Outside     |
| Brush C | Inside        | Inside | Inside  | Inside      | Inside      |
|         | Aligned       | Inside | Aligned | Inside      | Aligned     |
|         | Rev-Aligned   | Inside | Inside  | Rev-Aligned | Rev-Aligned |
|         | Outside       | Inside | Aligned | Rev-Aligned | Outside     |

Boolean operations are **baked** into routing table

using the boolean lookup tables

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

In practice we use **numerical values** instead of symbolic names

Inside = 0  
Aligned = 1  
Rev-Aligned = 2  
Outside = 3

The output of each row becomes an index to a row in the next section

Always be sure to keep the output of the last brush convertible back to categories

Polygon.index == 0

Lookup rows  
using a category  
stored on our  
polygon

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
|         | 1             | 0      | 1       | 0           | 1       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Polygon.index == 0

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example  
Brush A: **Rev-Aligned**  
Brush B: **Aligned**  
Brush C: **Outside**

Each **brush** will categorize our polygon, and this category is the **column**



Polygon.index == 0

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example  
Brush A: **Rev-Aligned**  
Brush B: **Aligned**  
Brush C: **Outside**

Using the **brush category** we find the **column**

Using the **polygon index** we find the **row**

Polygon.index == 2

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example

Brush A: **Rev-Aligned**

Brush B: **Aligned**

Brush C: **Outside**

The intersection of the **column** and **row** leads to the **output index**

Polygon.index == 2

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example  
Brush A: Rev-Aligned  
Brush B: **Aligned**  
Brush C: Outside

We do this with every brush ...

Polygon.index == 0

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example  
Brush A: Rev-Aligned  
Brush B: **Aligned**  
Brush C: Outside

We do this with every brush ...



Polygon.index == 0

Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example  
Brush A: Rev-Aligned  
Brush B: Aligned  
Brush C: Outside

Until we find the final index ...

Polygon.index == 0  
(final index)

Inside = 0  
Aligned = 1  
Rev-Aligned = 2  
Outside = 3

The final index  
can be  
converted  
back to a  
category

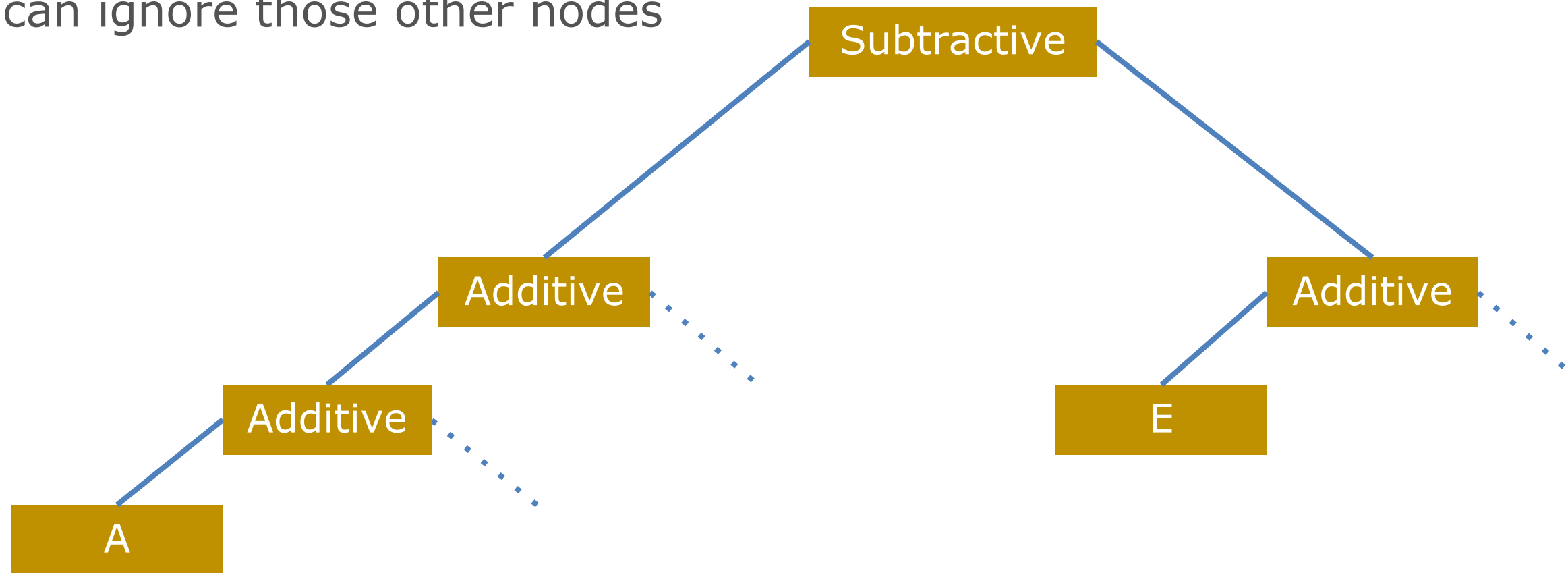
Routing table

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

Example  
Brush A: Rev-Aligned  
Brush B: Aligned  
Brush C: Outside

## Per brush CSG Tree

We can ignore those other nodes



Routing table **for brush B**

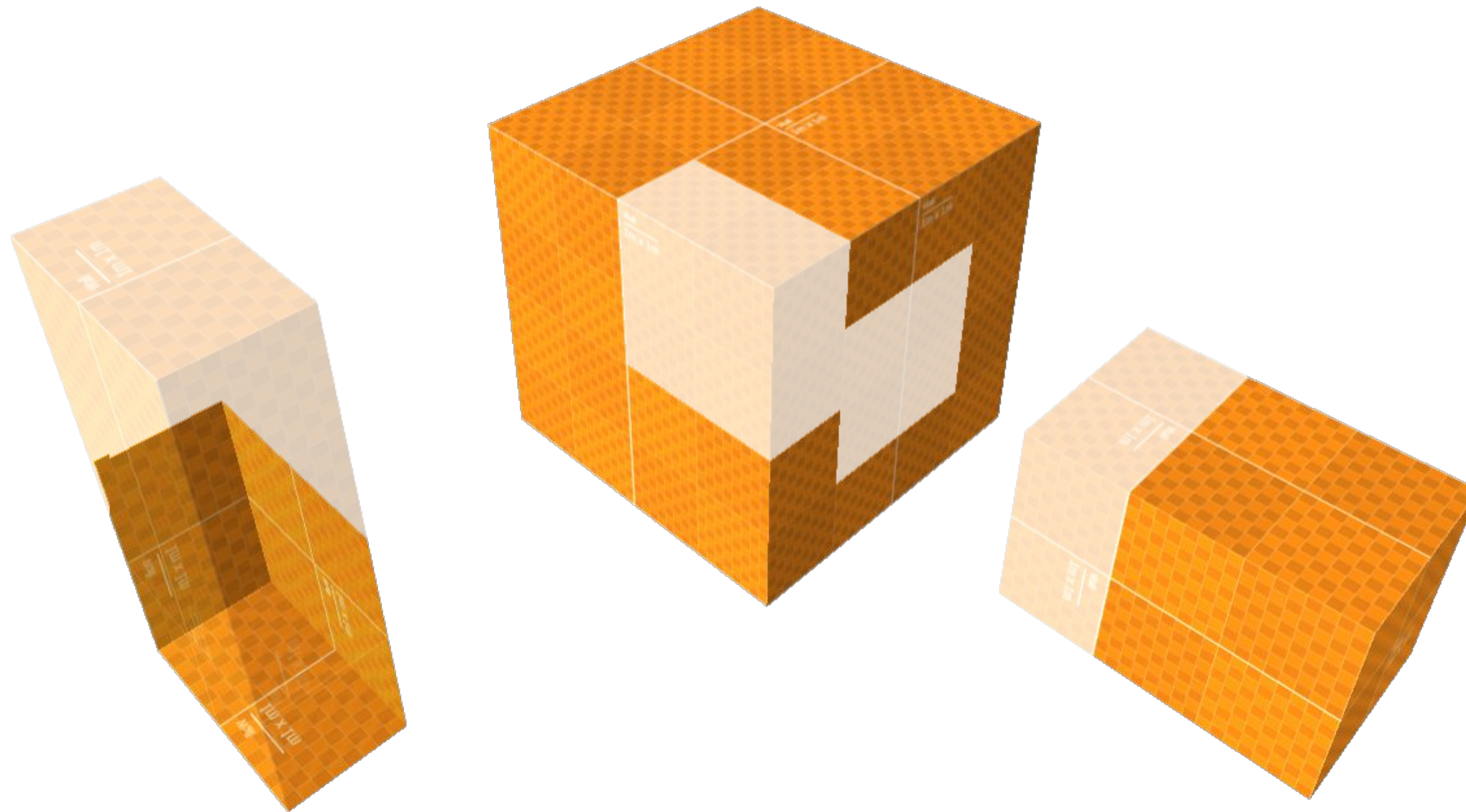
Routing tables  
are build  
**individually** for  
each brush

For example,  
this table is the  
routing table for  
**brush B**

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |







Which polygon piece is what,  
to the ***entire generated mesh?***

Routing table **for brush B**

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 2           | 3       |
| Brush B | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |

All categories of the brush this routing table is made for will always be aligned

Brush B can be optimized away

Routing table **for brush B**

All categories of the brush this routing table is made for will always be aligned

Brush B can be optimized away

|                    | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|--------------------|---------------|--------|---------|-------------|---------|
| Brush A            | 0             | 0      | 1       | 2           | 3       |
| <del>Brush B</del> | 0             | 0      | 0       | 0           | 0       |
|                    | 1             | 0      | 1       | 0           | 1       |
|                    | 2             | 0      | 0       | 2           | 2       |
|                    | 3             | 0      | 1       | 2           | 3       |
| Brush C            | 0             | 0      | 0       | 0           | 0       |
|                    | 1             | 0      | 1       | 0           | 1       |
|                    | 2             | 0      | 0       | 2           | 2       |
|                    | 3             | 0      | 1       | 2           | 3       |

Routing table **for brush B**

|         | Polygon Index | Inside | Aligned | Rev Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 0           | 1       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |

All outputs of **brush B**  
Are rewired to  
the output of  
**brush A**

This allows  
us to **remove**  
**brush B**



Routing table **for brush B**

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 0           | 1       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |

Not all outputs  
of **brush A** lead  
to **brush C**

We can remove  
outputs that we'll  
never use

Routing table **for brush B**

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 0           | 1       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |
|         |               |        |         |             |         |

Not all outputs  
of **brush A** lead  
to **brush C**

We can remove  
outputs that we'll  
never use

Routing table **for brush B**

When optimizing routing tables make sure all indices are **sequential** and **start with 0**

|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 1      | 3       | 1           | 3       |
| Brush B | 1             | 0      | 1       | 0           | 1       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush D | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |



Routing table **for brush B**

Remap outputs  
of previous  
brush to new  
rows to **fix gaps**

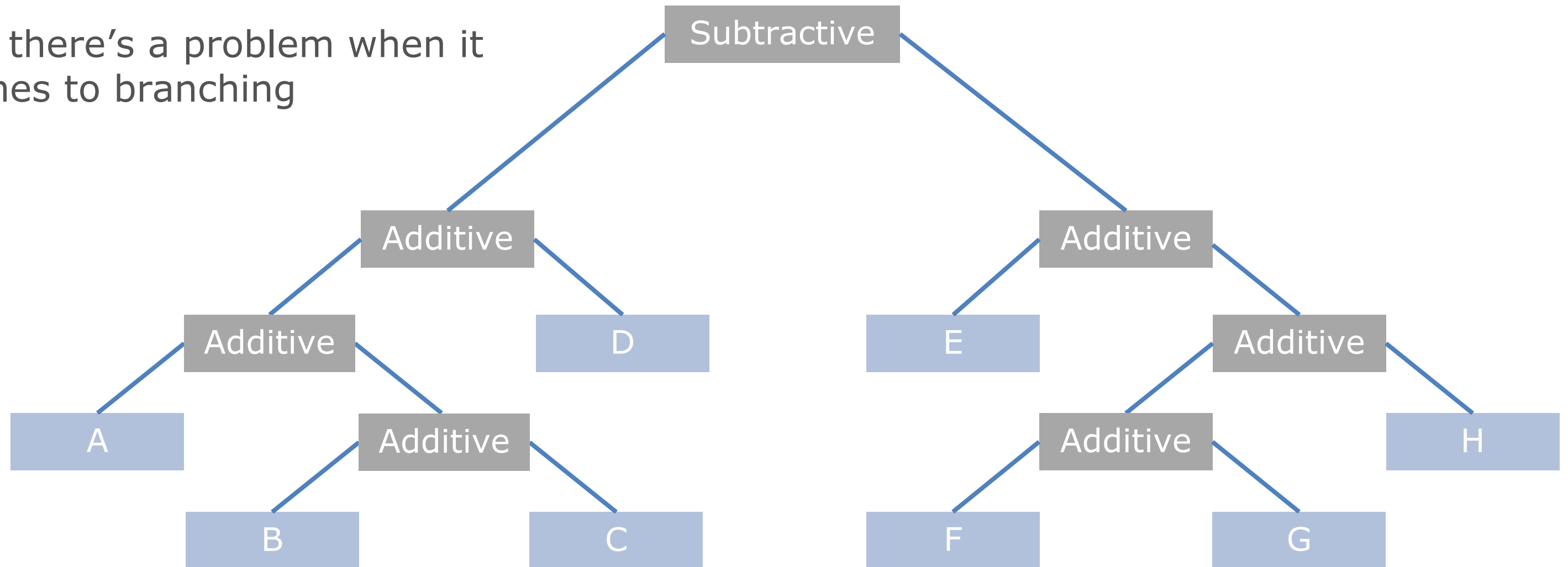
|         | Polygon Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|---------------|--------|---------|-------------|---------|
| Brush A | 0             | 0      | 1       | 0           | 1       |
| Brush B | 0             | 0      | 1       | 0           | 1       |
|         | 1             | 0      | 1       | 2           | 3       |
| Brush C | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |
|         | 2             | 0      | 0       | 2           | 2       |
|         | 3             | 0      | 1       | 2           | 3       |
| Brush D | 0             | 0      | 0       | 0           | 0       |
|         | 1             | 0      | 1       | 0           | 1       |





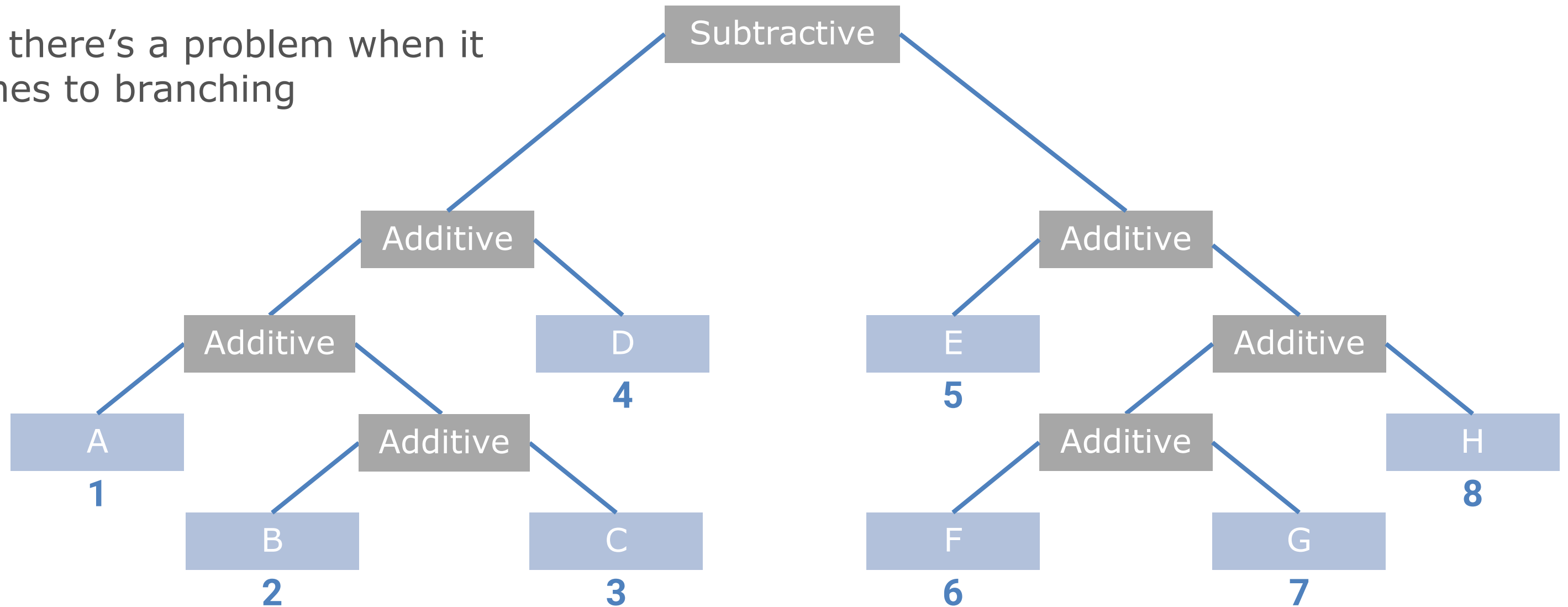
# Branching

But there's a problem when it comes to branching



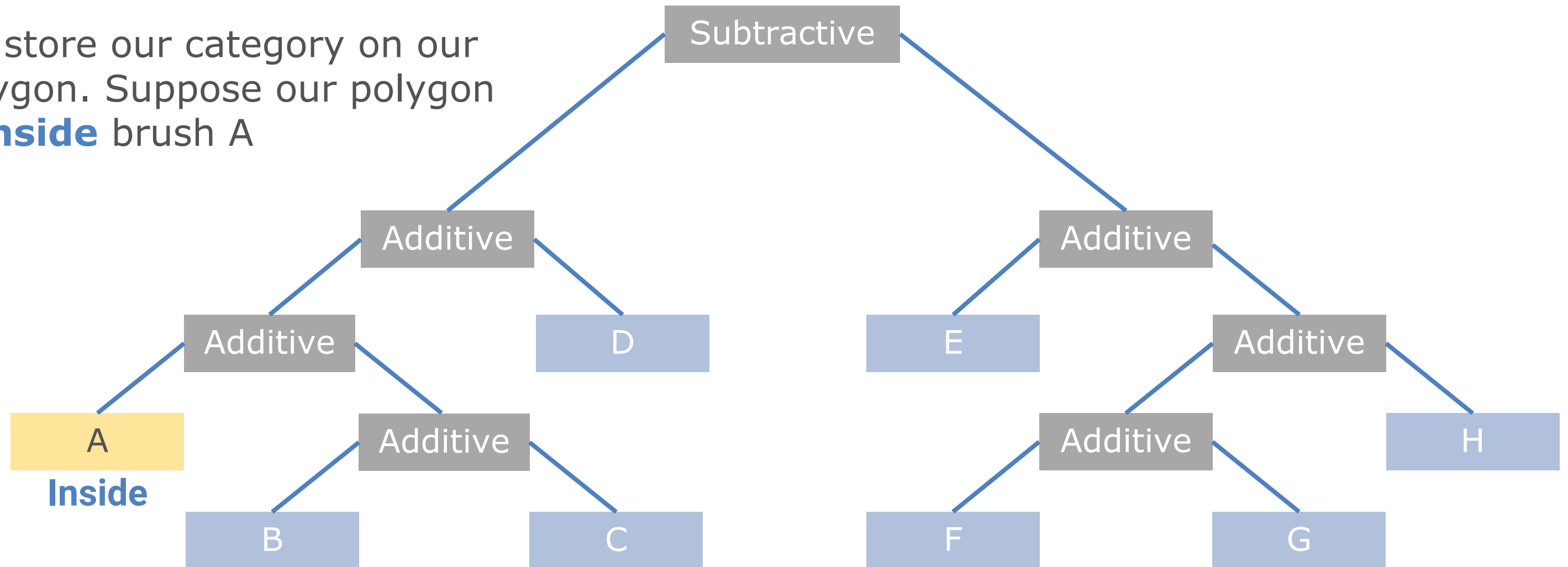
# Branching

But there's a problem when it comes to branching



## Branching

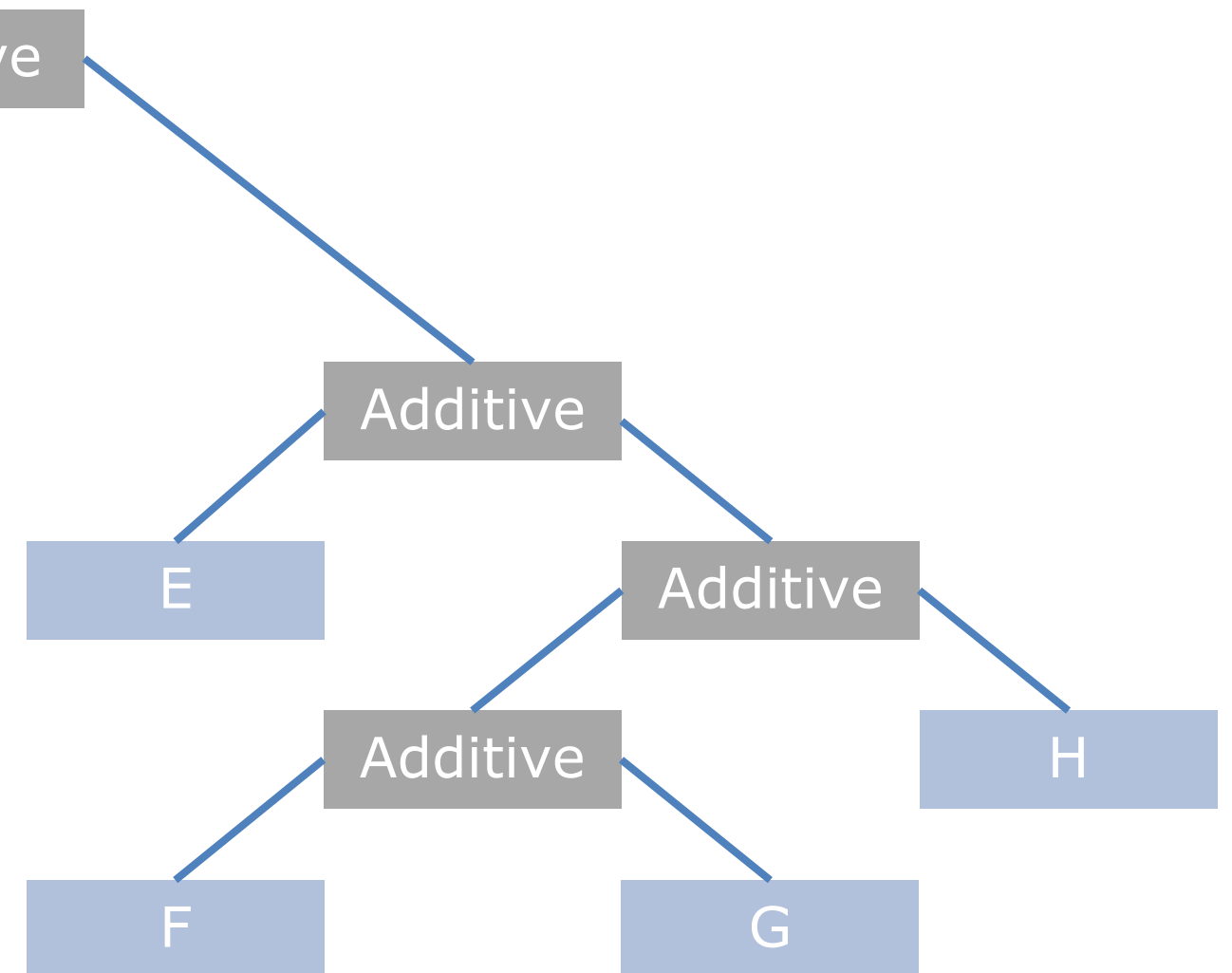
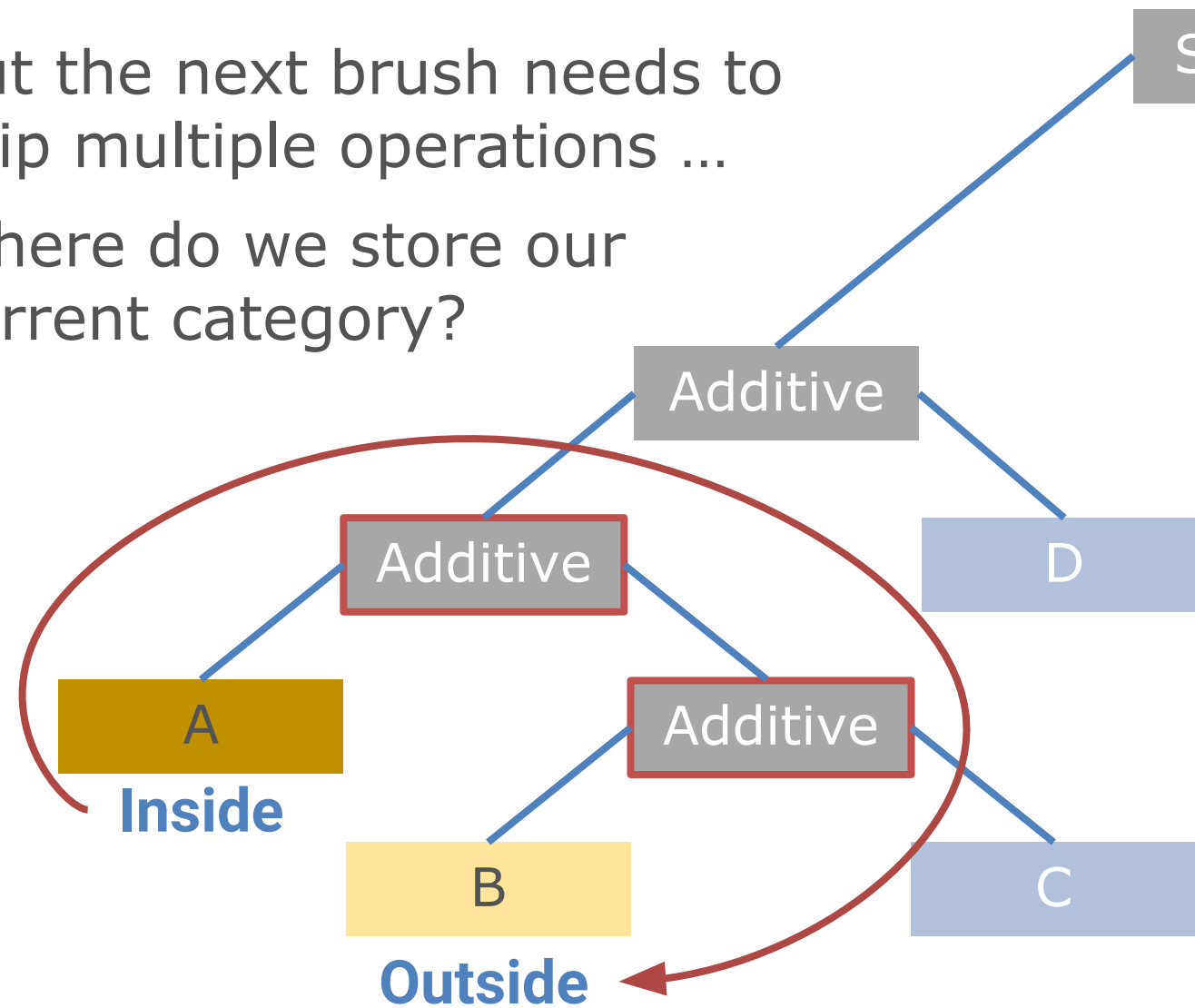
We store our category on our polygon. Suppose our polygon is **inside** brush A



# Branching

But the next brush needs to  
skip multiple operations ...

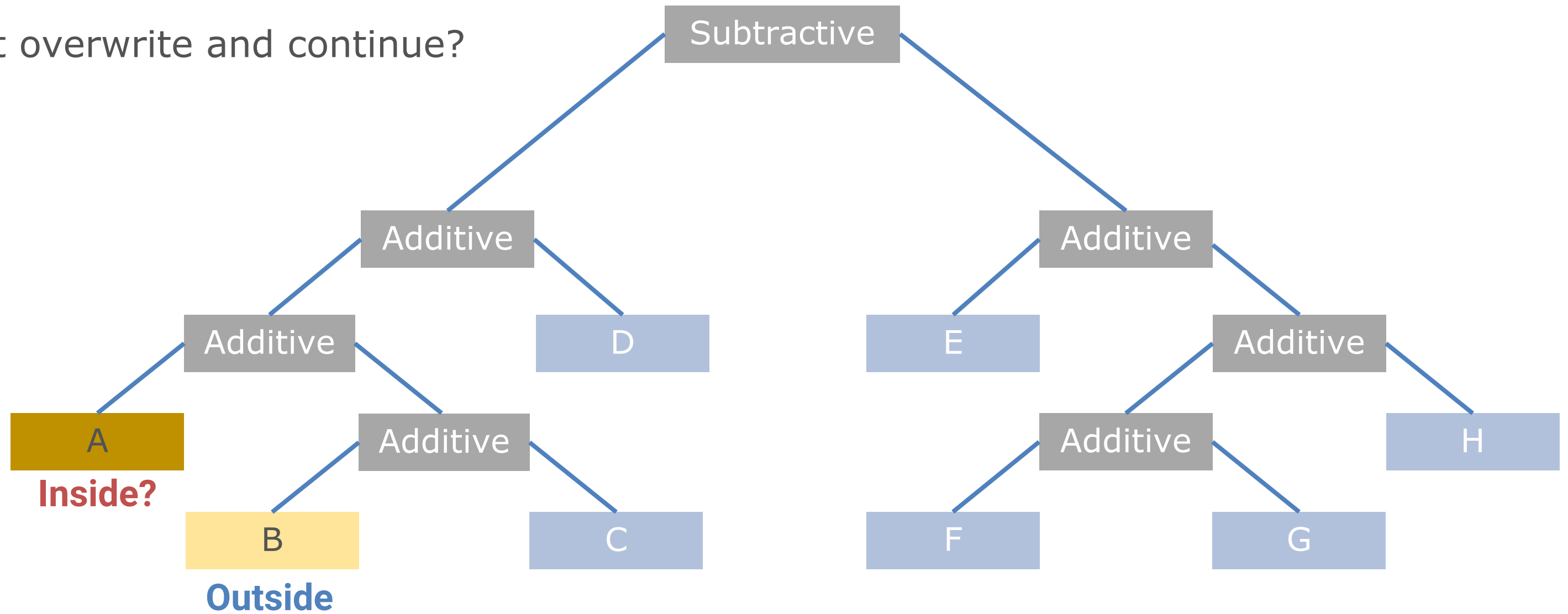
Where do we store our  
current category?





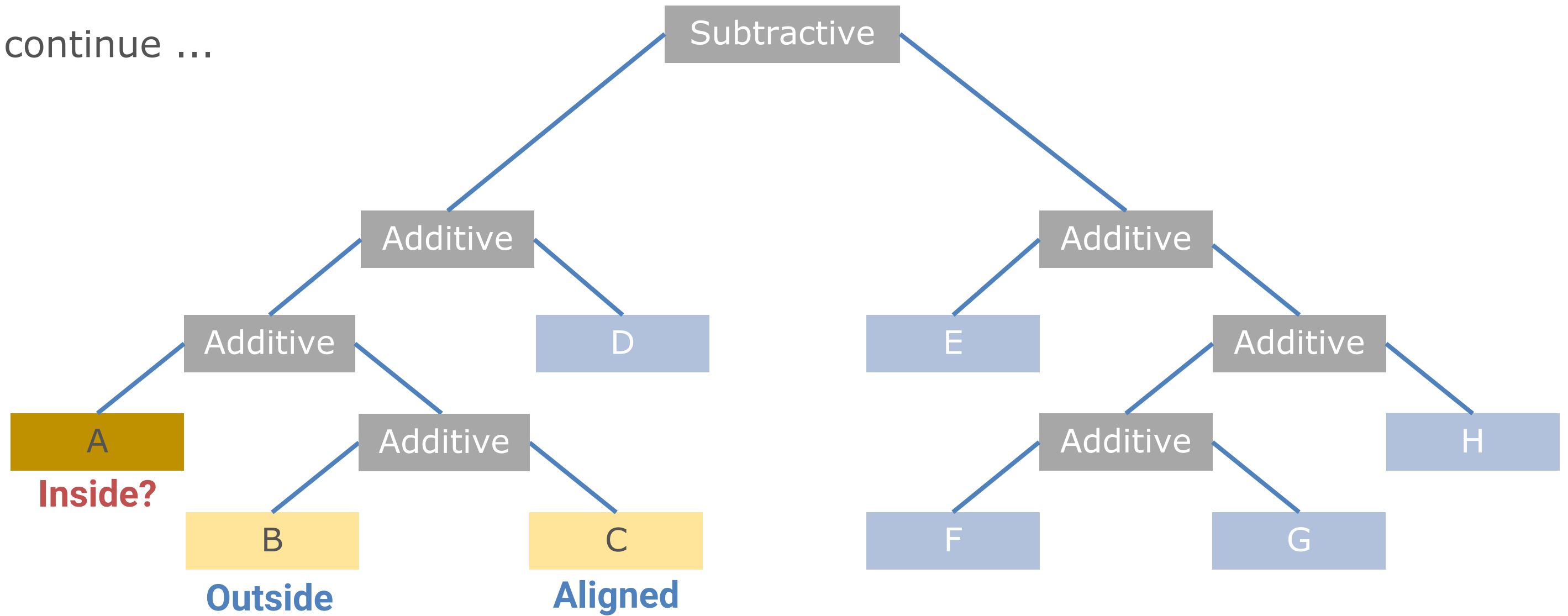
# Branching

Just overwrite and continue?



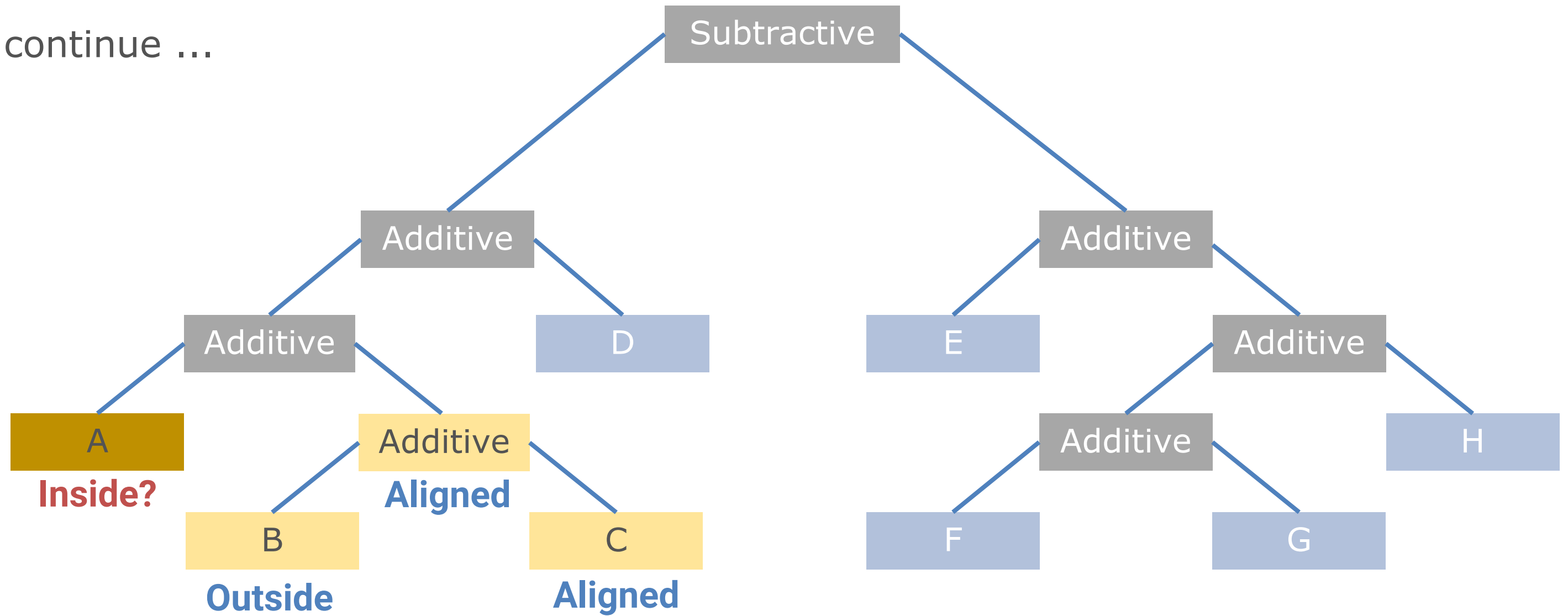
# Branching

We continue ...



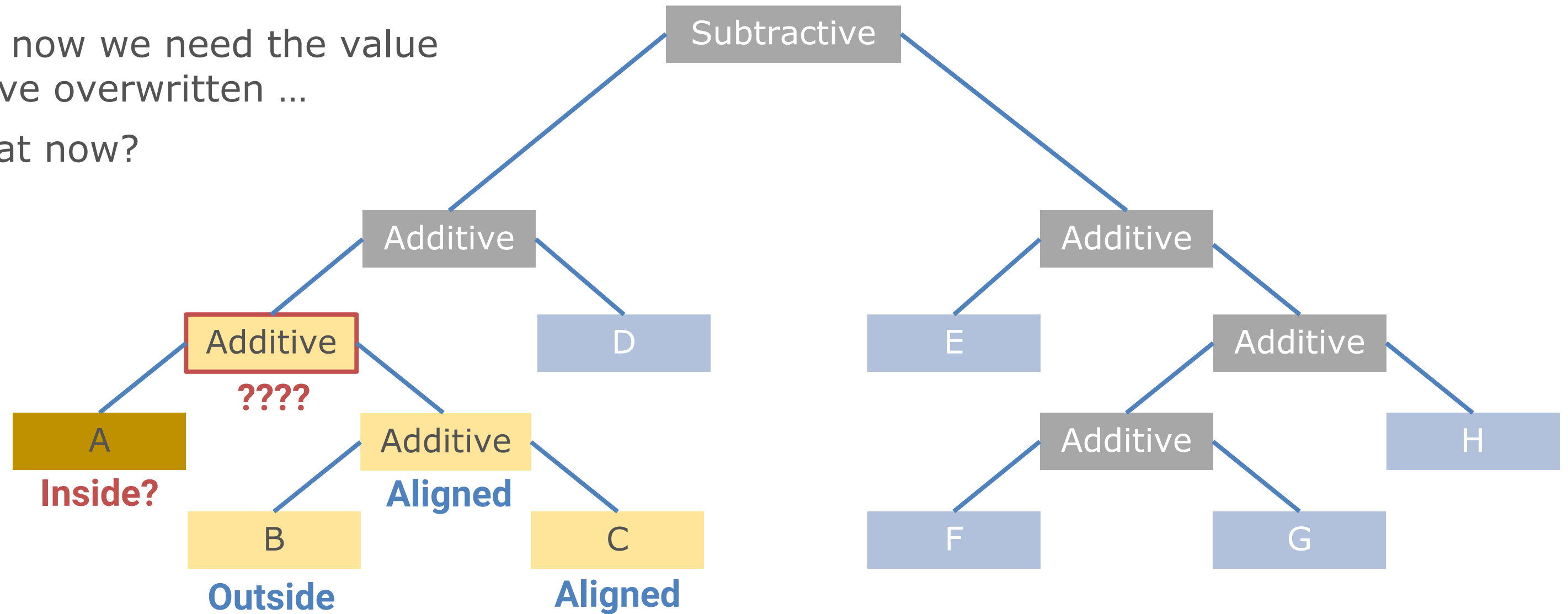
# Branching

We continue ...



# Branching

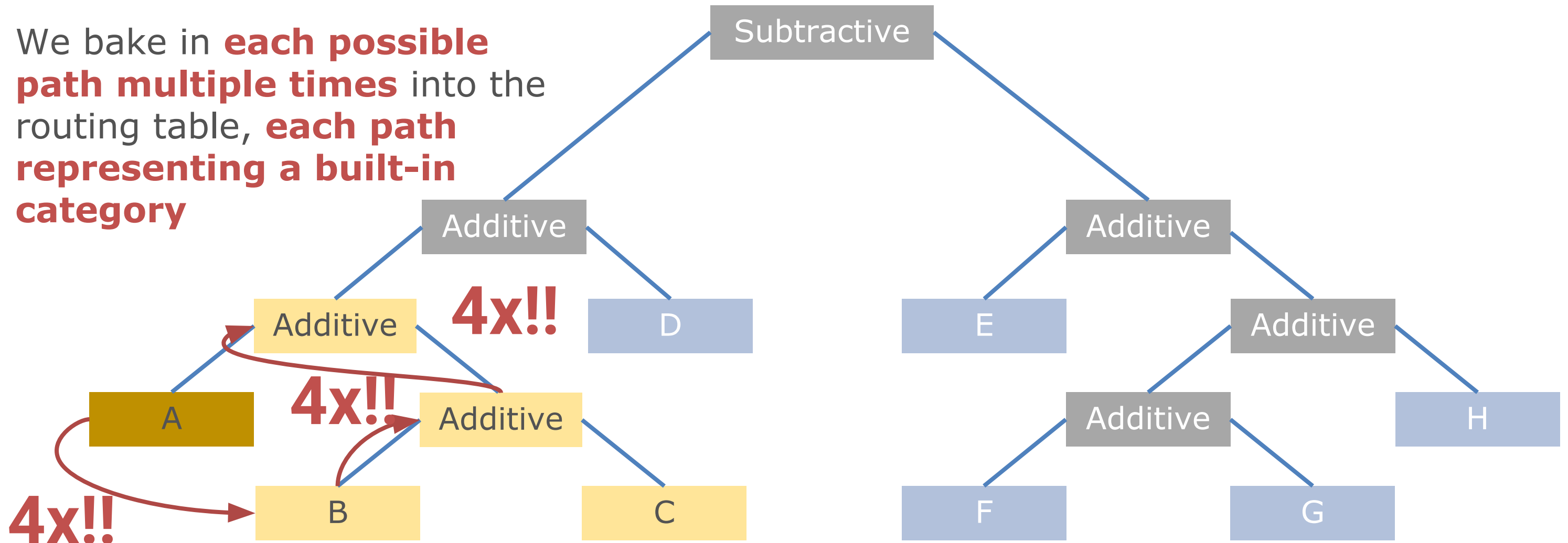
But now we need the value  
we've overwritten ...  
What now?

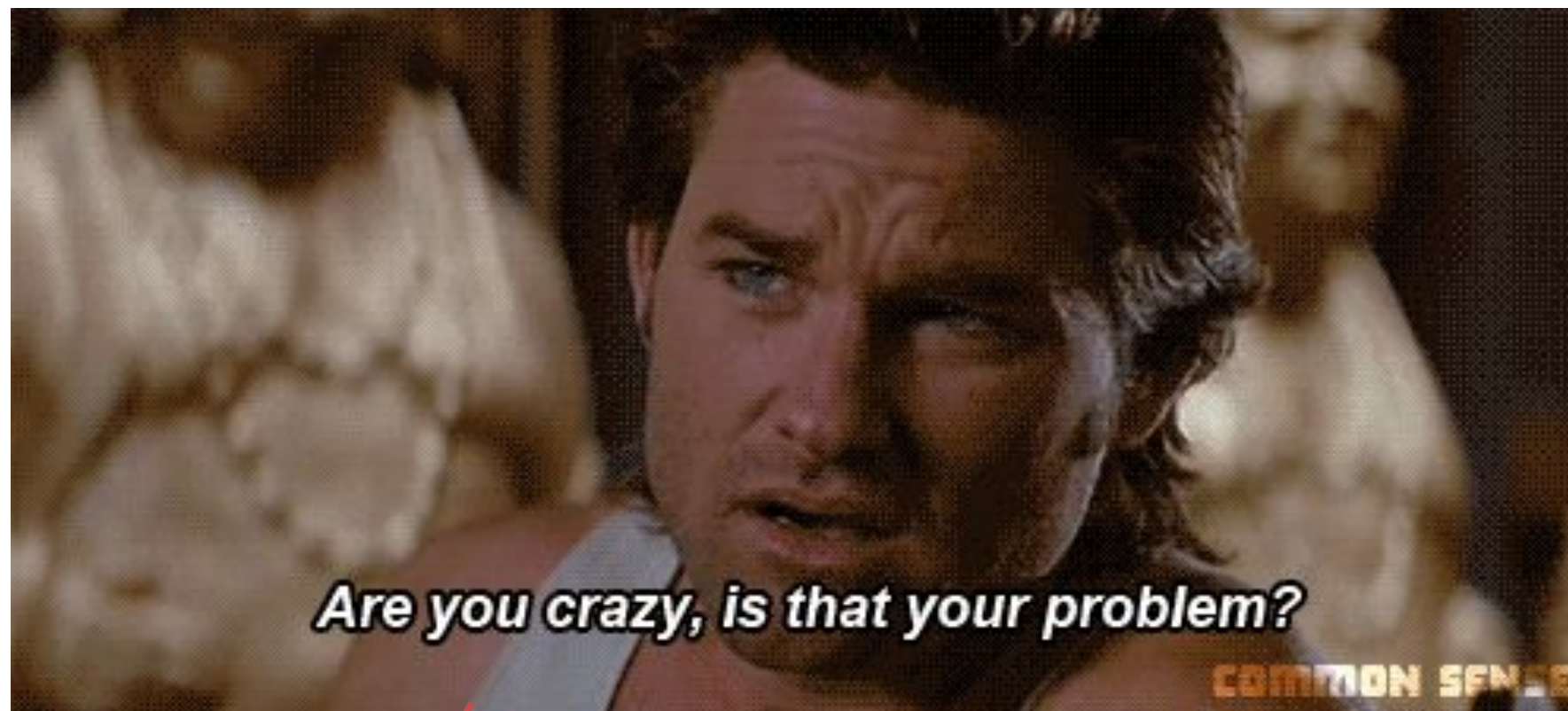




## Branching

We bake in **each possible path multiple times** into the routing table, **each path representing a built-in category**

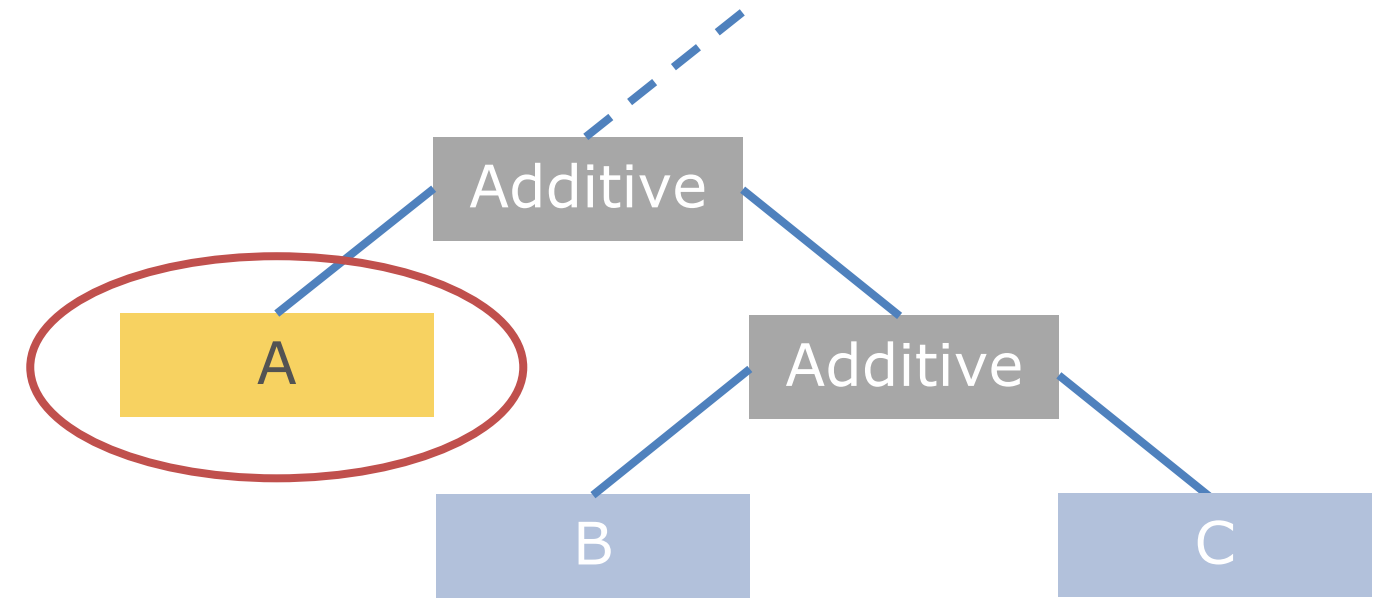




It's not as bad as it may seem™

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |

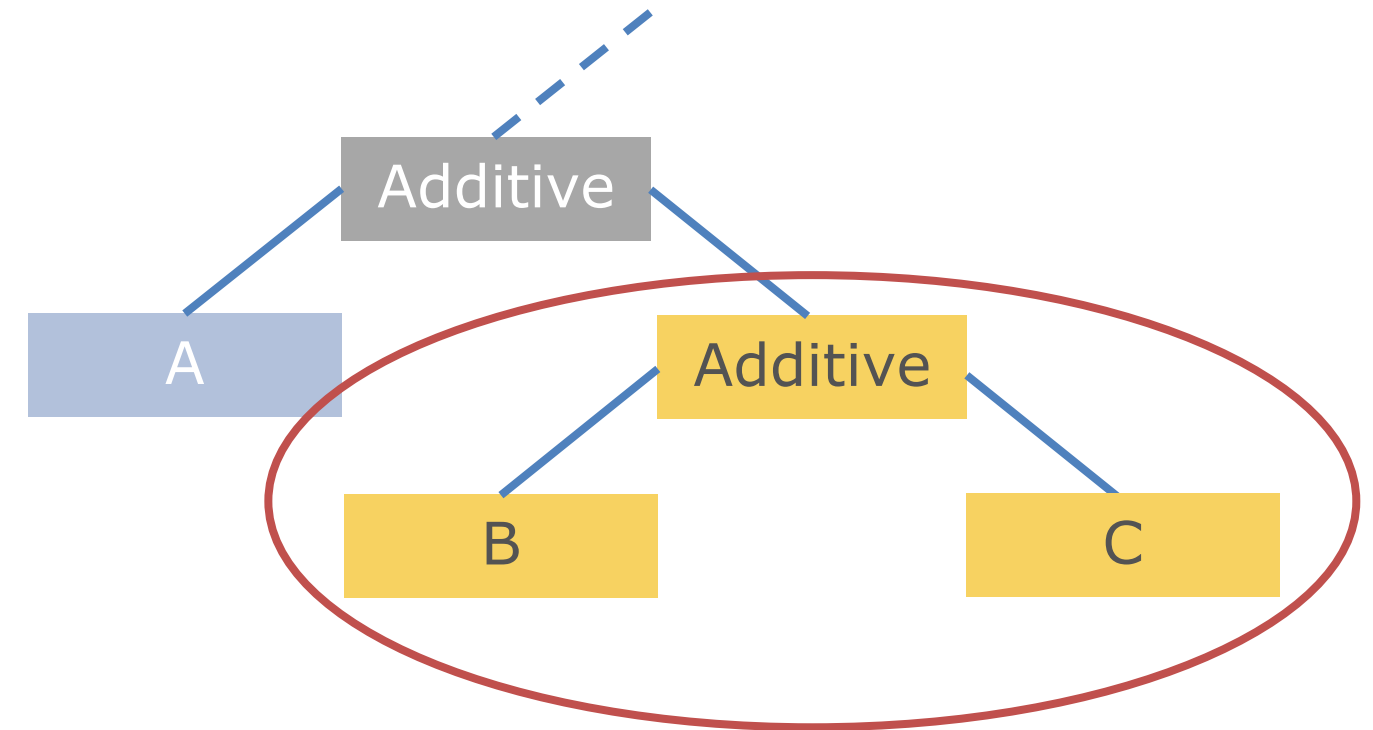




## Routing table

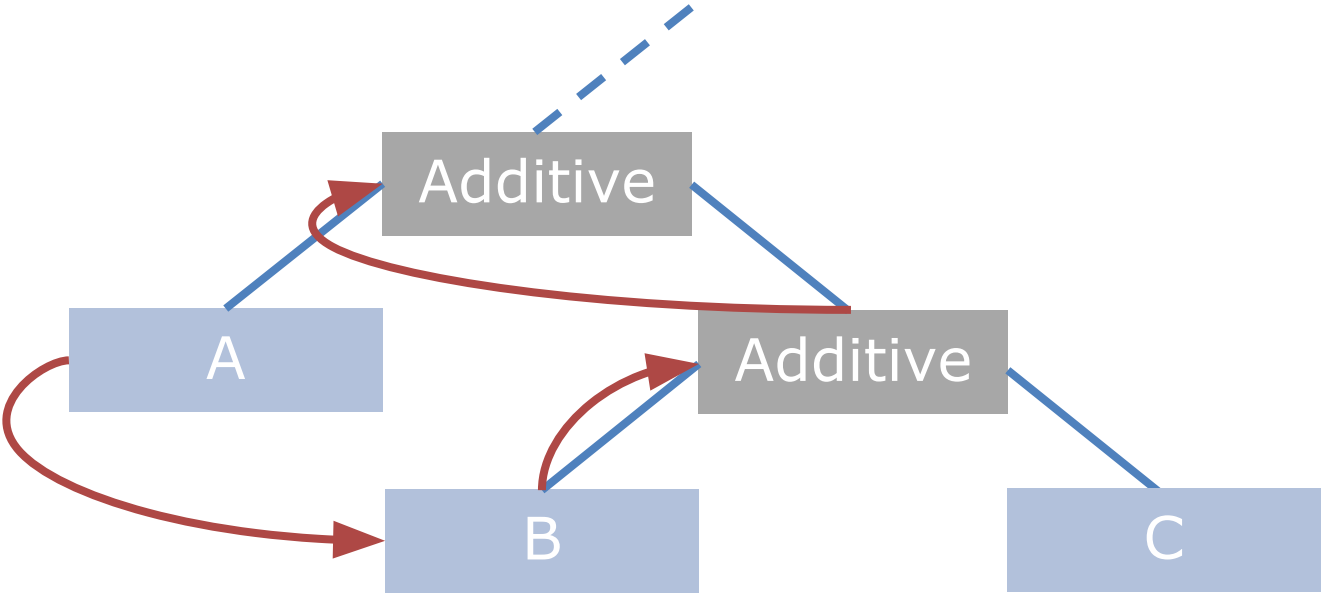
|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush B | 0     | 0      | 1       | 2           | 3       |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 1       | 0           | 1       |
|         | 2     | 0      | 0       | 2           | 2       |
|         | 3     | 0      | 1       | 2           | 3       |



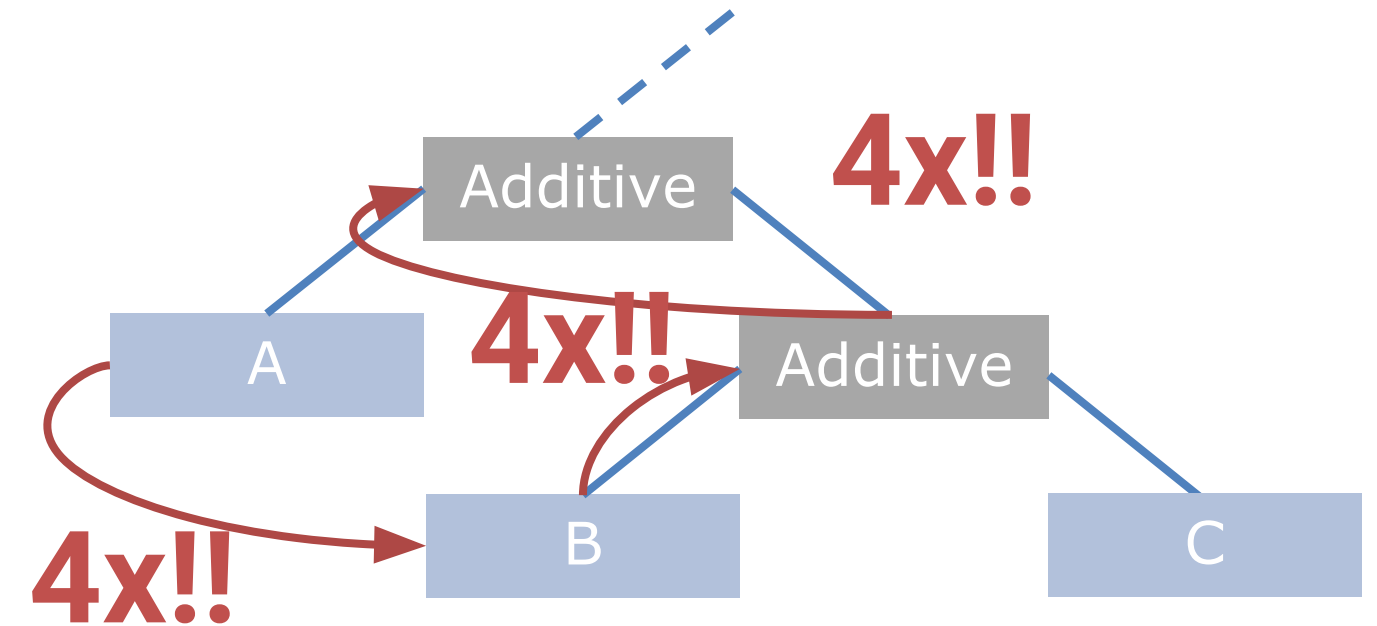
Routing table

|         | index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 1       | 2           | 3       |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 1       | 0           | 1       |
|         | 2     | 0      | 0       | 2           | 2       |
|         | 3     | 0      | 1       | 2           | 3       |



## Routing table

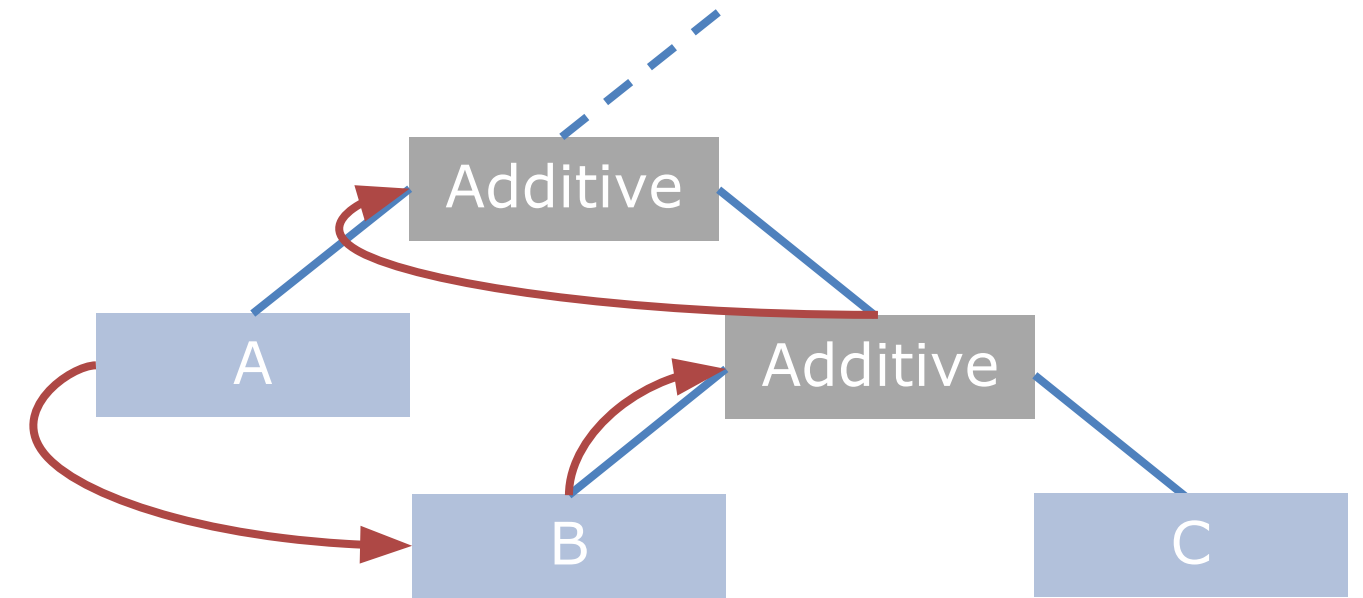
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        |       |        |         |             |         |
| (Rev-Aligned)    |       |        |         |             |         |
| (Outside)        |       |        |         |             |         |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Rev-Aligned)    |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Outside)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |



Duplicate all rows of all brushes in second routing table, once for each category

## Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 0     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 0     | 0      | 1       | 2           | 3       |
| (Outside)        | 0     | 0      | 1       | 2           | 3       |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Rev-Aligned)    |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Outside)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |

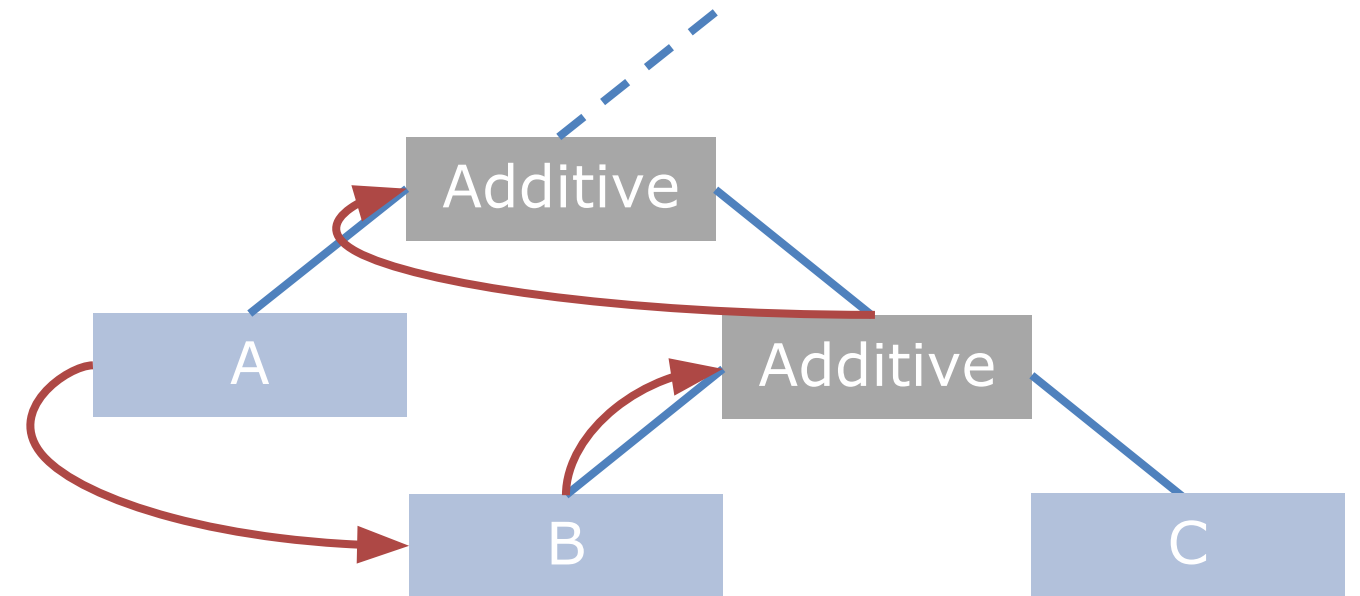


Duplicate all rows of all brushes in second routing table, once for each category



# Routing table

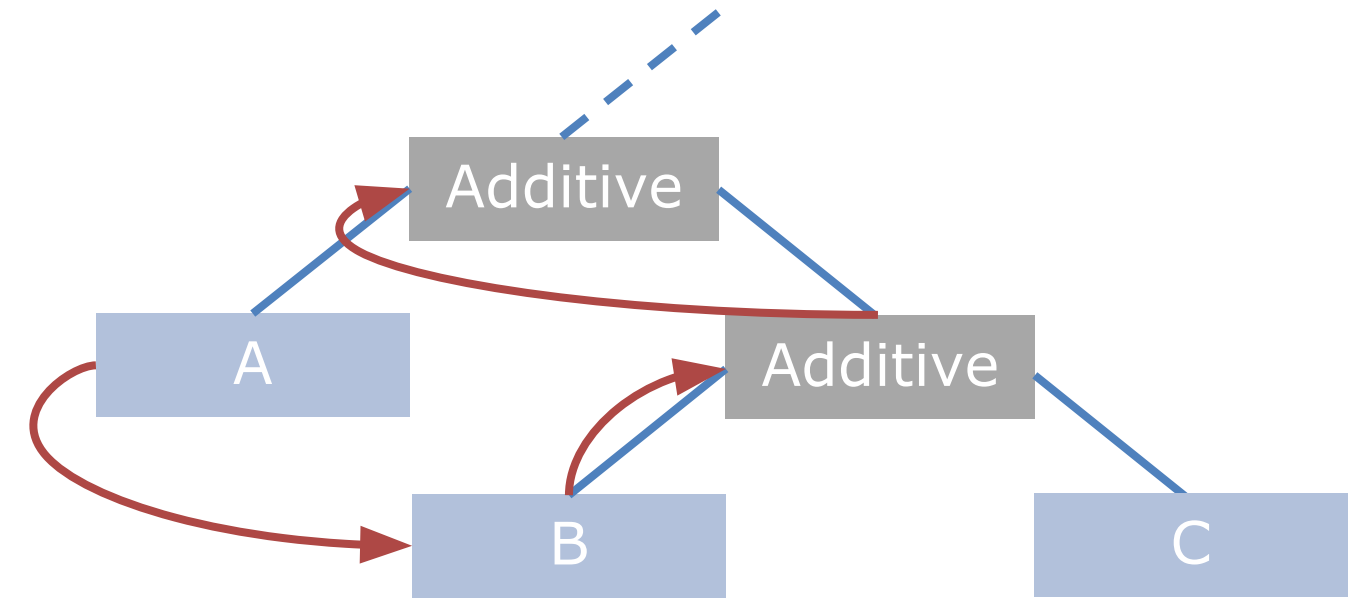
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 2     | 0      | 1       | 2           | 3       |
| (Outside)        | 3     | 0      | 1       | 2           | 3       |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Rev-Aligned)    |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Outside)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |



Give each row an unique index

# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Rev-Aligned)    |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |
| (Outside)        |       |        |         |             |         |
|                  |       |        |         |             |         |
|                  |       |        |         |             |         |



Make every output unique,  
add 4 for each duplicated row

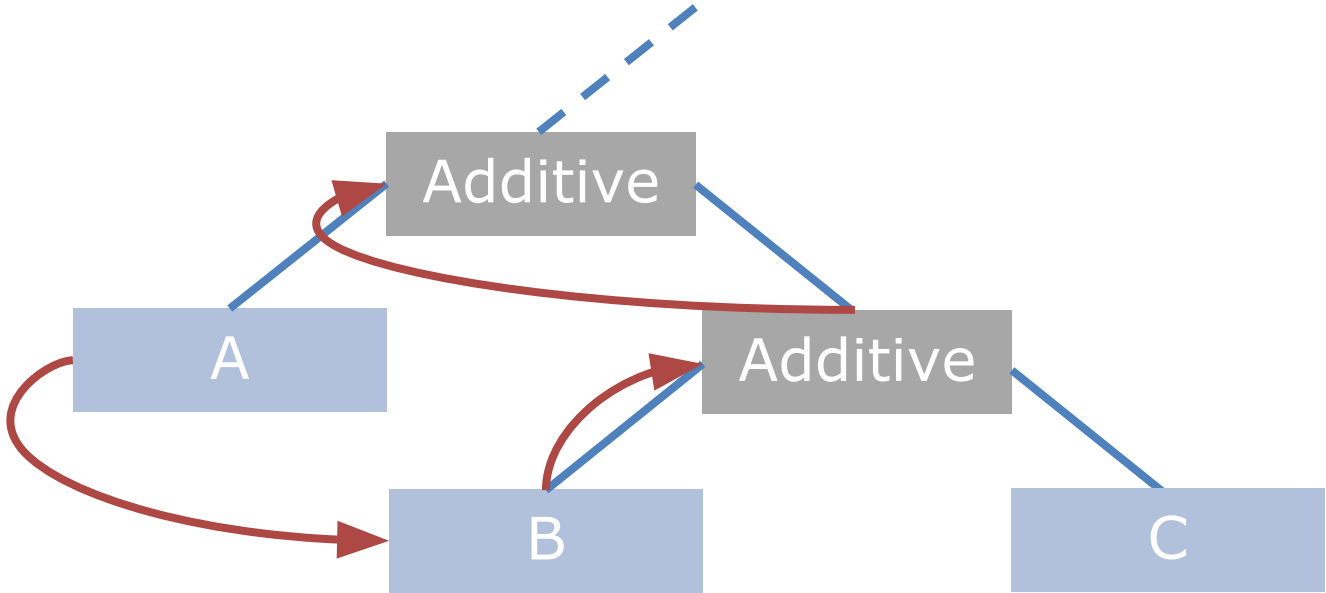
This gives is unique sequential  
values for all outputs

Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

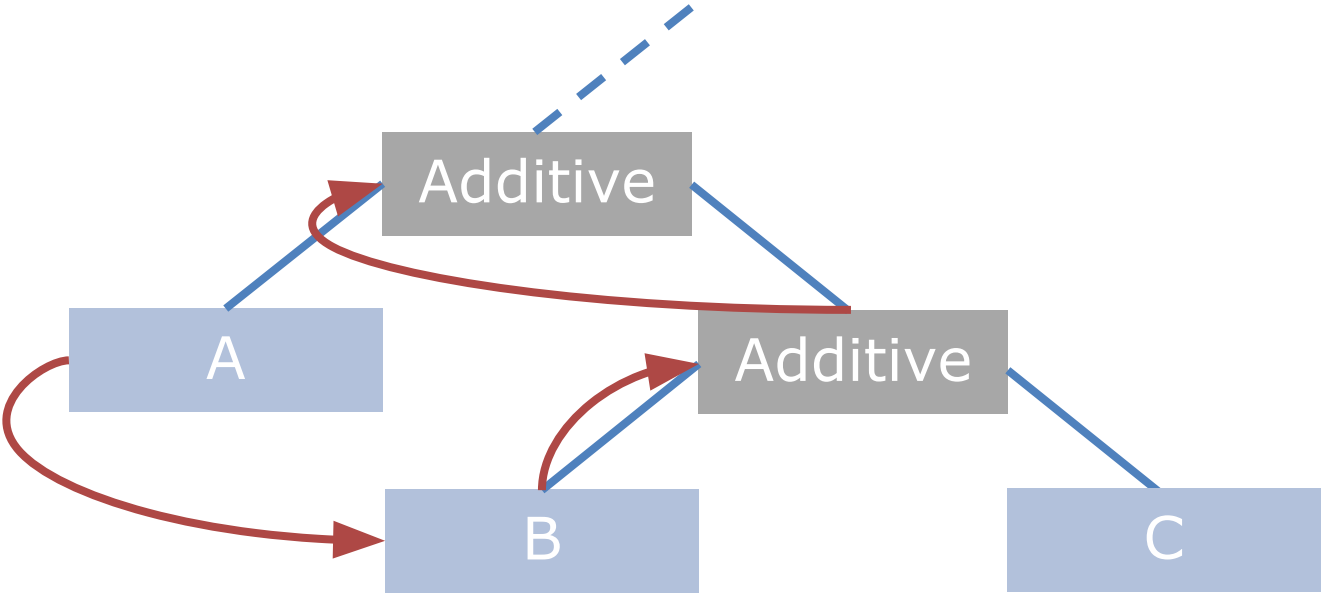
last brush  
in table

final output  
(don't modify output values)



# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

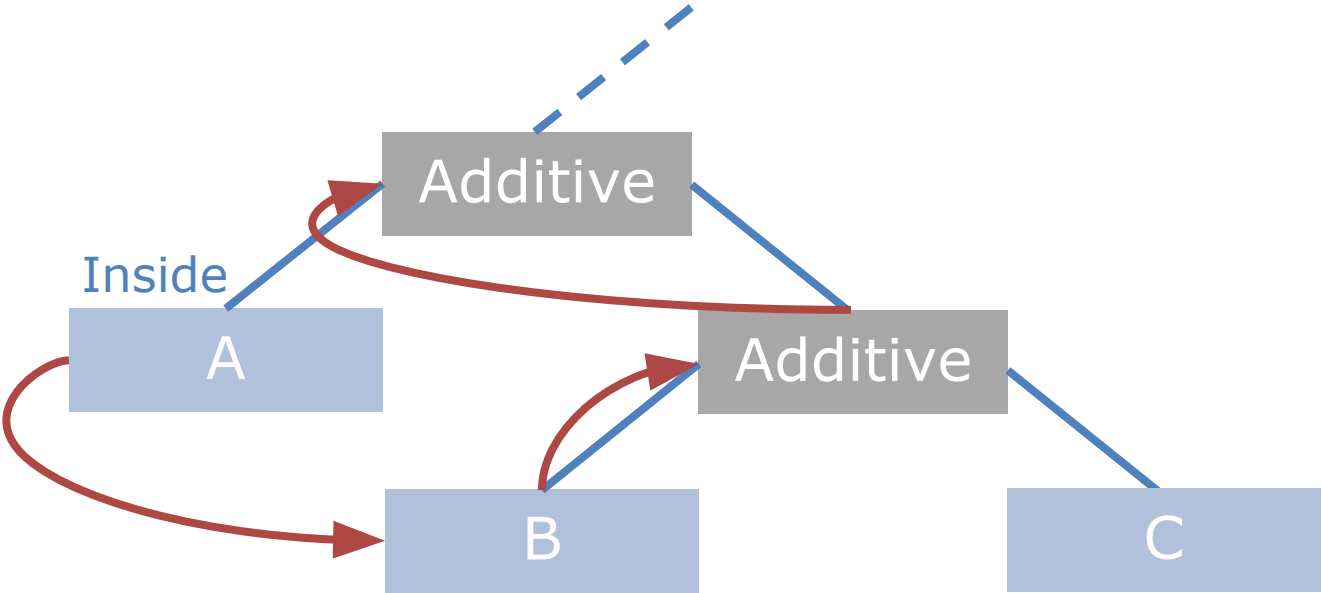


Each path from brush A represents a brush A category



Routing table

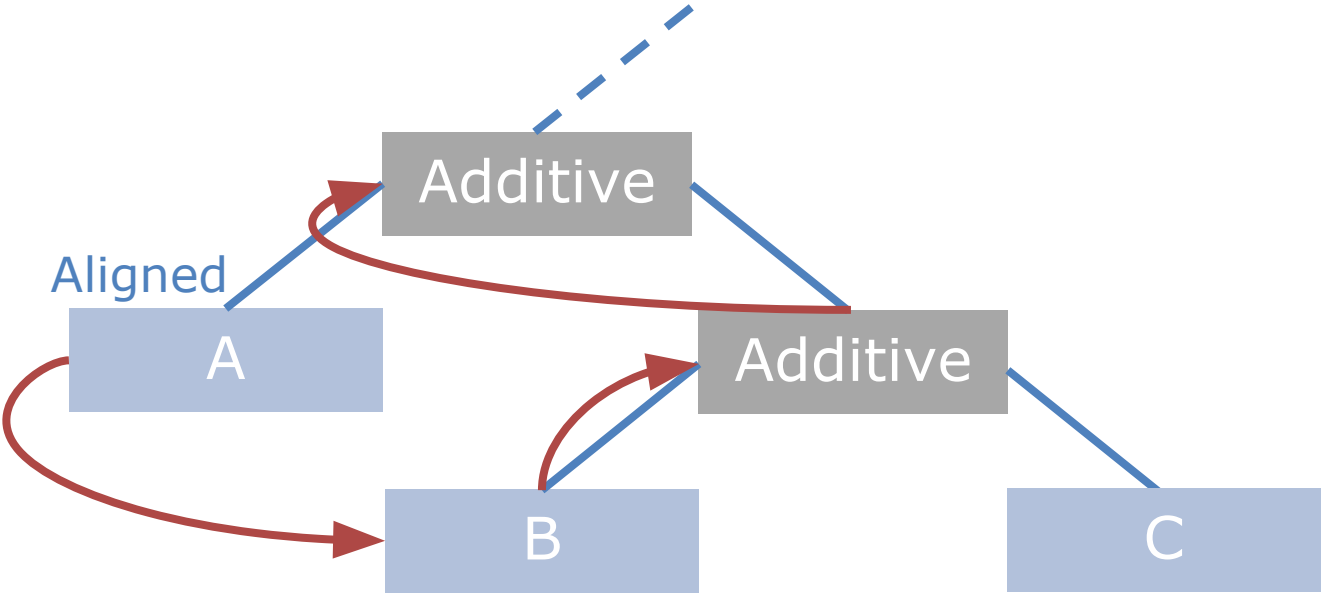
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



Each path from brush A represents a brush A category

# Routing table

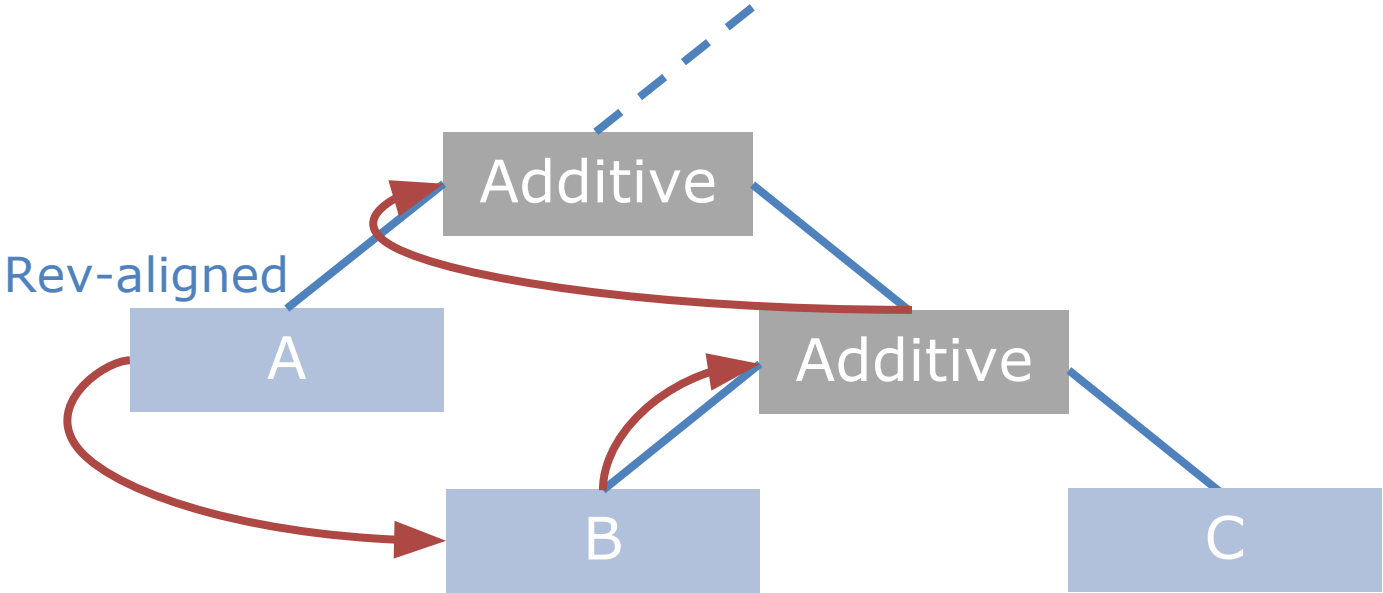
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



Each path from brush A represents a brush A category

# Routing table

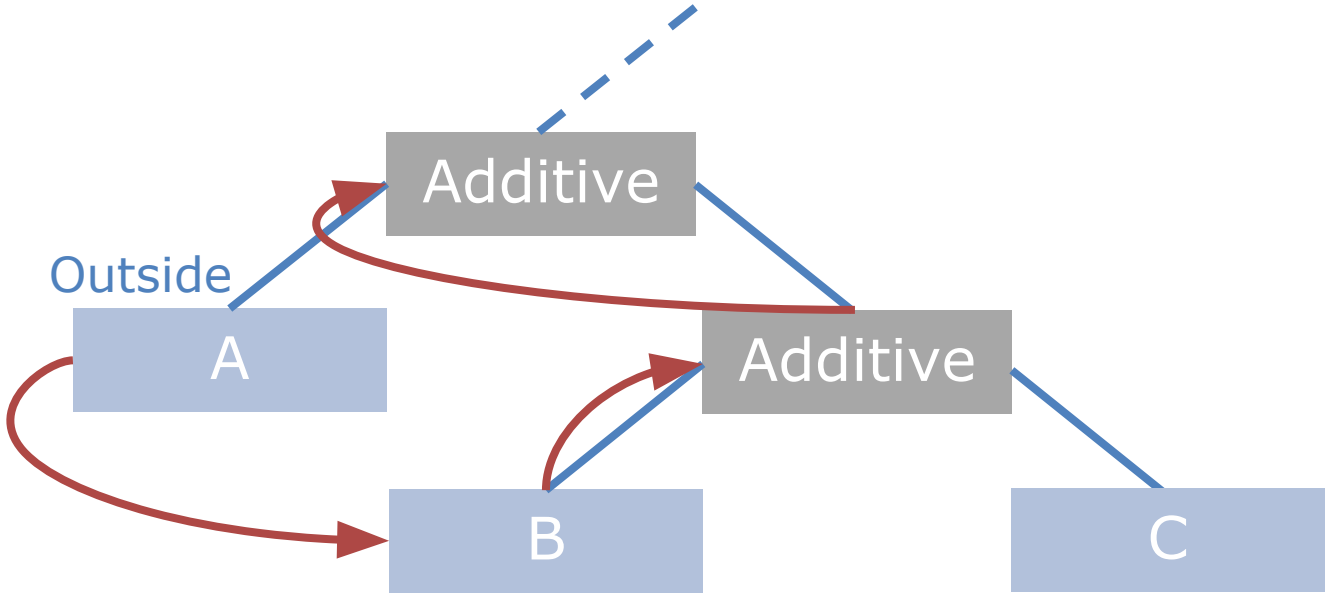
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



Each path from brush A represents a brush A category

# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

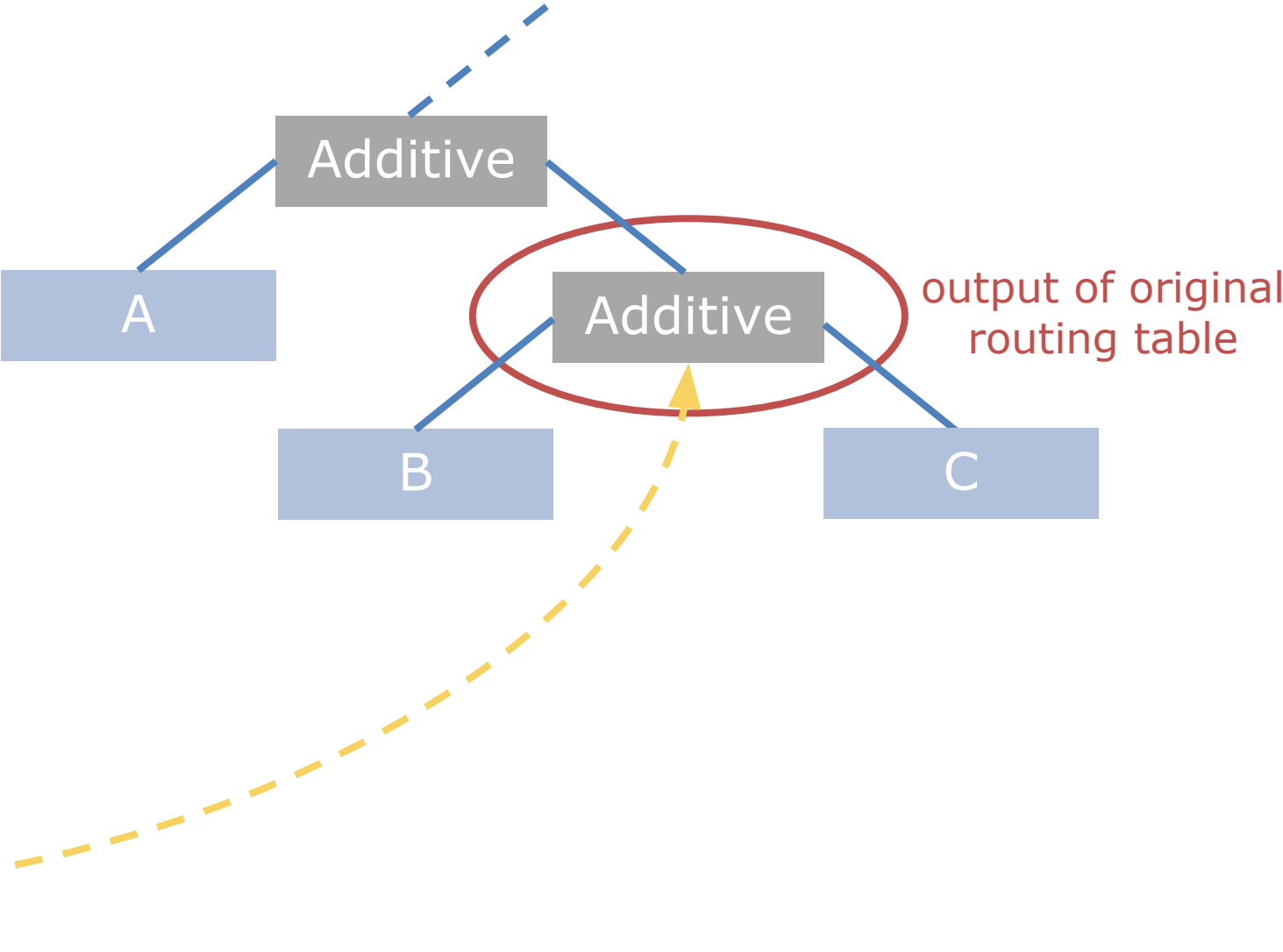


Each path from brush A represents a brush A category



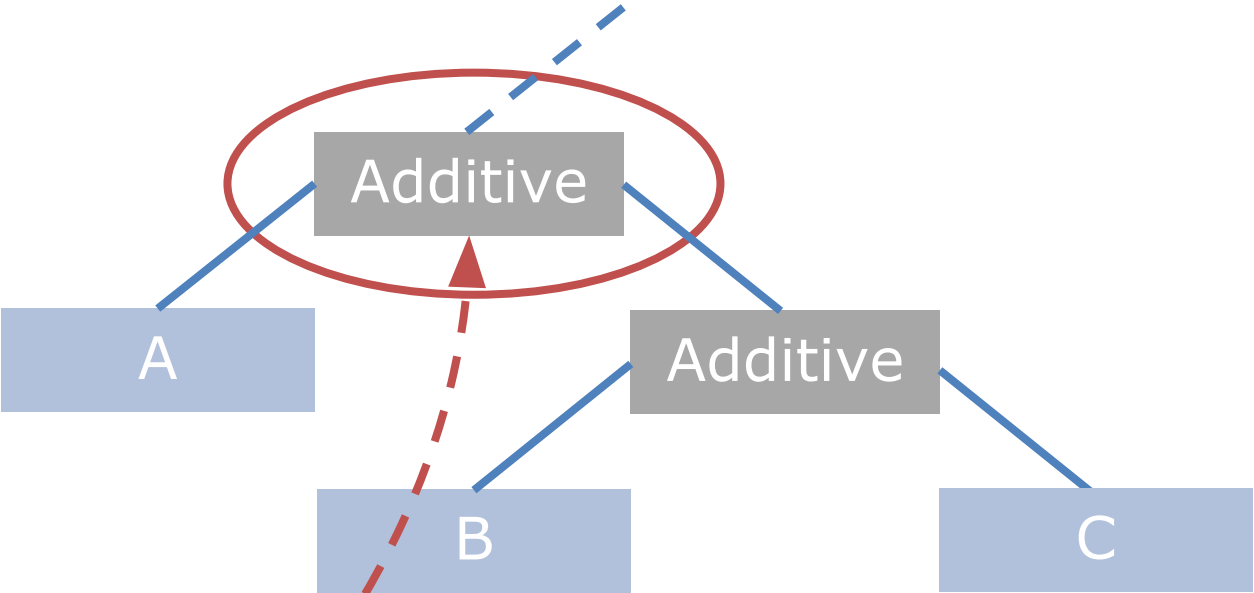
# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

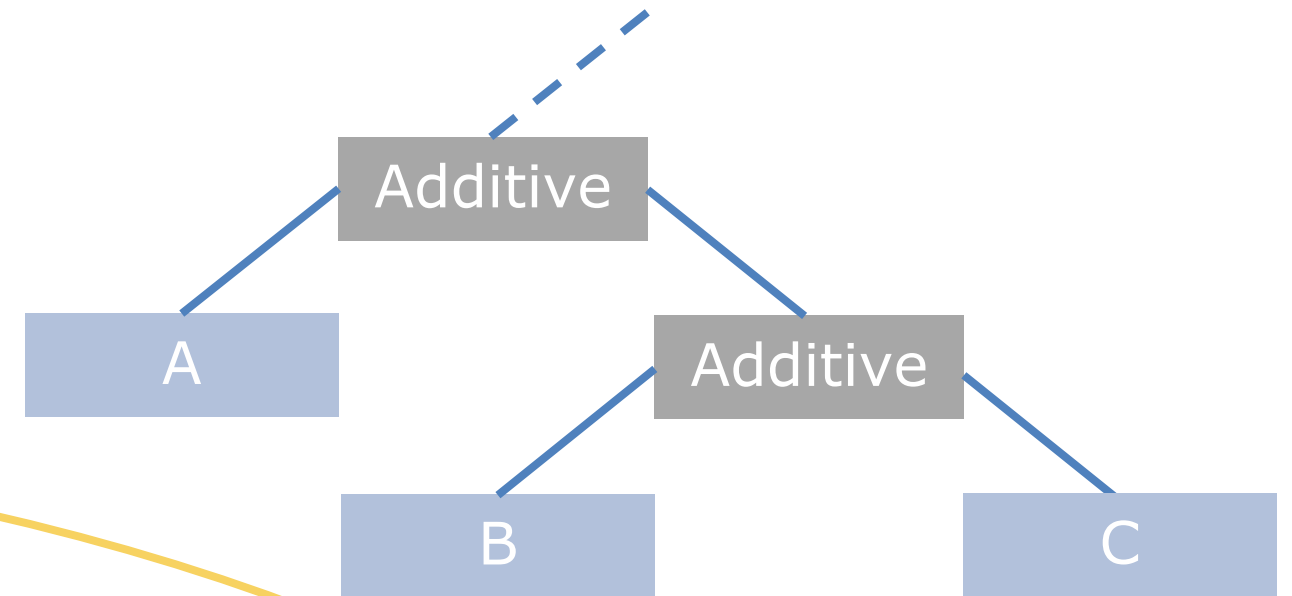


|             | Inside | Aligned | Rev-Aligned | Outside     |
|-------------|--------|---------|-------------|-------------|
| Inside      | Inside | Inside  | Inside      | Inside      |
| Aligned     | Inside | Aligned | Inside      | Aligned     |
| Rev-Aligned | Inside | Inside  | Rev-Aligned | Rev-Aligned |
| Outside     | Inside | Aligned | Rev-Aligned | Outside     |

Additive  
operation table

# Routing table

|               |         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------------|---------|-------|--------|---------|-------------|---------|
|               | Brush A | -     | 0      | 1       | 2           | 3       |
| (Inside)      | Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)     |         | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned) |         | 2     | 8      | 9       | 10          | 11      |
| (Outside)     |         | 3     | 12     | 13      | 14          | 15      |
| (Inside)      | Brush C | 0     | 0      | 0       | 0           | 0       |
|               |         | 1     | 0      | 1       | 0           | 0       |
|               |         | 2     | 0      | 0       | 2           | 2       |
|               |         | 3     | 0      | 1       | 2           | 3       |
| (Aligned)     |         | 4     | 0      | 0       | 0           | 0       |
|               |         | 5     | 0      | 1       | 0           | 0       |
|               |         | 6     | 0      | 0       | 2           | 2       |
|               |         | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned) |         | 8     | 0      | 0       | 0           | 0       |
|               |         | 9     | 0      | 1       | 0           | 0       |
|               |         | 10    | 0      | 0       | 2           | 2       |
|               |         | 11    | 0      | 1       | 2           | 3       |
| (Outside)     |         | 12    | 0      | 0       | 0           | 0       |
|               |         | 13    | 0      | 1       | 0           | 0       |
|               |         | 14    | 0      | 0       | 2           | 2       |
|               |         | 15    | 0      | 1       | 2           | 3       |

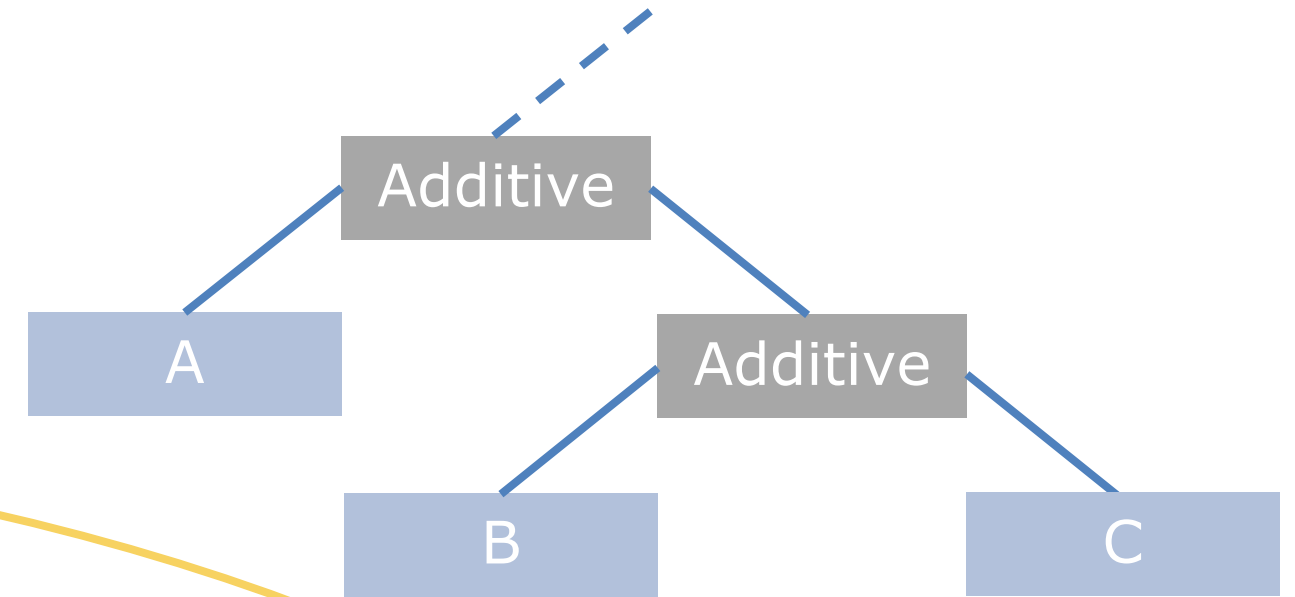


|             | Inside | Aligned | Rev-Aligned | Outside     |
|-------------|--------|---------|-------------|-------------|
| Inside      | Inside | Inside  | Inside      | Inside      |
| Aligned     | Inside | Aligned | Inside      | Aligned     |
| Rev-Aligned | Inside | Inside  | Rev-Aligned | Rev-Aligned |
| Outside     | Inside | Aligned | Rev-Aligned | Outside     |

Additive  
operation table

# Routing table

|               |         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------------|---------|-------|--------|---------|-------------|---------|
|               | Brush A | -     | 0      | 1       | 2           | 3       |
| (Inside)      | Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)     |         | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned) |         | 2     | 8      | 9       | 10          | 11      |
| (Outside)     |         | 3     | 12     | 13      | 14          | 15      |
| (Inside)      | Brush C | 0     | 0      | 0       | 0           | 0       |
|               |         | 1     | 0      | 1       | 0           | 0       |
|               |         | 2     | 0      | 0       | 2           | 2       |
|               |         | 3     | 0      | 1       | 2           | 3       |
| (Aligned)     |         | 4     | 0      | 0       | 0           | 0       |
|               |         | 5     | 0      | 1       | 0           | 0       |
|               |         | 6     | 0      | 0       | 2           | 2       |
|               |         | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned) |         | 8     | 0      | 0       | 0           | 0       |
|               |         | 9     | 0      | 1       | 0           | 0       |
|               |         | 10    | 0      | 0       | 2           | 2       |
|               |         | 11    | 0      | 1       | 2           | 3       |
| (Outside)     |         | 12    | 0      | 0       | 0           | 0       |
|               |         | 13    | 0      | 1       | 0           | 0       |
|               |         | 14    | 0      | 0       | 2           | 2       |
|               |         | 15    | 0      | 1       | 2           | 3       |

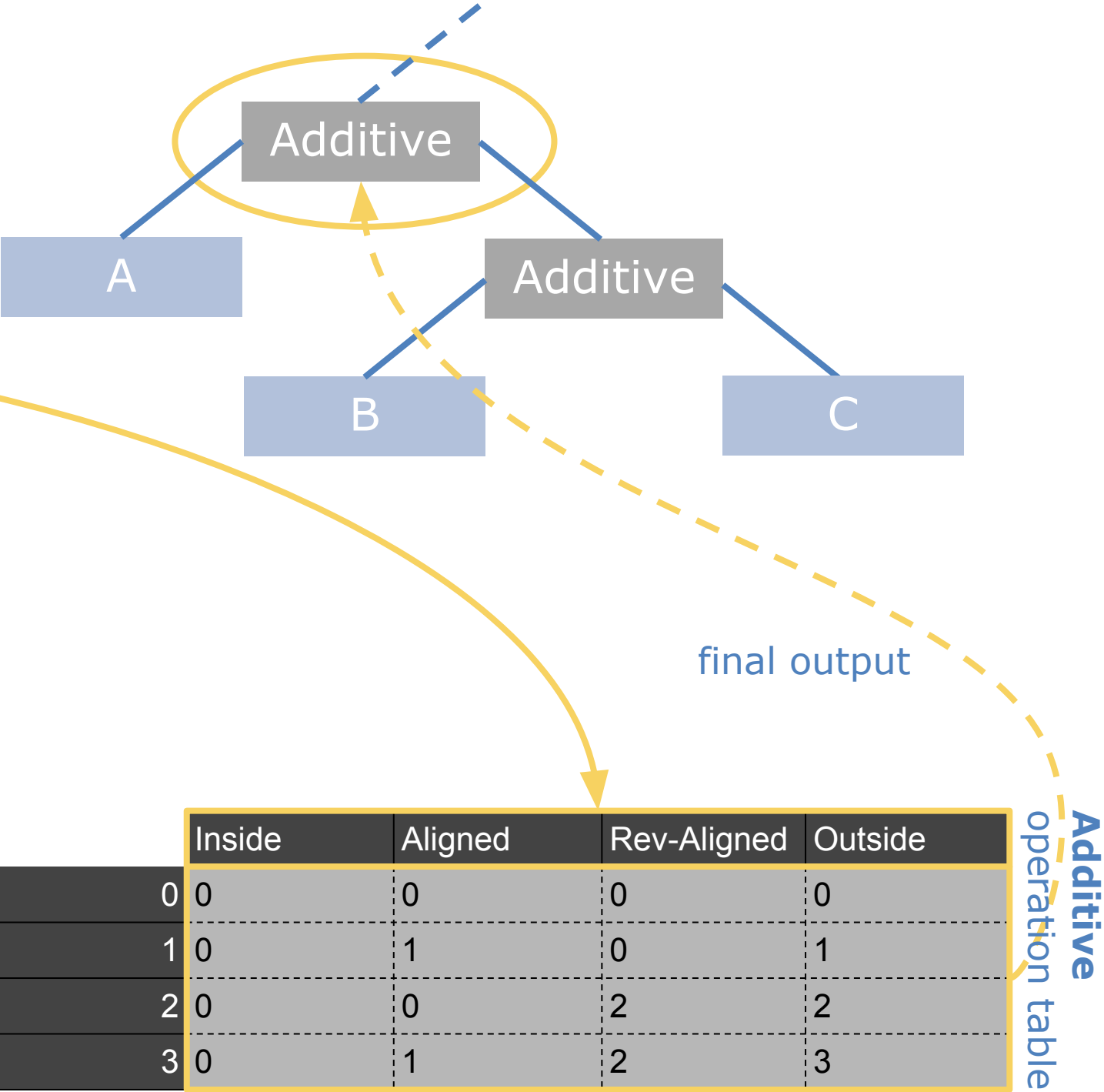


|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

Additive  
operation table

Routing table

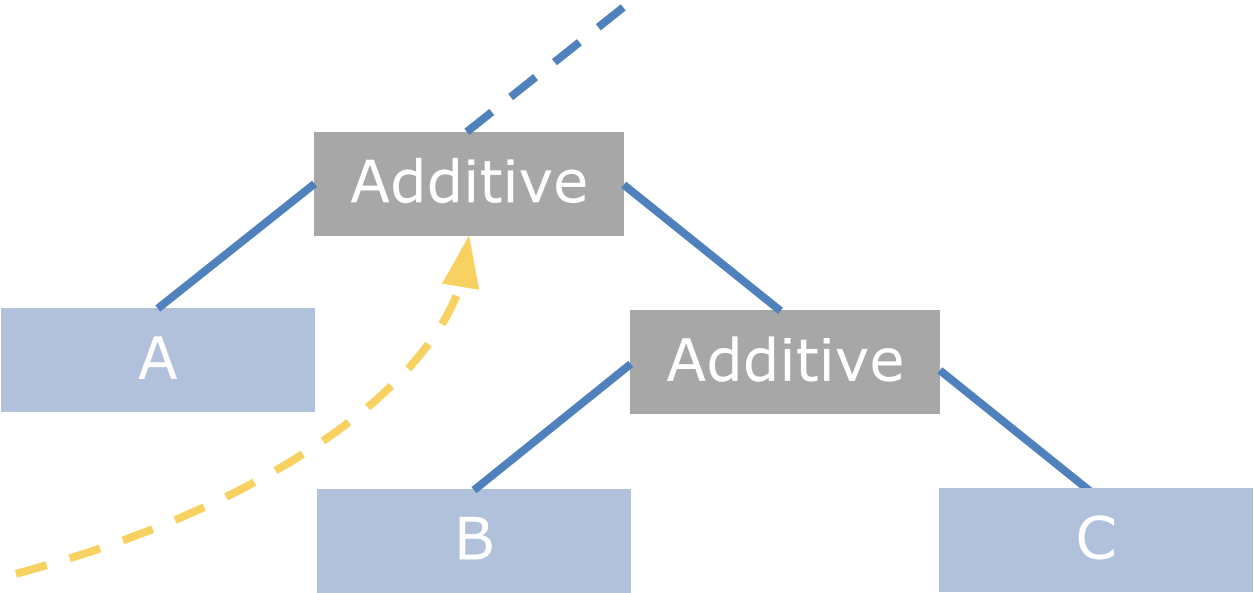
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |





# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 0       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



final output

write

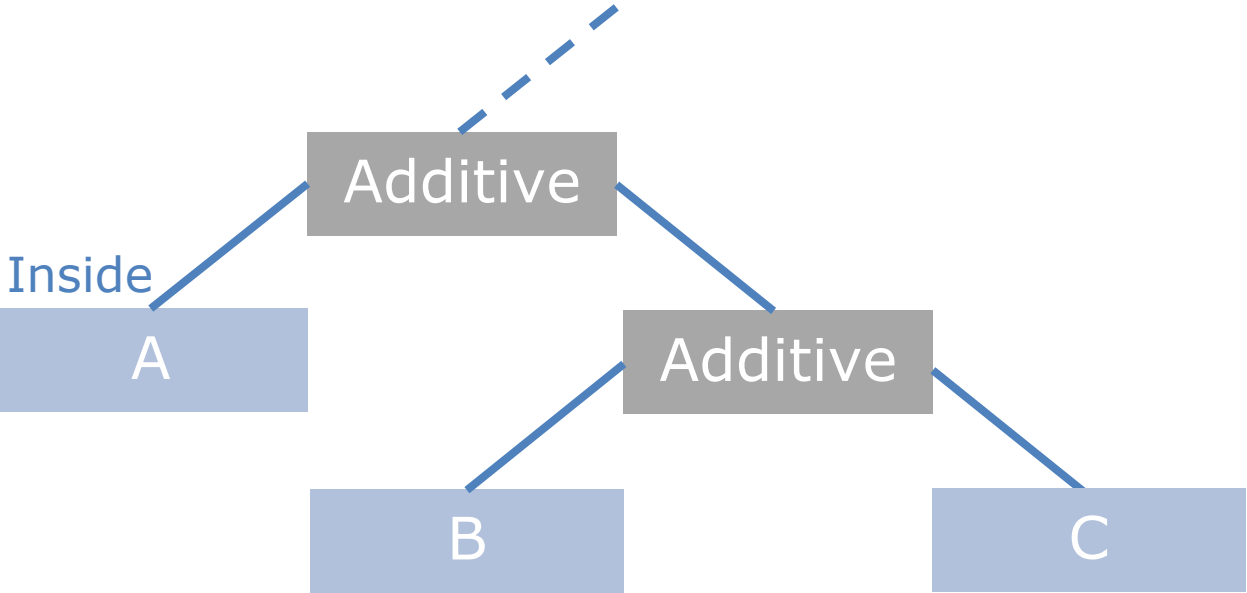
read

|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

Additive  
operation table

# Routing table

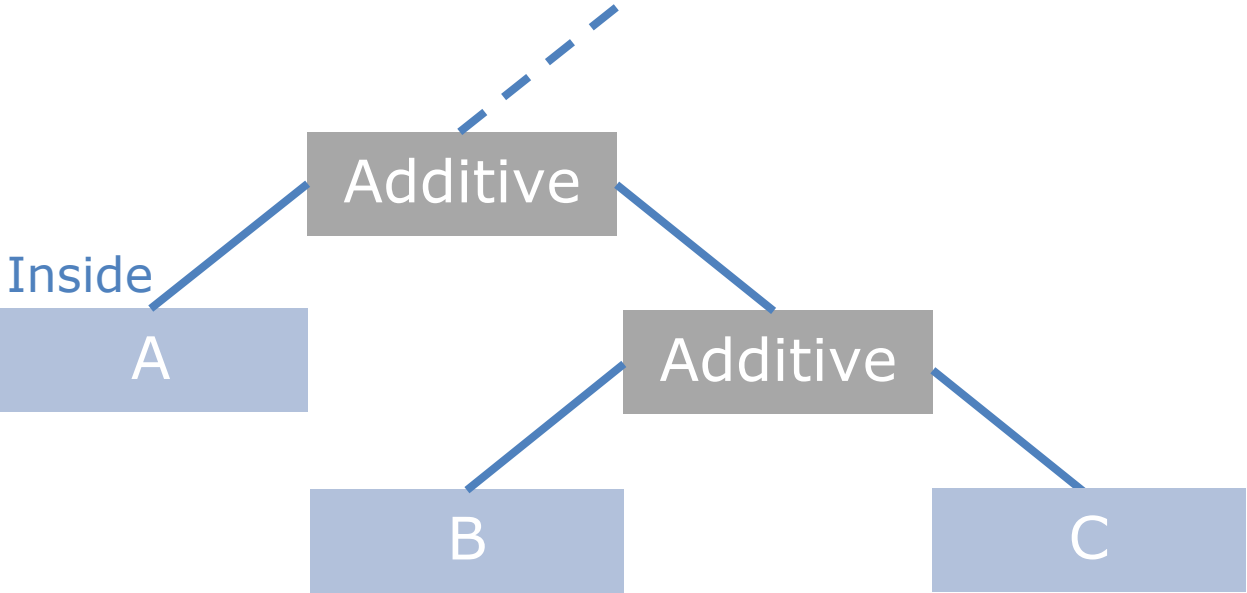
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 4      | 5       | 6           | 7       |
| (Aligned)        | 1     | 8      | 9       | 10          | 11      |
| (Rev-Aligned)    | 2     | 12     | 13      | 14          | 15      |
| (Outside)        | 3     | 16     | 17      | 18          | 19      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 1       | 0           | 1       |
|                  | 2     | 0      | 0       | 2           | 2       |
|                  | 3     | 0      | 1       | 2           | 3       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 4      | 5       | 6           | 7       |
| (Aligned)        | 1     | 8      | 9       | 10          | 11      |
| (Rev-Aligned)    | 2     | 12     | 13      | 14          | 15      |
| (Outside)        | 3     | 16     | 17      | 18          | 19      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

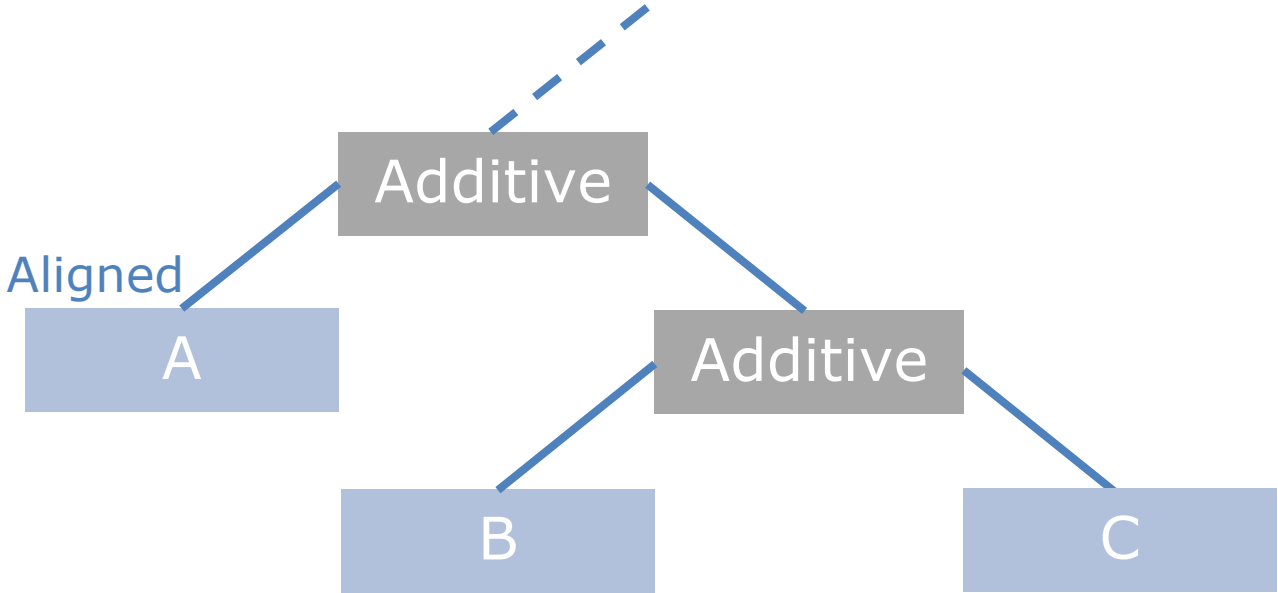


|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

Additive operation table

Routing table

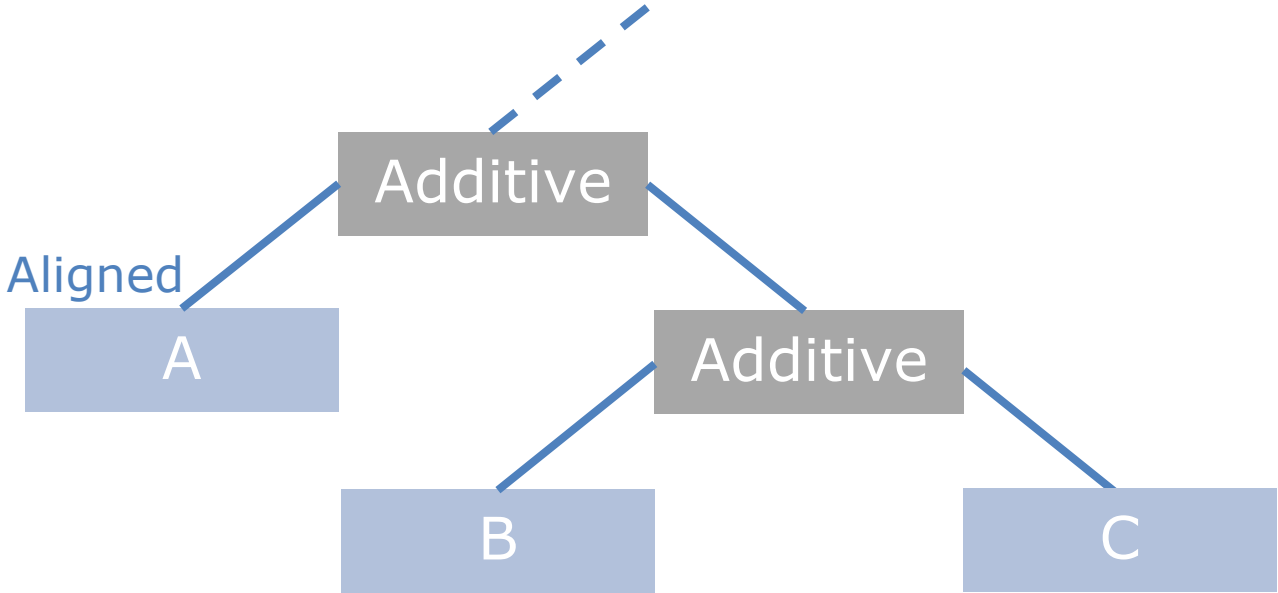
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 2           | 2       |
|                  | 7     | 0      | 1       | 2           | 3       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 0           | 0       |
|                  | 7     | 0      | 1       | 0           | 1       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



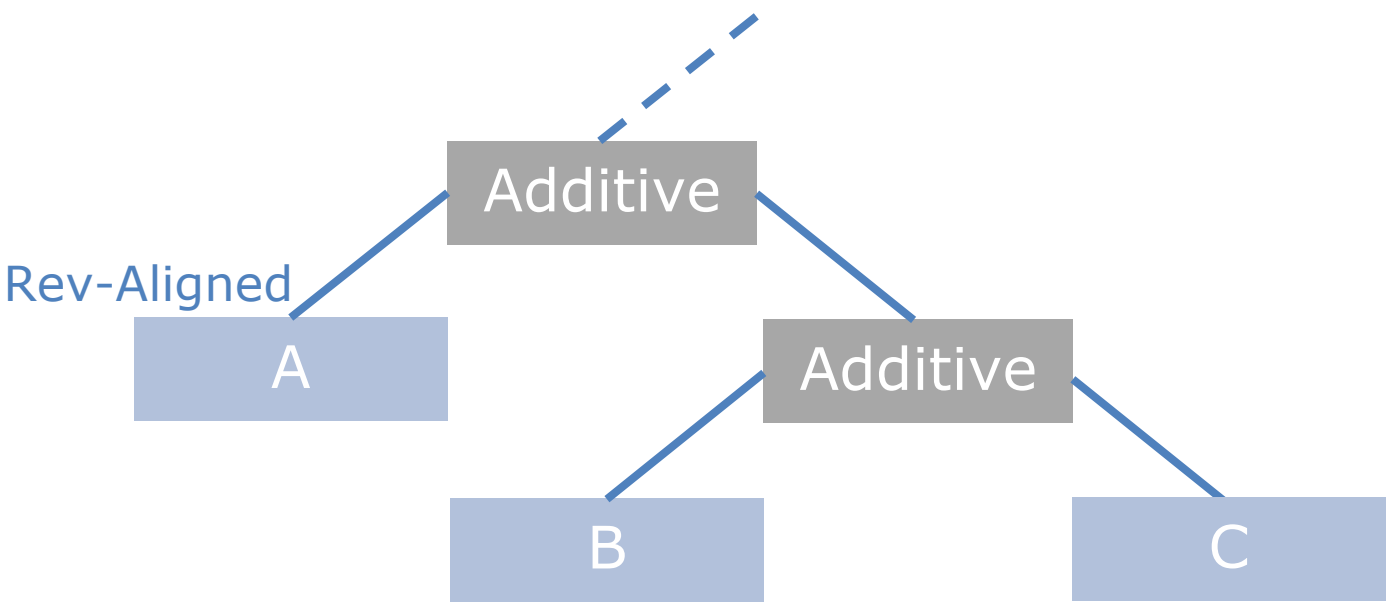
|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

operation table



# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 0           | 0       |
|                  | 7     | 0      | 1       | 0           | 1       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 1       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 1       | 2           | 3       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

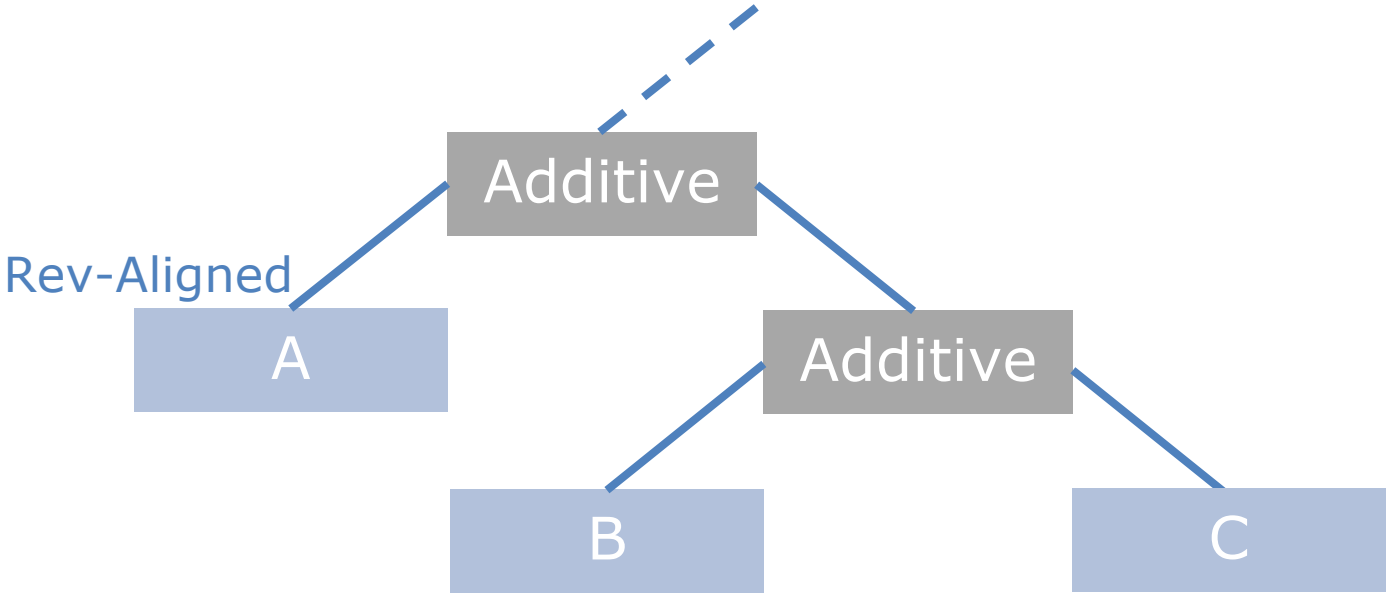


|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

operation table

# Routing table

|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 0           | 0       |
|                  | 7     | 0      | 1       | 0           | 1       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 0       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 0       | 2           | 2       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |

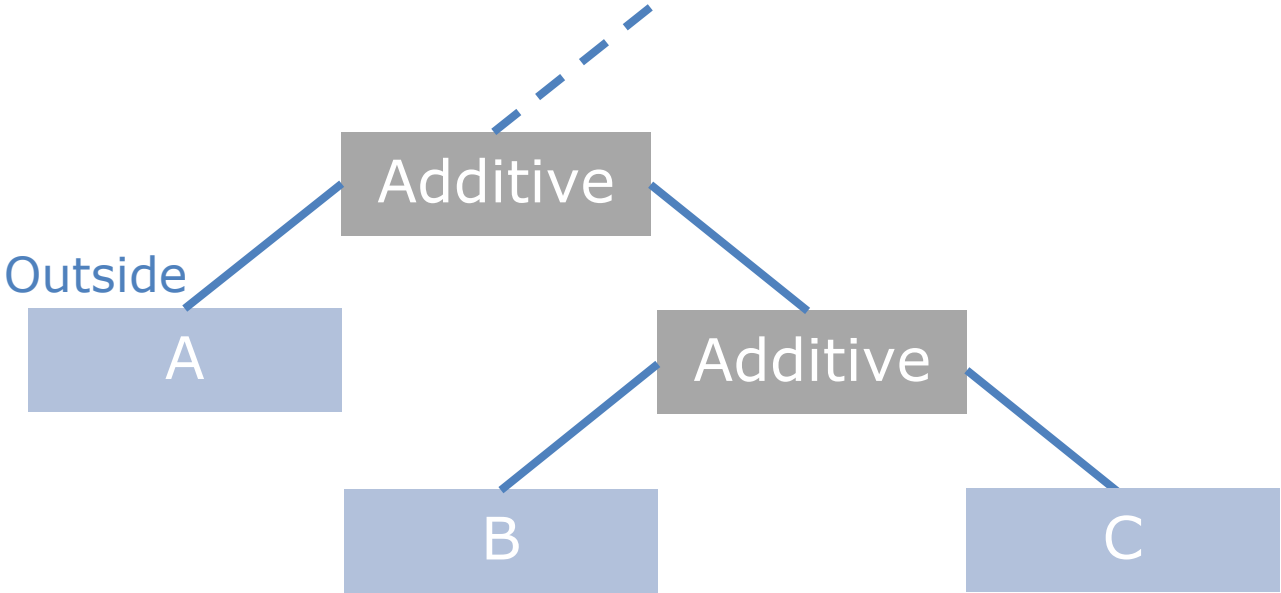


|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

operation table

# Routing table

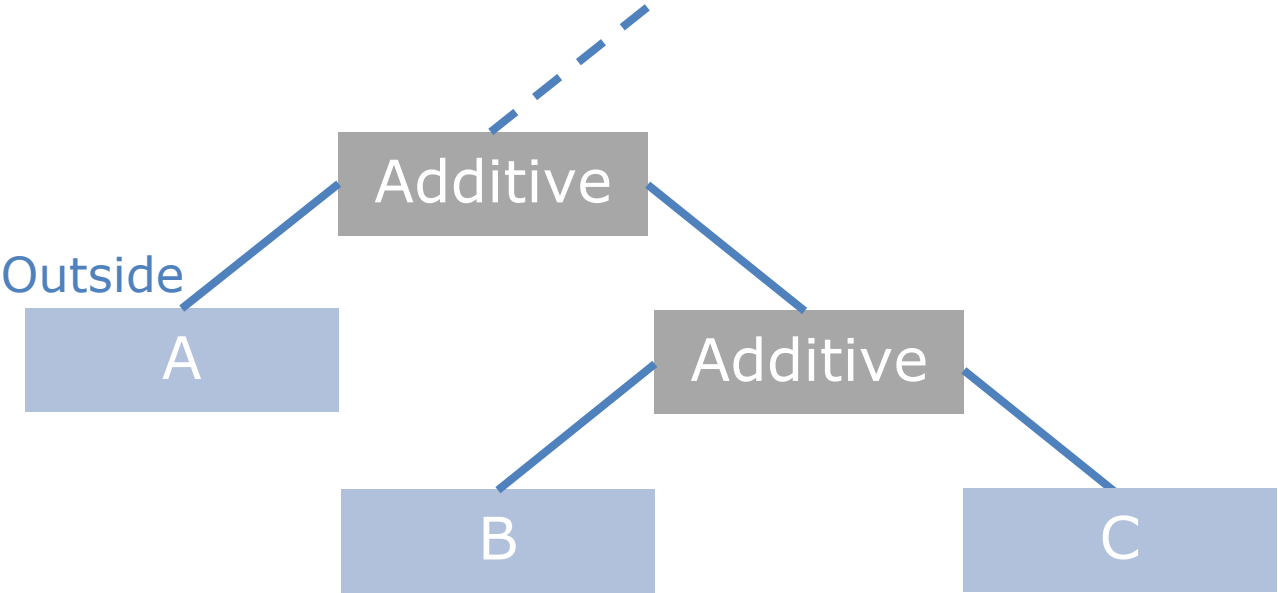
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 0           | 0       |
|                  | 7     | 0      | 1       | 0           | 1       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 0       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 0       | 2           | 2       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

# Routing table

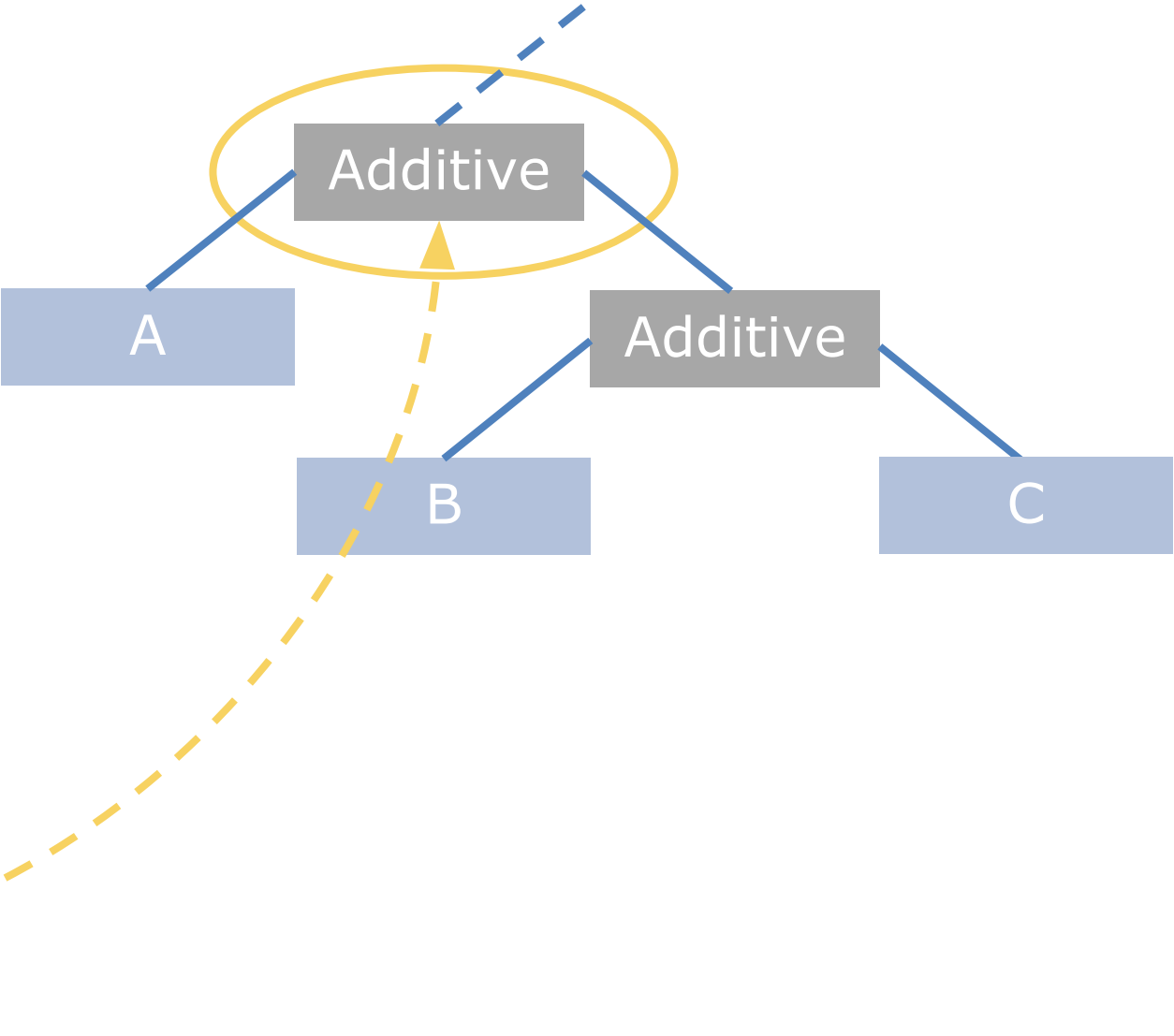
|                  | Index | Inside | Aligned | Rev-Aligned | Outside |
|------------------|-------|--------|---------|-------------|---------|
| Brush A          | -     | 0      | 1       | 2           | 3       |
| (Inside) Brush B | 0     | 0      | 1       | 2           | 3       |
| (Aligned)        | 1     | 4      | 5       | 6           | 7       |
| (Rev-Aligned)    | 2     | 8      | 9       | 10          | 11      |
| (Outside)        | 3     | 12     | 13      | 14          | 15      |
| (Inside) Brush C | 0     | 0      | 0       | 0           | 0       |
|                  | 1     | 0      | 0       | 0           | 0       |
|                  | 2     | 0      | 0       | 0           | 0       |
|                  | 3     | 0      | 0       | 0           | 0       |
| (Aligned)        | 4     | 0      | 0       | 0           | 0       |
|                  | 5     | 0      | 1       | 0           | 0       |
|                  | 6     | 0      | 0       | 0           | 0       |
|                  | 7     | 0      | 1       | 0           | 1       |
| (Rev-Aligned)    | 8     | 0      | 0       | 0           | 0       |
|                  | 9     | 0      | 0       | 0           | 0       |
|                  | 10    | 0      | 0       | 2           | 2       |
|                  | 11    | 0      | 0       | 2           | 2       |
| (Outside)        | 12    | 0      | 0       | 0           | 0       |
|                  | 13    | 0      | 1       | 0           | 0       |
|                  | 14    | 0      | 0       | 2           | 2       |
|                  | 15    | 0      | 1       | 2           | 3       |



|   | Inside | Aligned | Rev-Aligned | Outside |
|---|--------|---------|-------------|---------|
| 0 | 0      | 0       | 0           | 0       |
| 1 | 0      | 1       | 0           | 1       |
| 2 | 0      | 0       | 2           | 2       |
| 3 | 0      | 1       | 2           | 3       |

Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 1       | 2           | 3       |
|         | 1     | 4      | 5       | 6           | 7       |
|         | 2     | 8      | 9       | 10          | 11      |
|         | 3     | 12     | 13      | 14          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 0       | 0           | 0       |
|         | 2     | 0      | 0       | 0           | 0       |
|         | 3     | 0      | 0       | 0           | 0       |
|         | 4     | 0      | 0       | 0           | 0       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | 6     | 0      | 0       | 0           | 0       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | 8     | 0      | 0       | 0           | 0       |
|         | 9     | 0      | 0       | 0           | 0       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | 11    | 0      | 0       | 2           | 2       |
|         | 12    | 0      | 0       | 0           | 0       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | 14    | 0      | 0       | 2           | 2       |
|         | 15    | 0      | 1       | 2           | 3       |





# Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 1       | 2           | 3       |
|         | 1     | 4      | 5       | 6           | 7       |
|         | 2     | 8      | 9       | 10          | 11      |
|         | 3     | 12     | 13      | 14          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 0       | 0           | 0       |
|         | 2     | 0      | 0       | 0           | 0       |
|         | 3     | 0      | 0       | 0           | 0       |
|         | 4     | 0      | 0       | 0           | 0       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | 6     | 0      | 0       | 0           | 0       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | 8     | 0      | 0       | 0           | 0       |
|         | 9     | 0      | 0       | 0           | 0       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | 11    | 0      | 0       | 2           | 2       |
|         | 12    | 0      | 0       | 0           | 0       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | 14    | 0      | 0       | 2           | 2       |
|         | 15    | 0      | 1       | 2           | 3       |

# Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 1       | 2           | 3       |
|         | 1     | 4      | 5       | 6           | 7       |
|         | 2     | 8      | 9       | 10          | 11      |
|         | 3     | 12     | 13      | 14          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 0       | 0           | 0       |
|         | 2     | 0      | 0       | 0           | 0       |
|         | 3     | 0      | 0       | 0           | 0       |
|         | 4     | 0      | 0       | 0           | 0       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | 6     | 0      | 0       | 0           | 0       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | 8     | 0      | 0       | 0           | 0       |
|         | 9     | 0      | 0       | 0           | 0       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | 11    | 0      | 0       | 2           | 2       |
|         | 12    | 0      | 0       | 0           | 0       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | 14    | 0      | 0       | 2           | 2       |
|         | 15    | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 1       | 2           | 3       |
|         | 1     | 4      | 5       | 6           | 7       |
|         | 2     | 8      | 9       | 10          | 11      |
|         | 3     | 12     | 13      | 14          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     |        |         |             |         |
|         | 2     |        |         |             |         |
|         | 3     |        |         |             |         |
|         | 4     |        |         |             |         |
|         | 5     | 0      | 1       | 0           | 0       |
|         | 6     |        |         |             |         |
|         | 7     | 0      | 1       | 0           | 1       |
|         | 8     | -      | -       | -           | -       |
|         | 9     | -      | -       | -           | -       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | 11    | 0      | 0       | 2           | 2       |
|         | 12    | -      | -       | -           | -       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | 14    | 0      | 0       | 2           | 2       |
|         | 15    | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 5       | 0           | 7       |
|         | 2     | 0      | 0       | 10          | 11      |
|         | 3     | 0      | 13      | 14          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | 11    | 0      | 0       | 2           | 2       |
|         | -     | -      | -       | -           | -       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | 14    | 0      | 0       | 2           | 2       |
|         | 15    | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 5       | 0           | 7       |
|         | 2     | 0      | 0       | 10          | 11      |
|         | 3     | 0      | 13      | 14          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | 11    | 0      | 0       | 2           | 2       |
|         | -     | -      | -       | -           | -       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | 14    | 0      | 0       | 2           | 2       |
|         | 15    | 0      | 1       | 2           | 3       |



## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 5       | 0           | 7       |
|         | 2     | 0      | 0       | 10          | 10      |
|         | 3     | 0      | 13      | 10          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 15    | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 5       | 0           | 7       |
|         | 2     | 0      | 0       | 10          | 10      |
|         | 3     | 0      | 13      | 10          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 13    | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 15    | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 5       | 0           | 7       |
|         | 2     | 0      | 0       | 10          | 10      |
|         | 3     | 0      | 5       | 10          | 15      |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 5     | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 7     | 0      | 1       | 0           | 1       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 10    | 0      | 0       | 2           | 2       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 15    | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 1       | 0           | 2       |
|         | 2     | 0      | 0       | 3           | 3       |
|         | 3     | 0      | 1       | 3           | 4       |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 1     | 0      | 1       | 0           | 0       |
|         | -     | -      | -       | -           | -       |
|         | 2     | 0      | 1       | 0           | 1       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 3     | 0      | 0       | 2           | 2       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | -     | -      | -       | -           | -       |
|         | 4     | 0      | 1       | 2           | 3       |

## Routing table

|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 2           | 3       |
| Brush B | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 1       | 0           | 2       |
|         | 2     | 0      | 0       | 3           | 3       |
|         | 3     | 0      | 1       | 3           | 4       |
| Brush C | 0     | 0      | 0       | 0           | 0       |
|         | 1     | 0      | 1       | 0           | 0       |
|         | 2     | 0      | 1       | 0           | 1       |
|         | 3     | 0      | 0       | 2           | 2       |
|         | 4     | 0      | 1       | 2           | 3       |

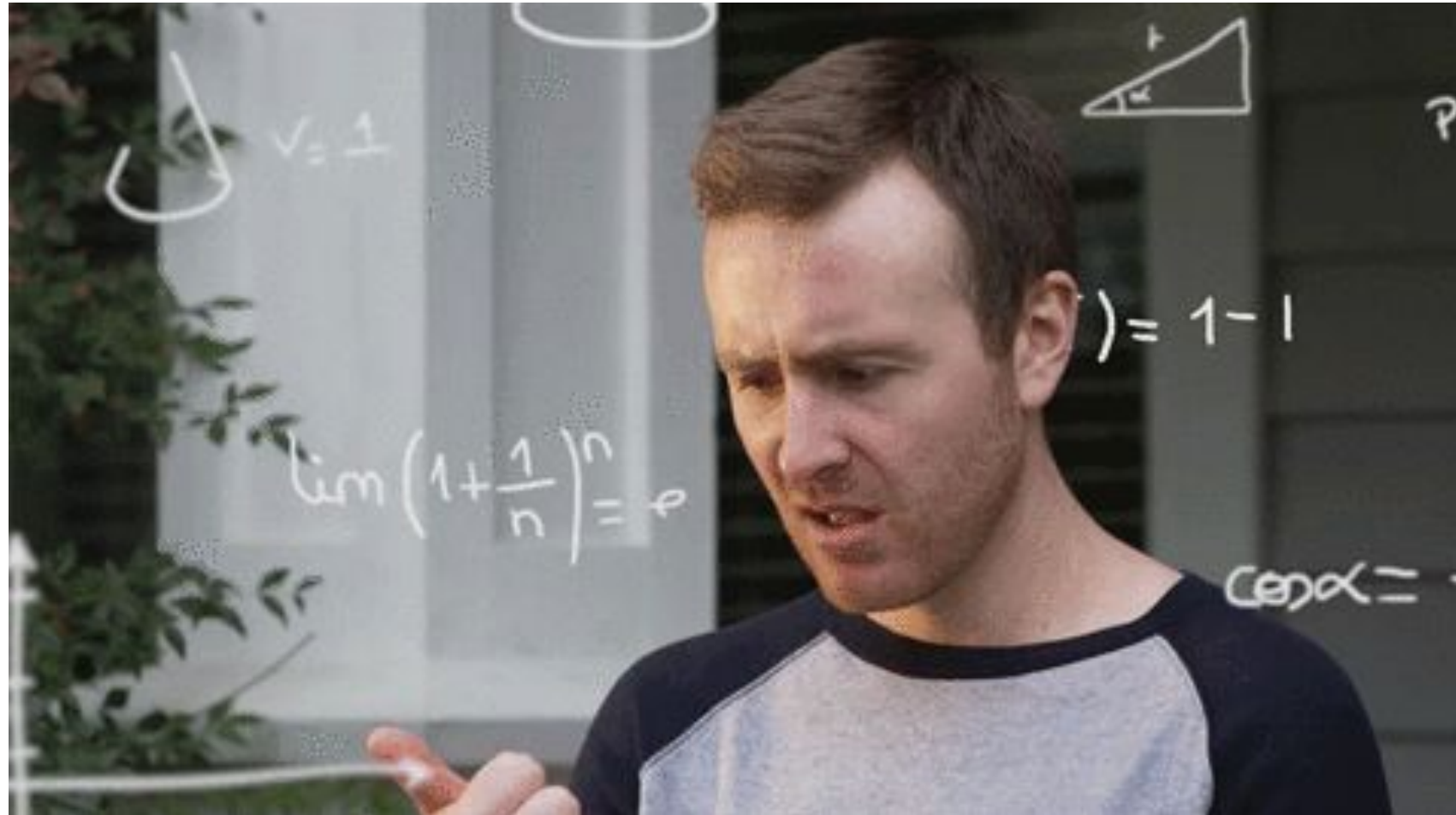
Compact routing table

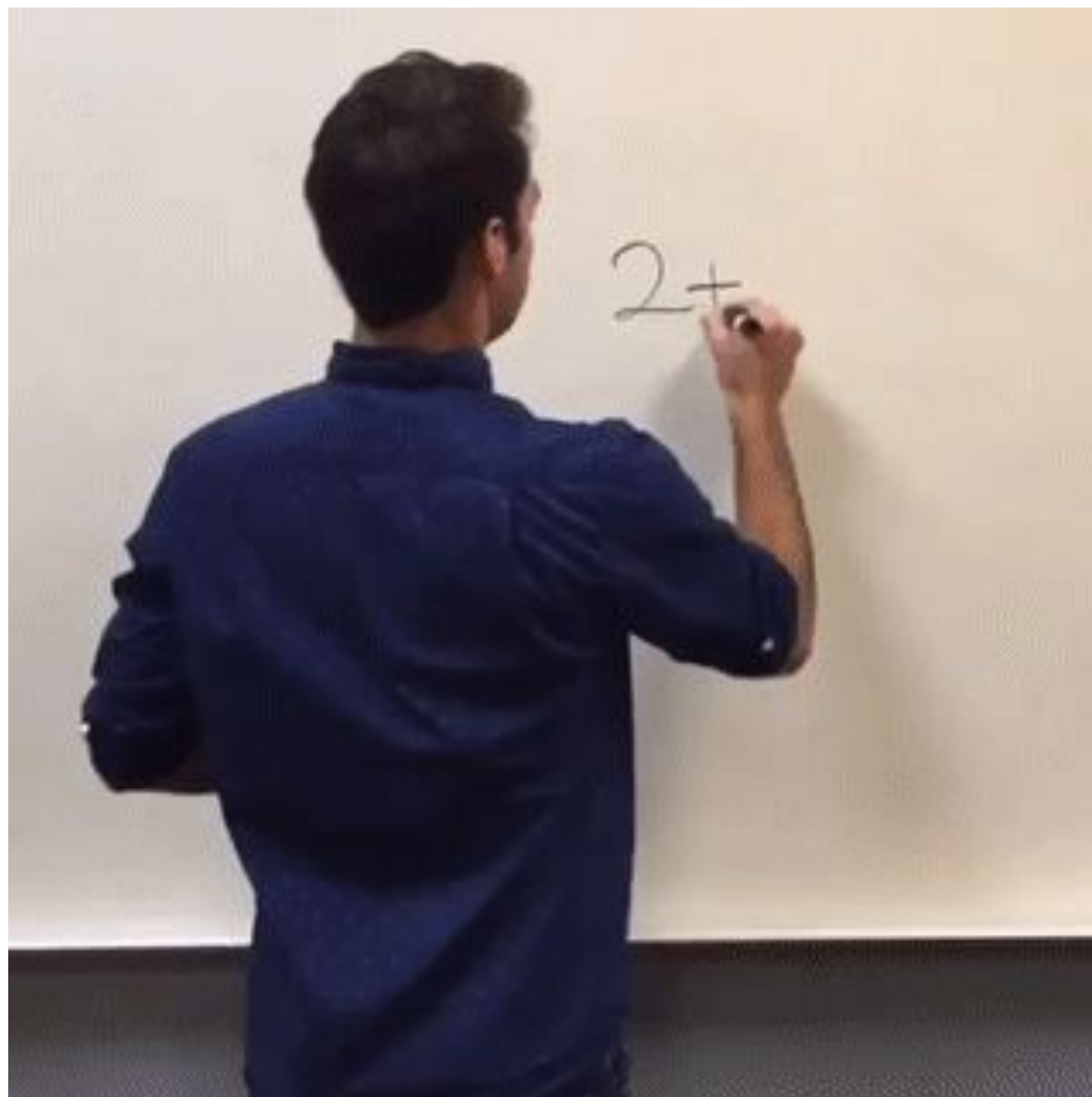
Cacheable per brush

Number of rows per brush < 255

- 4 output values, 0-3, 2 bits \* 4 = 8
- Row can be stored as 4 bytes
- More than 6 rows is rare
- Not all row output combinations make sense, or can be generated by operations
  - Theoretical maximum is probably *a lot* lower

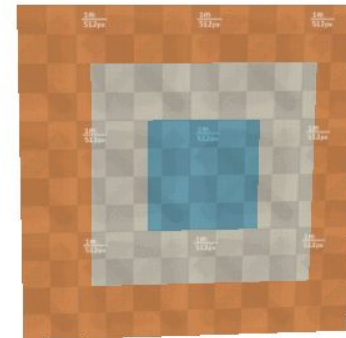
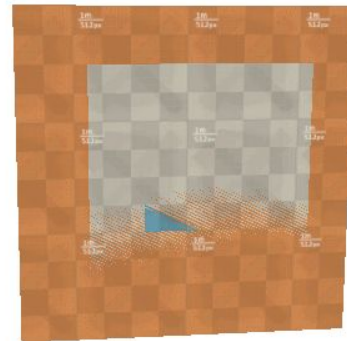






## Brush categorization

Can have **multiple** brushes overlapping on the **same polygon area**



Solution: Make every brush **remove** the area of the **previous** brushes

## Brush categorization

Solution:

Switch to variation of our operation tables that removes polygons that overlap by returning the **outside** category.

We use this on each brush beyond the brush the routing table belongs to.

Note: we keep using the original operation tables when combining routing tables

|         |             | Brush A |         |             |             |
|---------|-------------|---------|---------|-------------|-------------|
|         |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B | Inside      | Inside  | Inside  | Inside      | Inside      |
|         | Aligned     | Inside  | Outside | Inside      | Aligned     |
|         | Rev-Aligned | Inside  | Inside  | Outside     | Rev-Aligned |
|         | Outside     | Inside  | Outside | Outside     | Outside     |

Used to be Aligned

Used to be Rev-Aligned

# Brush categorization

Solution:

Switch to variation of our operation tables that removes polygons that overlap by returning the **outside** category.

We use this on each brush beyond the brush the routing table belongs to.  
Note: we keep using the original operation tables when combining routing tables

| Subtractive<br>Operation Table |             | Brush A |         |             |             |
|--------------------------------|-------------|---------|---------|-------------|-------------|
|                                |             | Inside  | Aligned | Rev-Aligned | Outside     |
| Brush B                        | Inside      | Outside | Outside | Outside     | Inside      |
|                                | Aligned     | Outside | Outside | Outside     | Aligned     |
|                                | Rev-Aligned | Outside | Outside | Outside     | Rev-Aligned |
|                                | Outside     | Outside | Outside | Outside     | Outside     |

Used to be Aligned

Used to be Rev-Aligned



## Brush categorization

Solution:

Switch to variation of our operation tables that removes polygons that overlap by returning the **outside** category.

We use this on each brush beyond the brush the routing table belongs to.

Note: we keep using the original operation tables when combining routing tables

| Intersecting<br>Operation Table |             | Brush A     |         |             |         |
|---------------------------------|-------------|-------------|---------|-------------|---------|
|                                 |             | Inside      | Aligned | Rev-Aligned | Outside |
| Brush B                         | Inside      | Inside      | Outside | Outside     | Outside |
|                                 | Aligned     | Aligned     | Outside | Outside     | Outside |
|                                 | Rev-Aligned | Rev-Aligned | Outside | Outside     | Outside |
|                                 | Outside     | Outside     | Outside | Outside     | Outside |

Used to be Aligned

Used to be Rev-Aligned

# Overview

## 1. History of CSG

### The algorithm

## 2. Iterative updates

## 3. Intersections

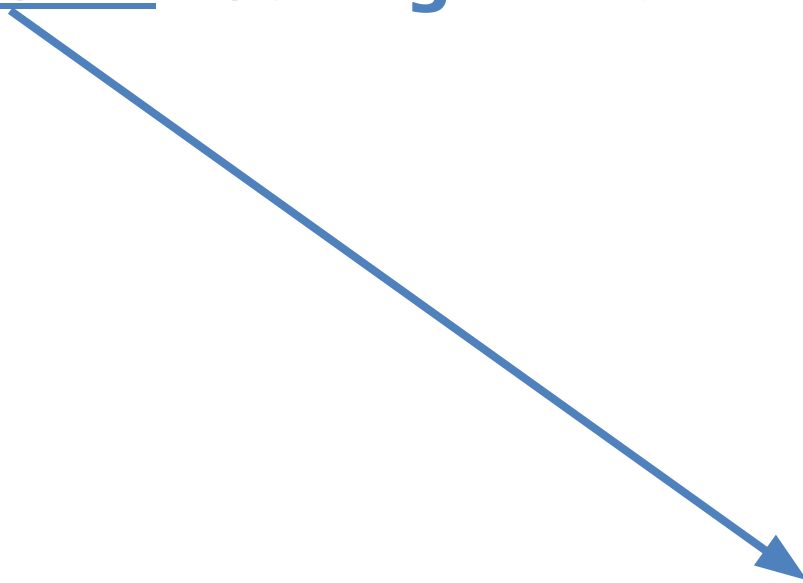
## 4. Mesh Generation

## 5. Polygon categories, Routing & Operation tables

## 6. **Putting it all together**

## Using the routing table

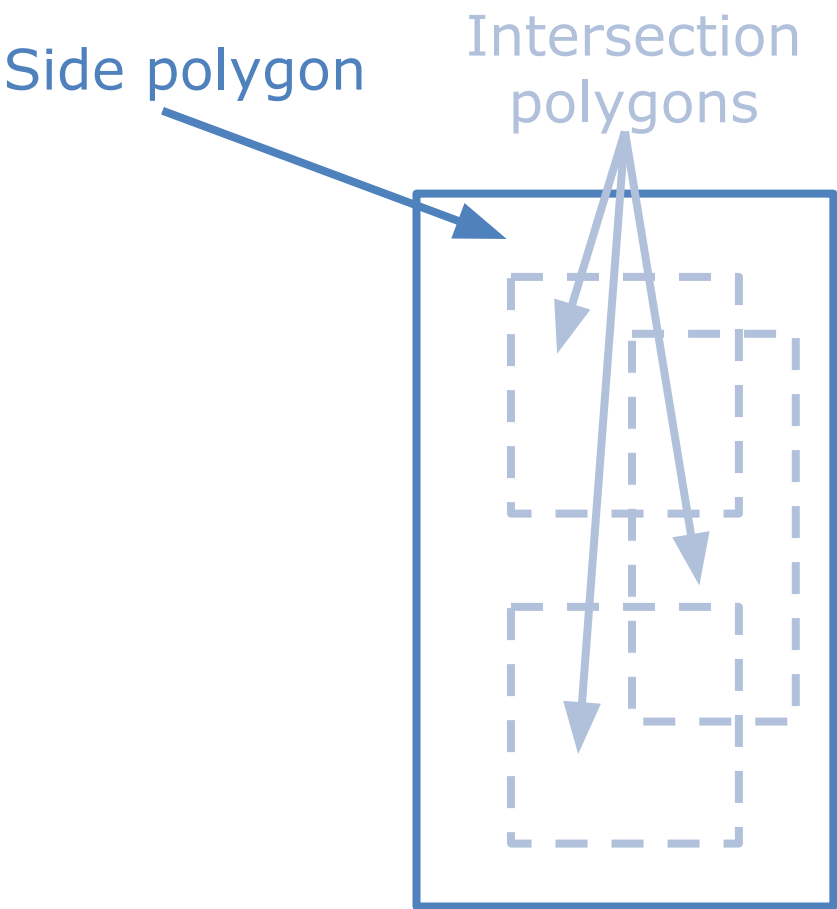
For each brush in the CSG tree, loop through the **brushes on its own routing table**



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

# Using the routing table

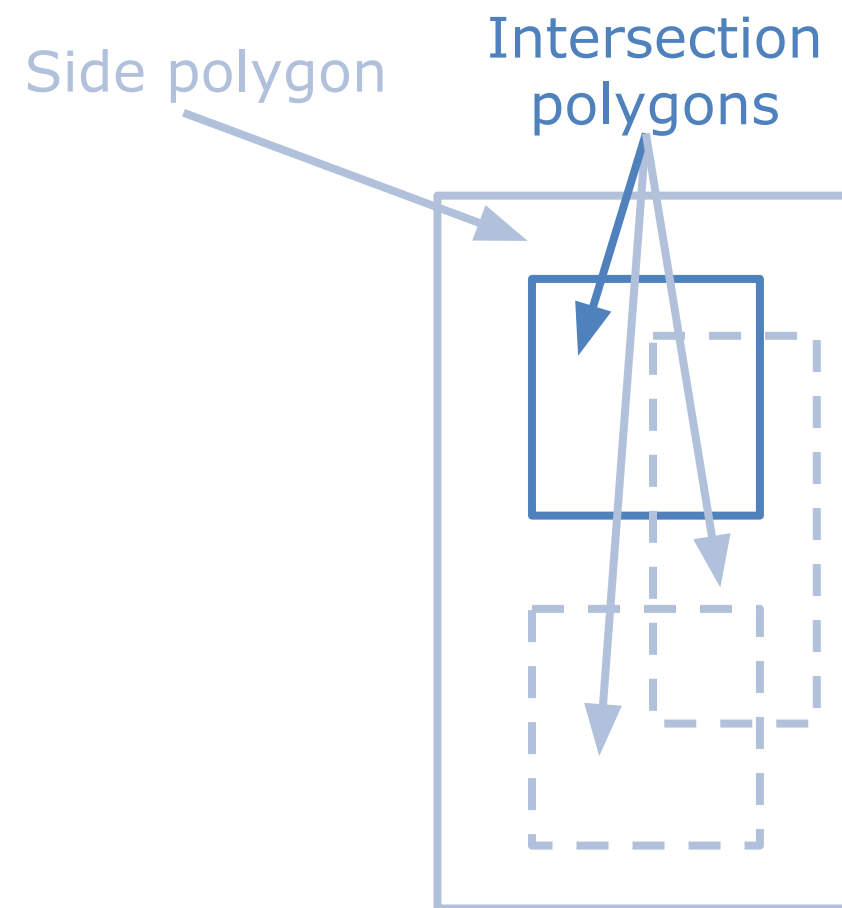
We do this for each side polygon of the brush we're processing



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

## Using the routing table

Each **intersection polygon** represents an **intersection** between the processed brush and a brush that's represented in the routing table

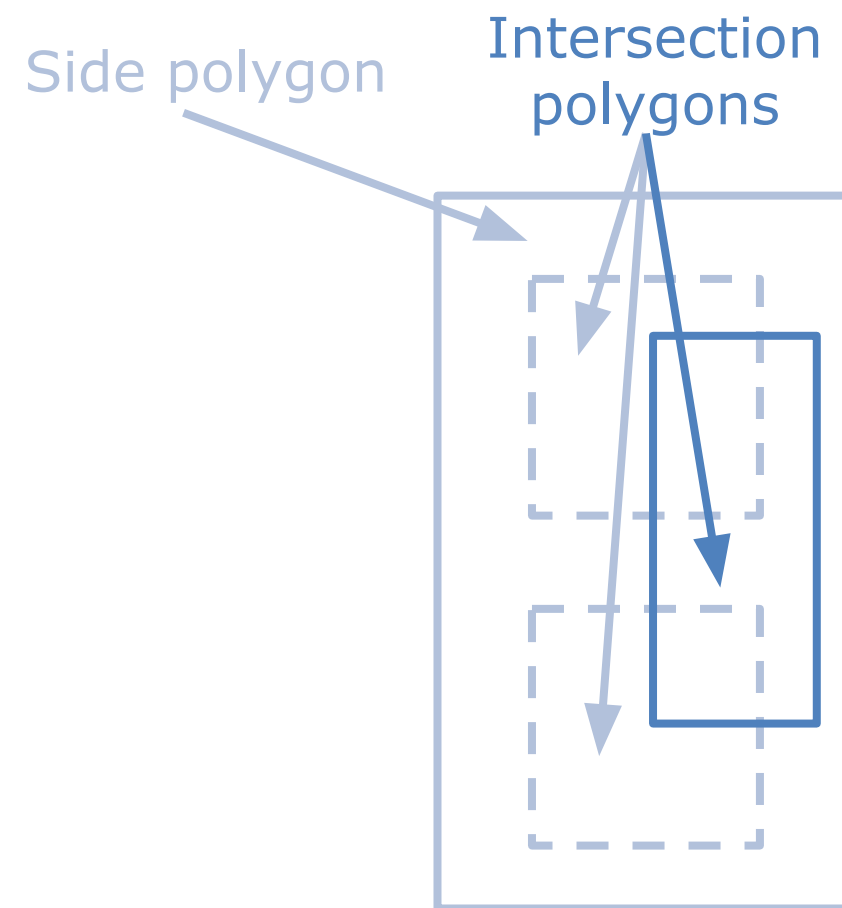


|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |



## Using the routing table

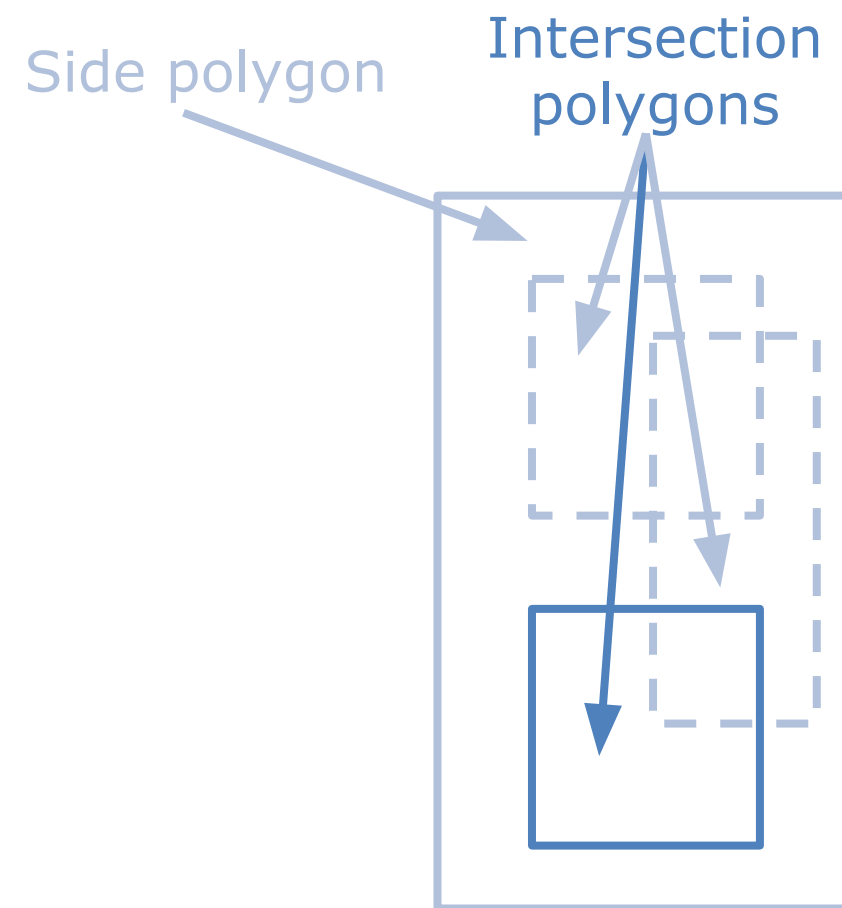
Each **intersection polygon** represents an **intersection** between the processed brush and a brush that's represented in the routing table



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

## Using the routing table

Each **intersection polygon** represents an **intersection** between the processed brush and a brush that's represented in the routing table



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

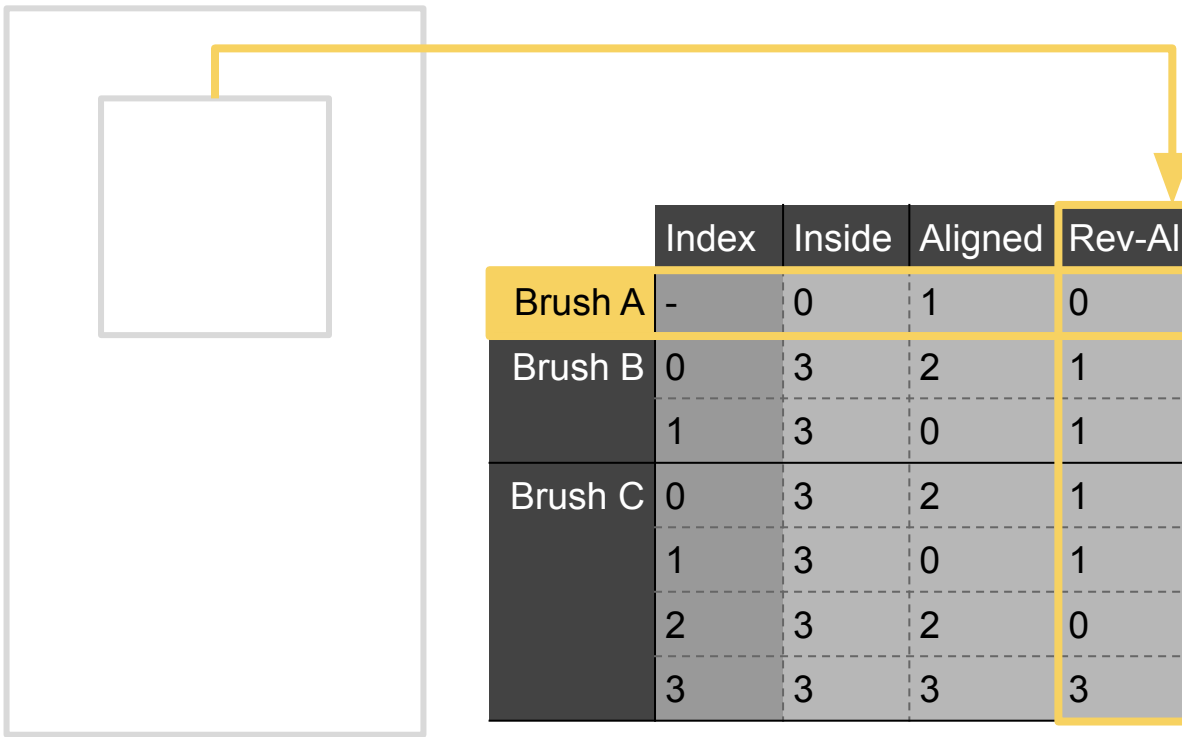
Determine which polygons to show

- Use categories on intersection polygons
  - Use *interior category* (inside polygon)

Determine which polygons to show

- Use categories on intersection polygons
  - Use *interior category* (inside polygon)

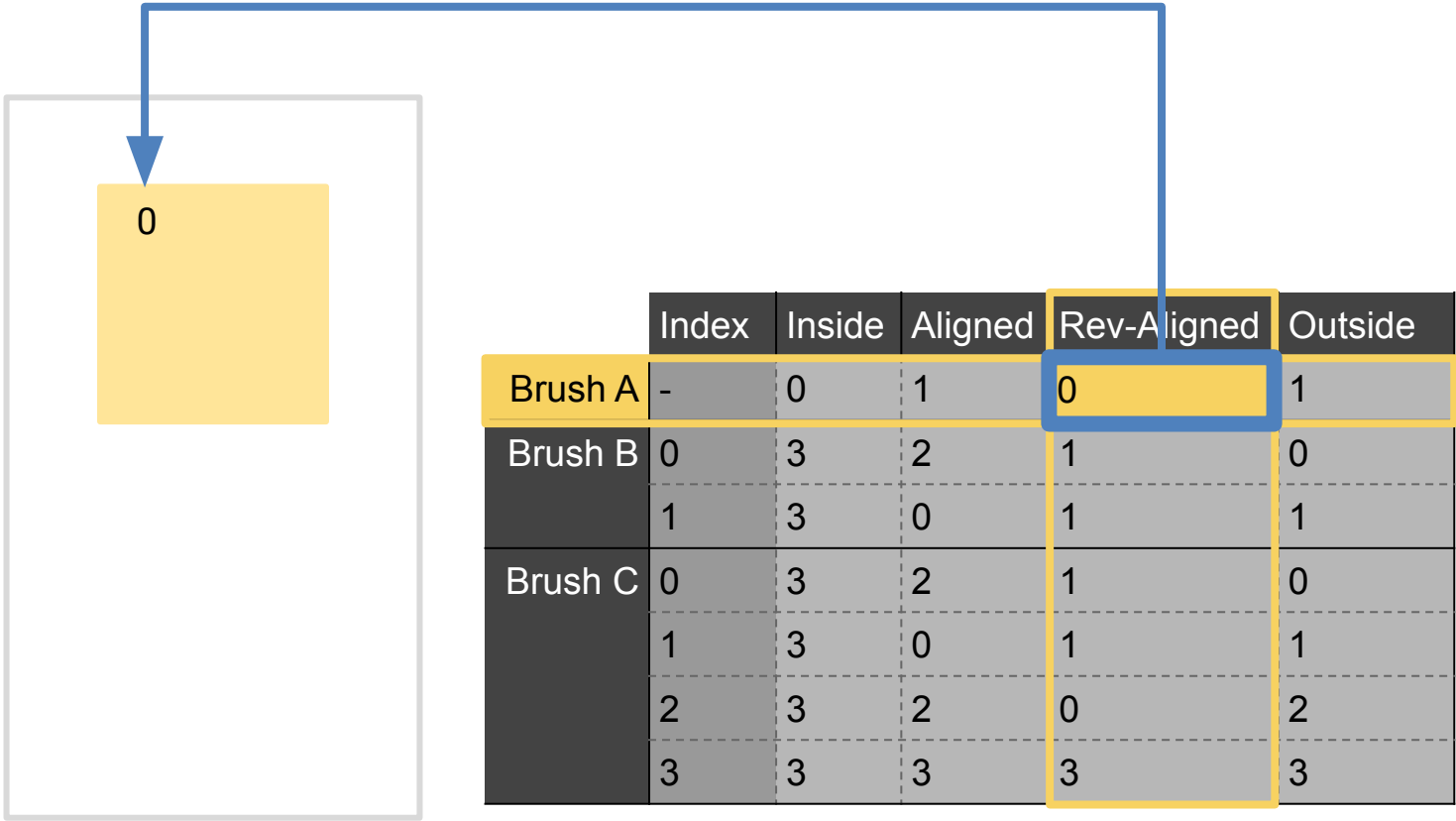
Interior intersection polygon:  
**Rev-Aligned**



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)

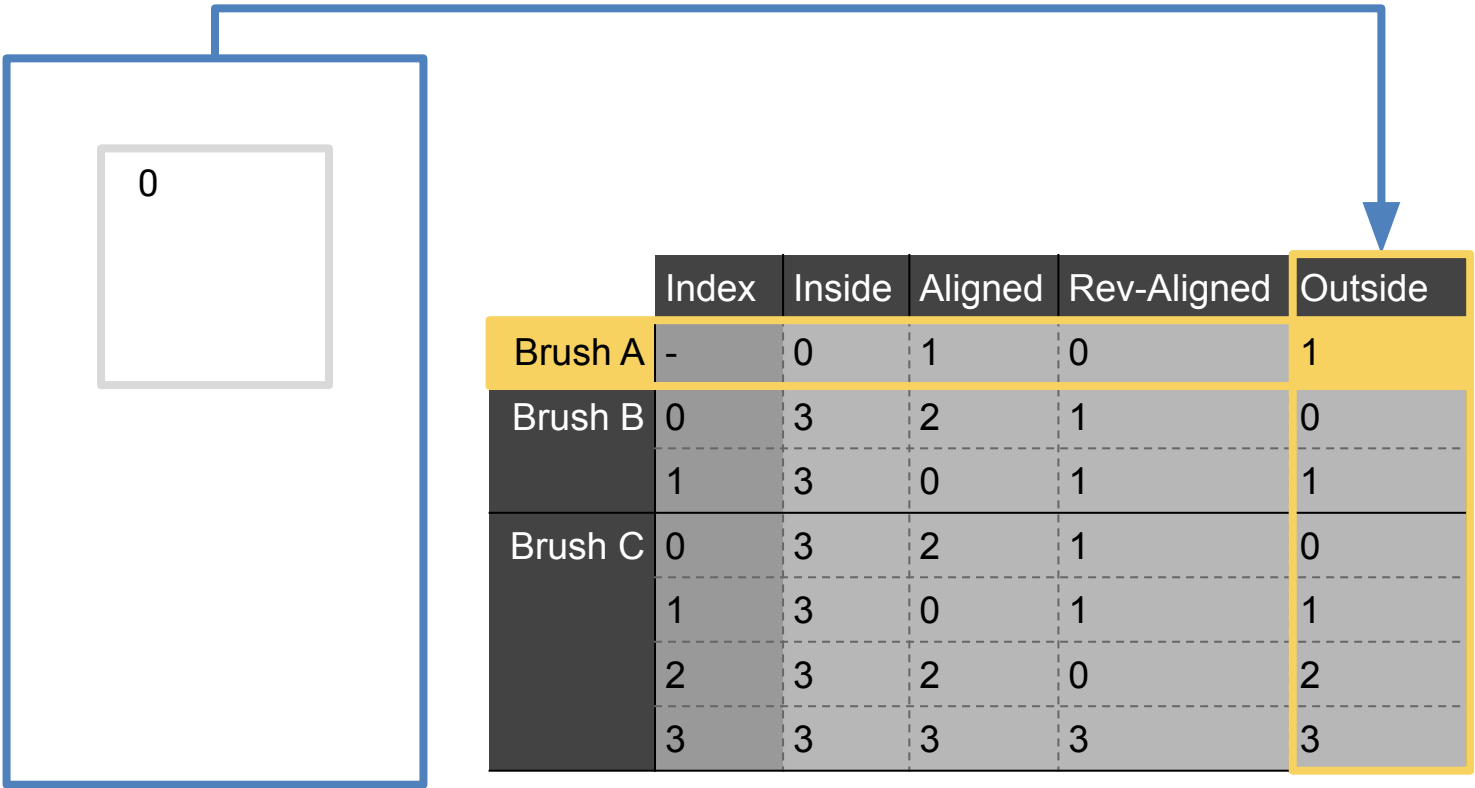




Determine which polygons to show

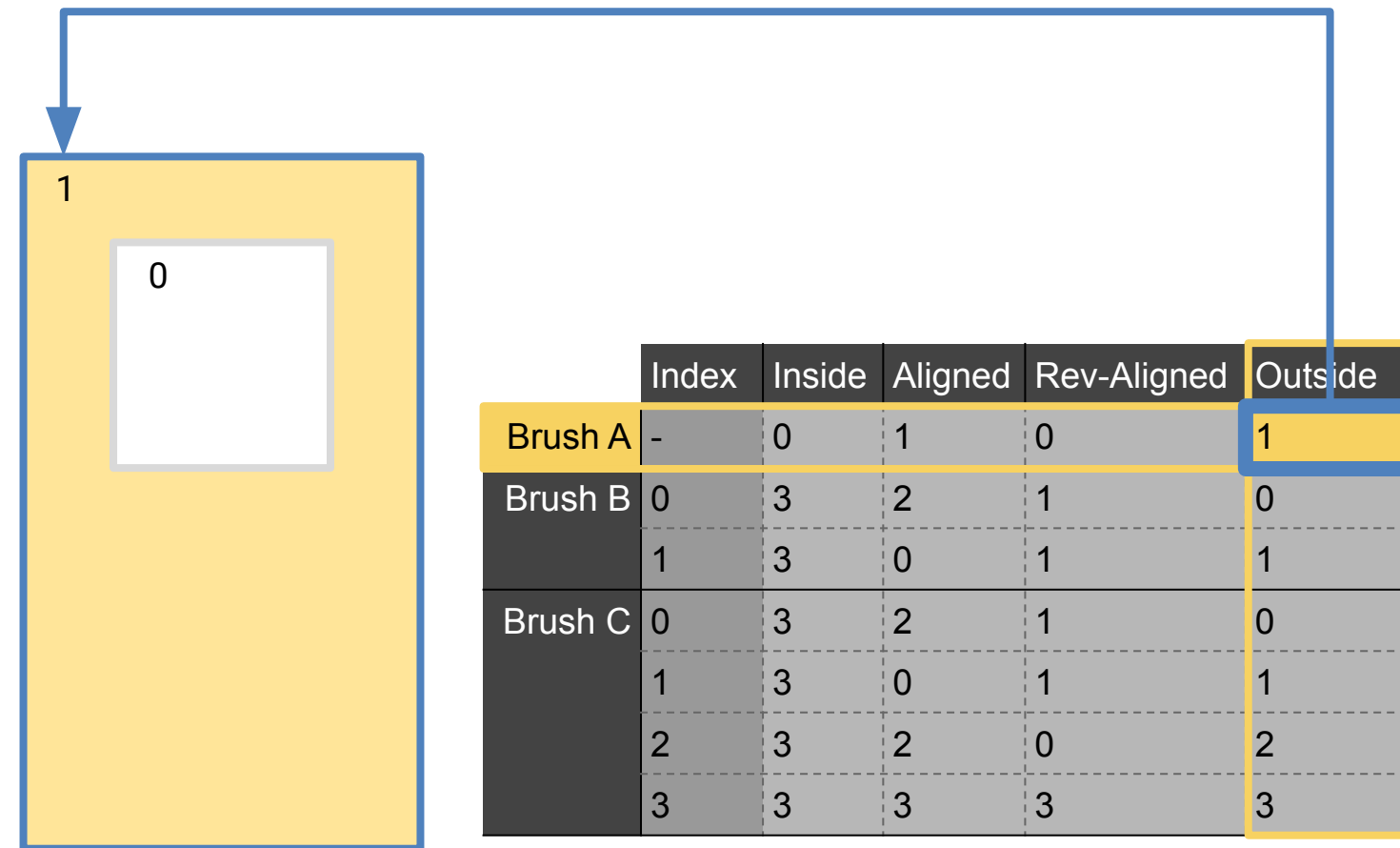
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else

Outside intersection category:  
*Outside* (always)



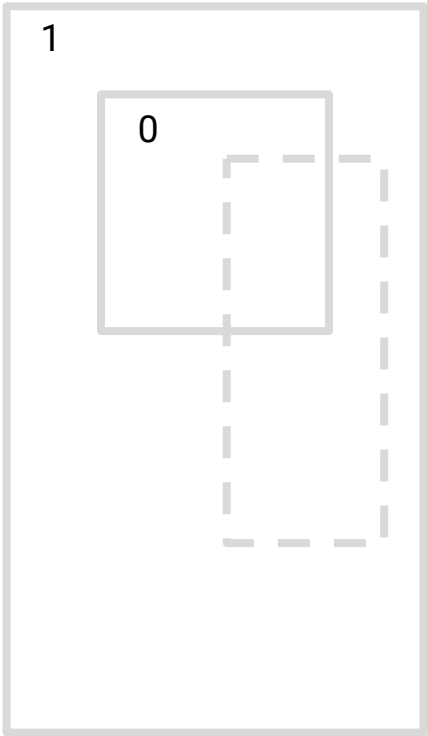
Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



Determine which polygons to show

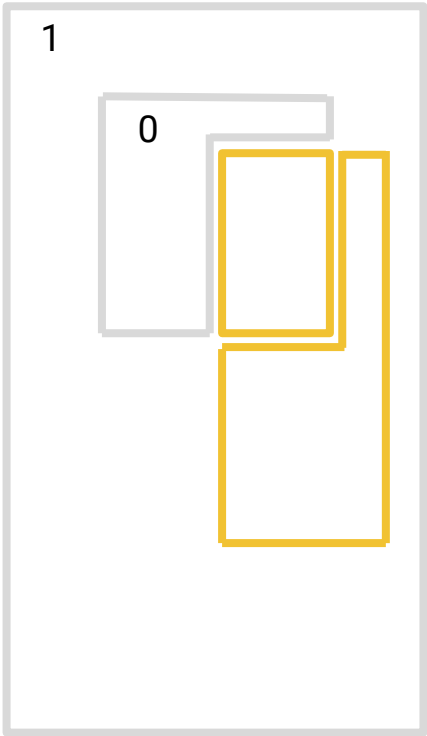
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

Determine which polygons to show

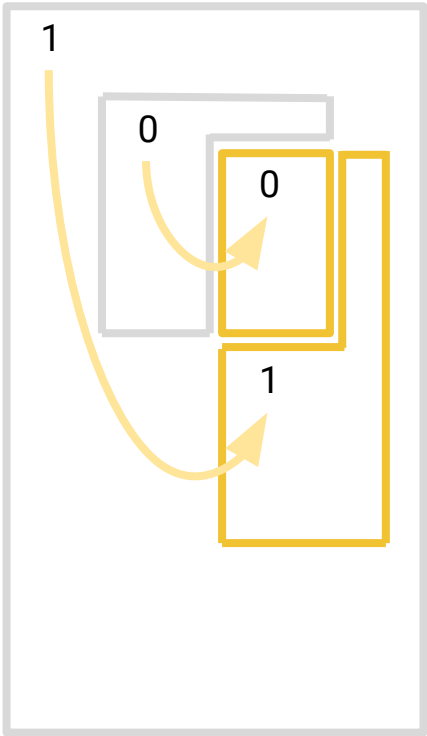
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

# Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else

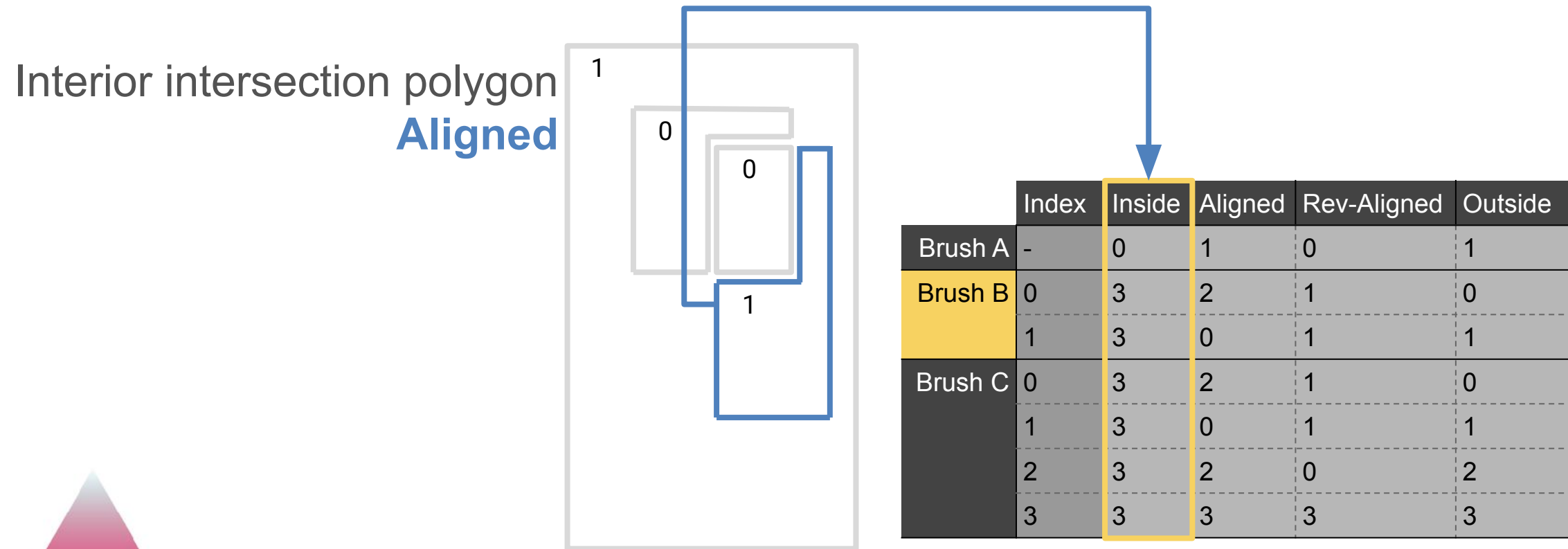


|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |



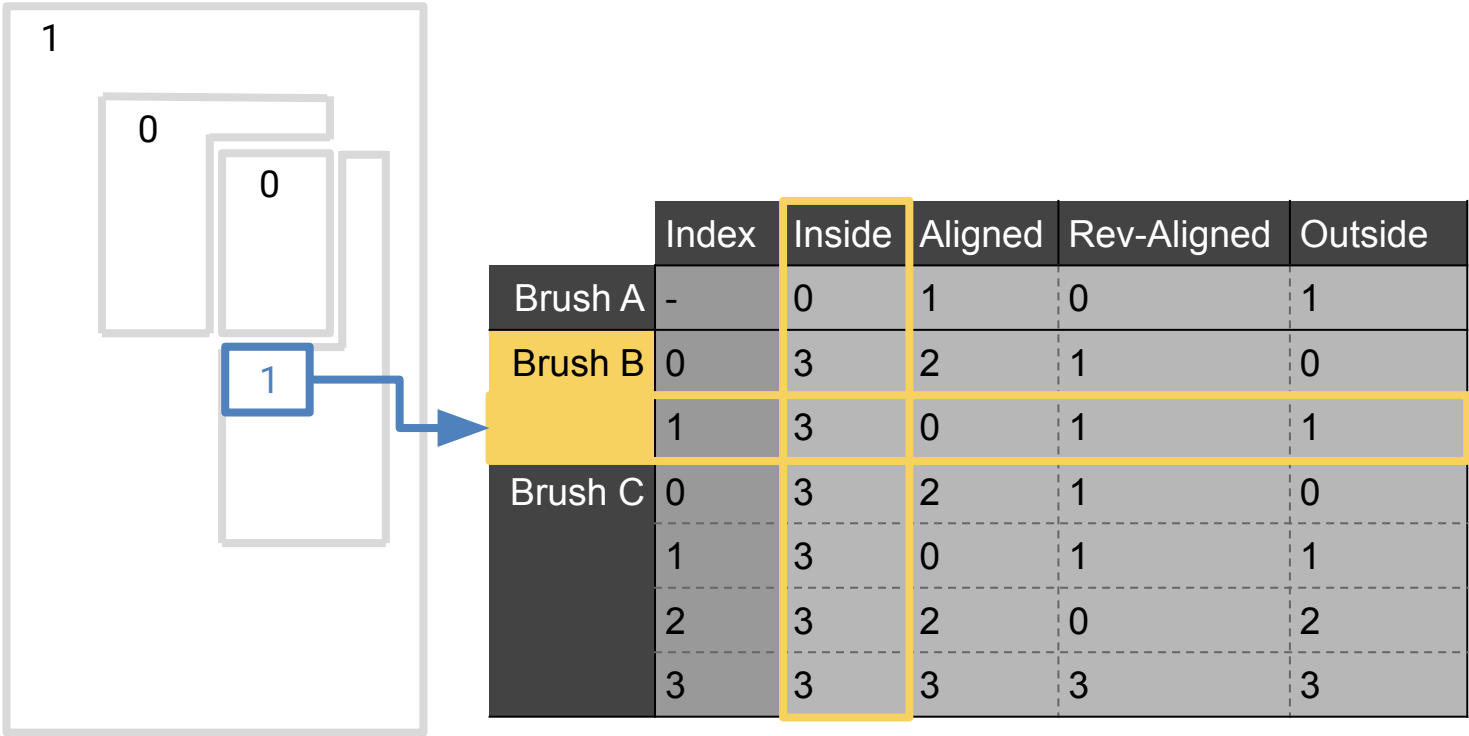
Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



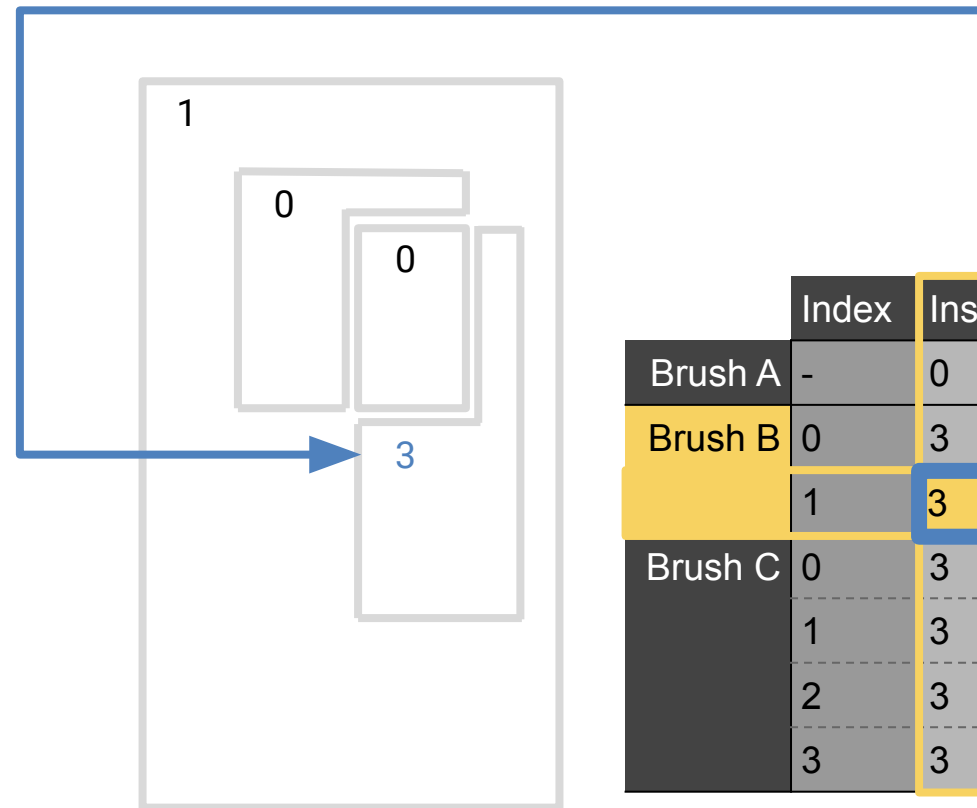
Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



Determine which polygons to show

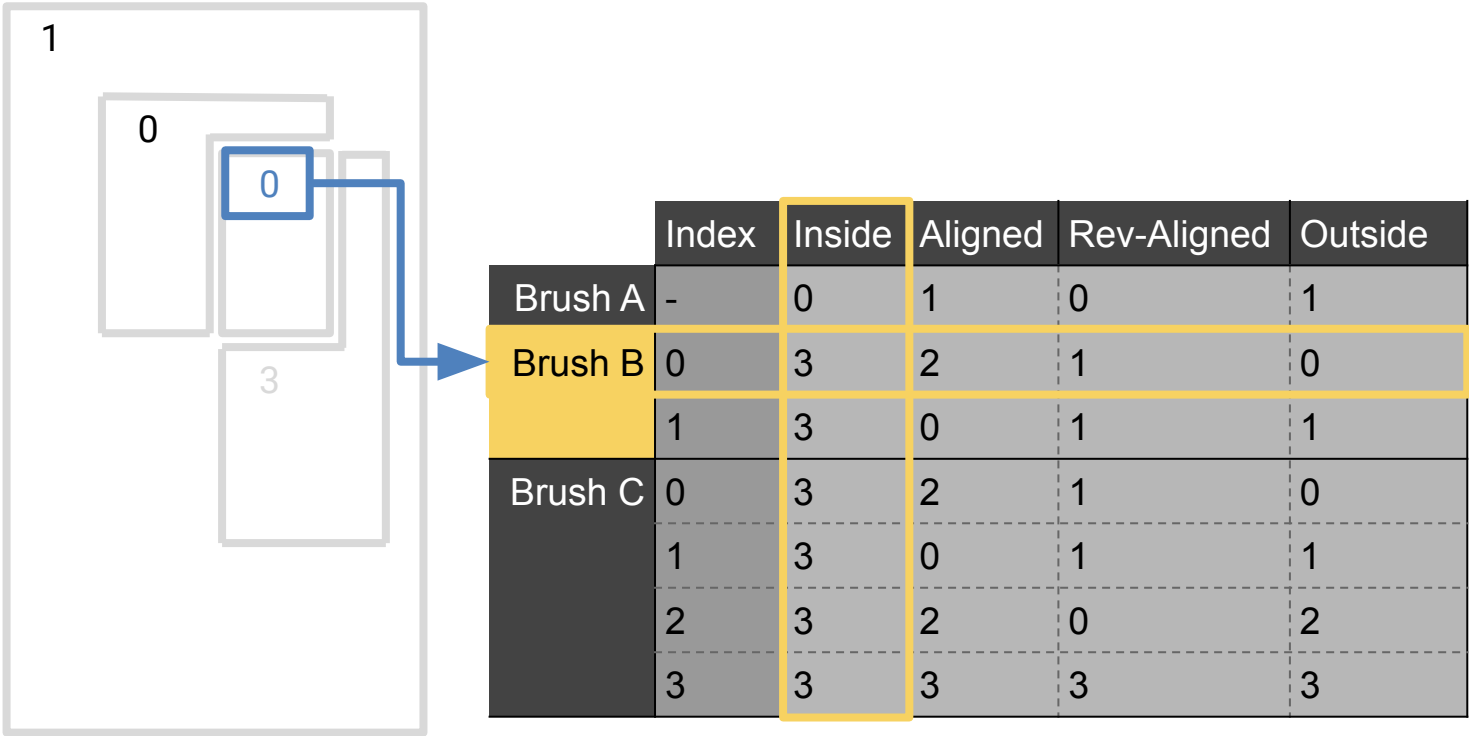
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

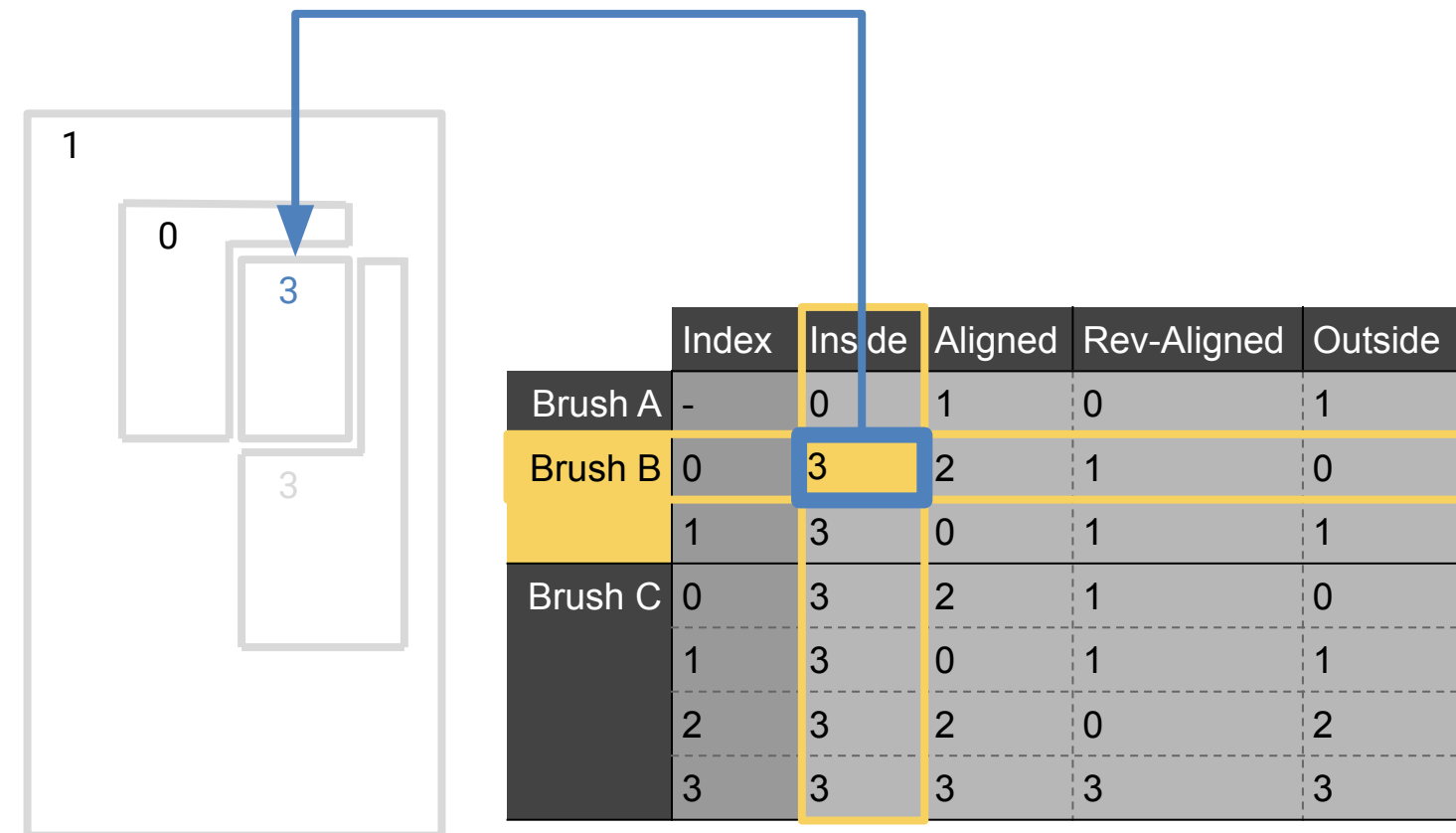
Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else

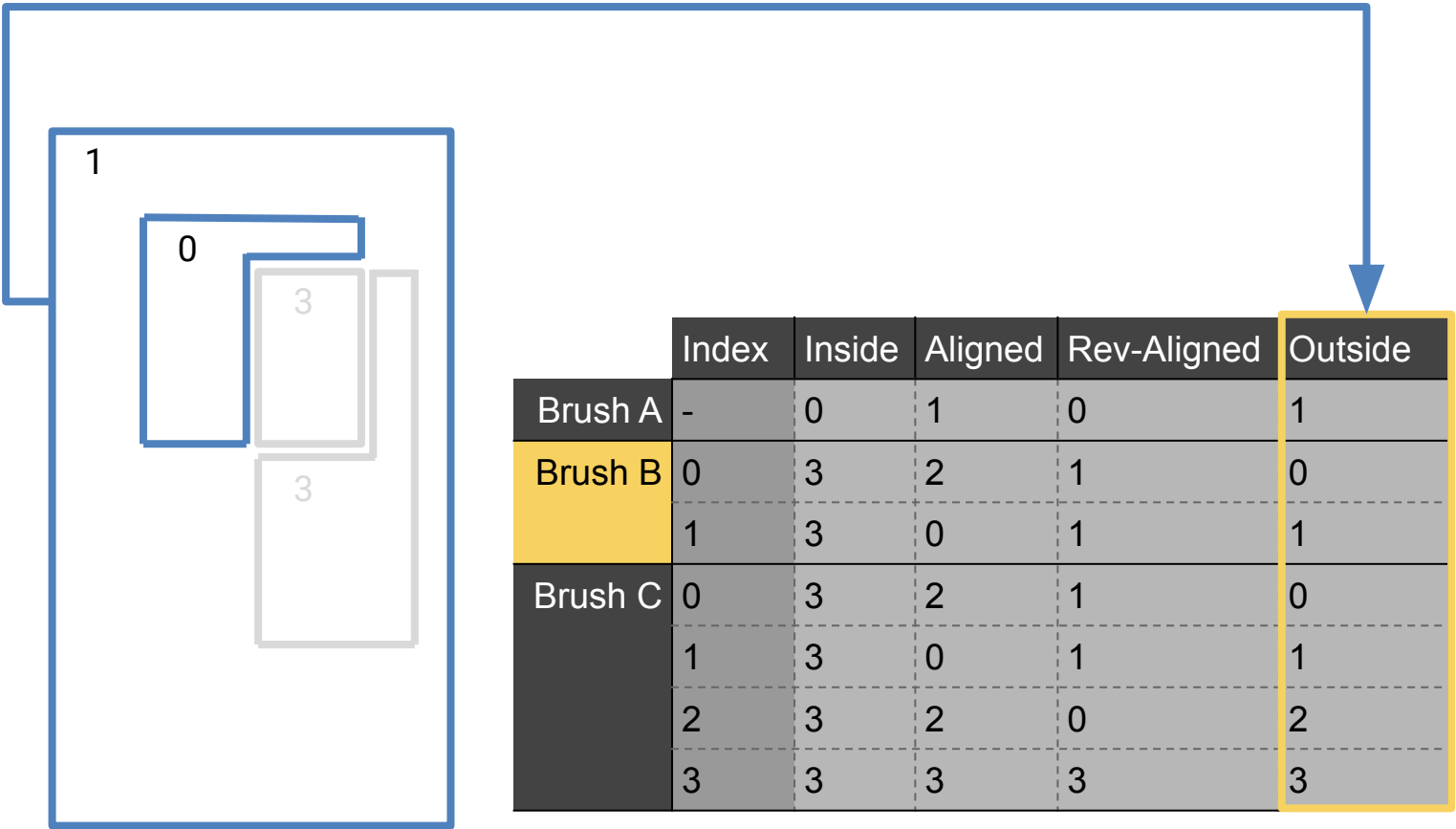




Determine which polygons to show

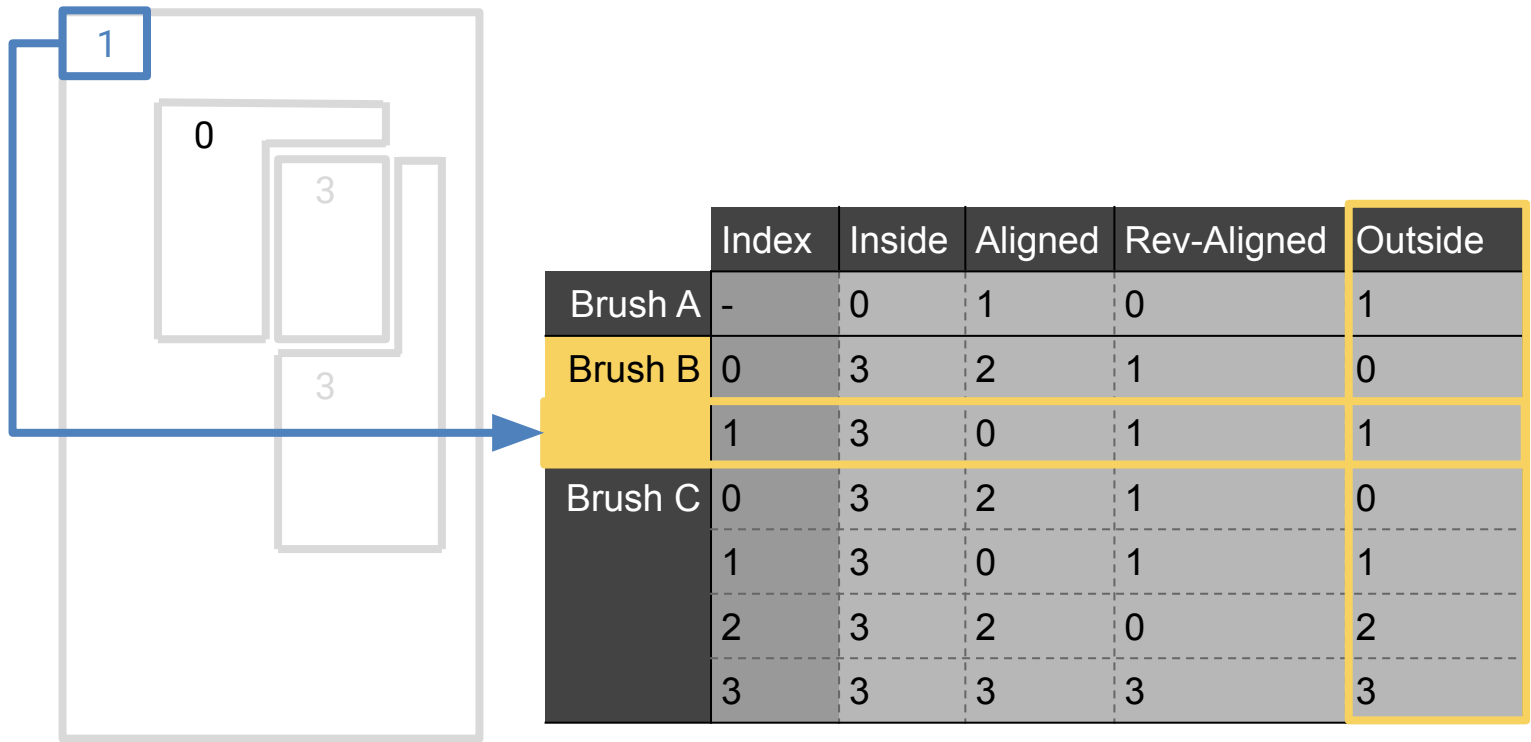
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else

Outside intersection polygon:  
**Outside** (always)



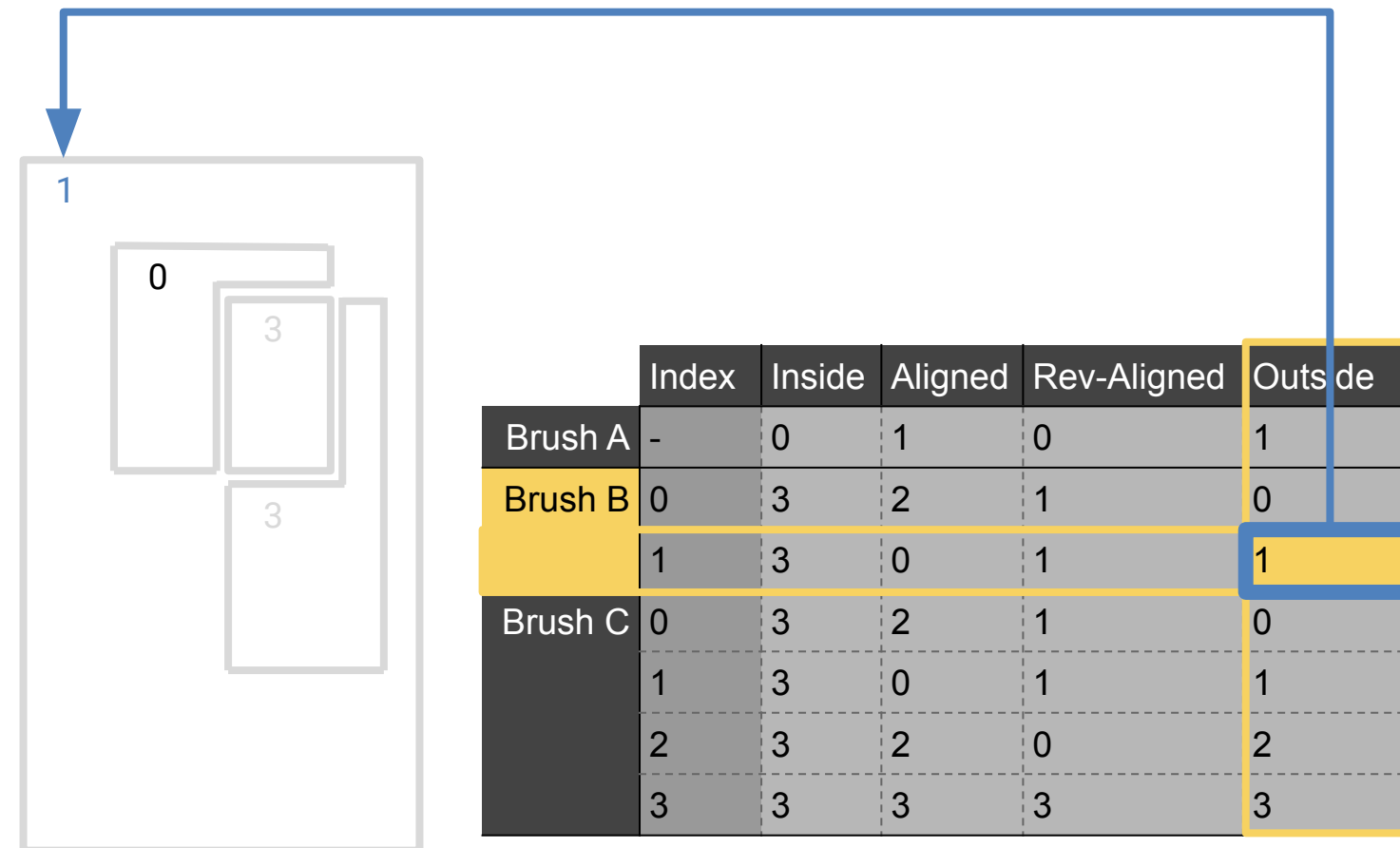
Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



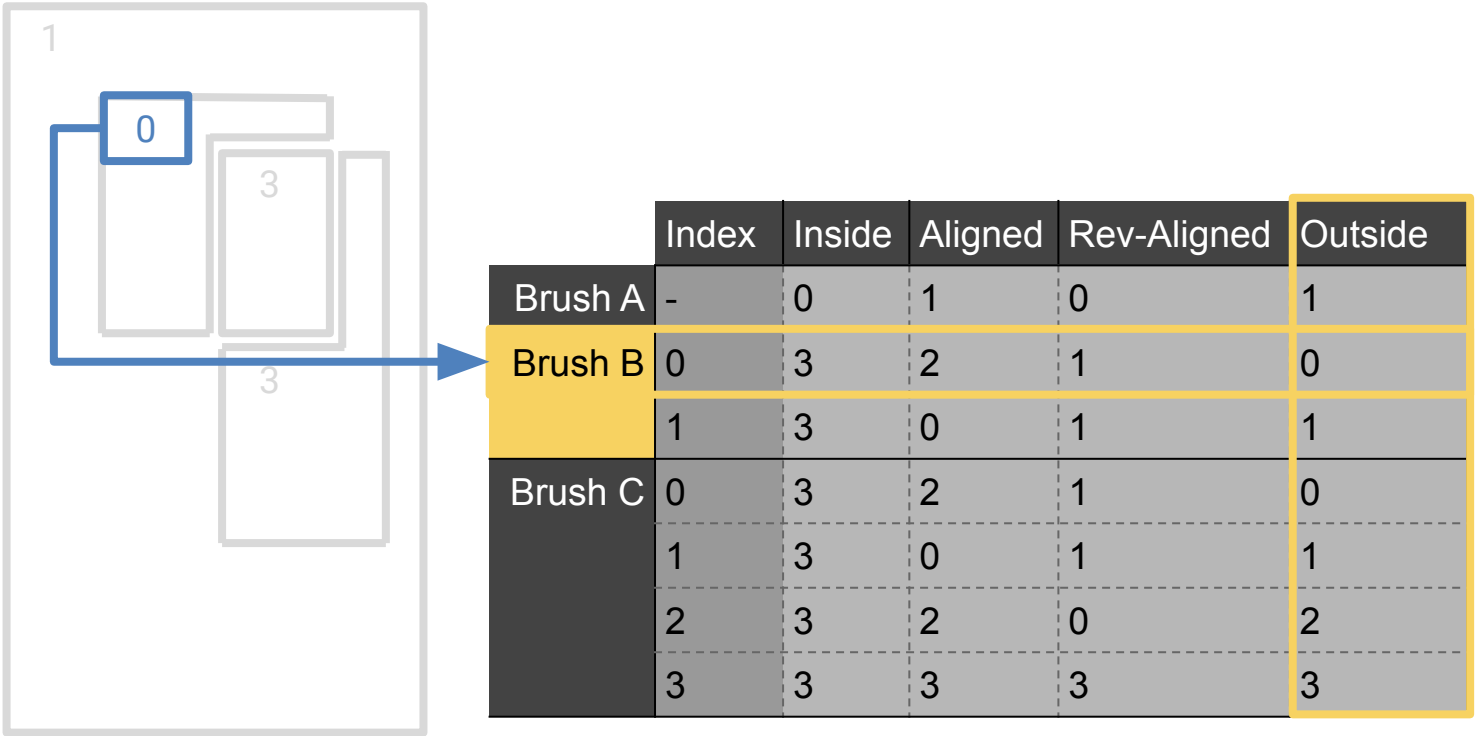
## Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



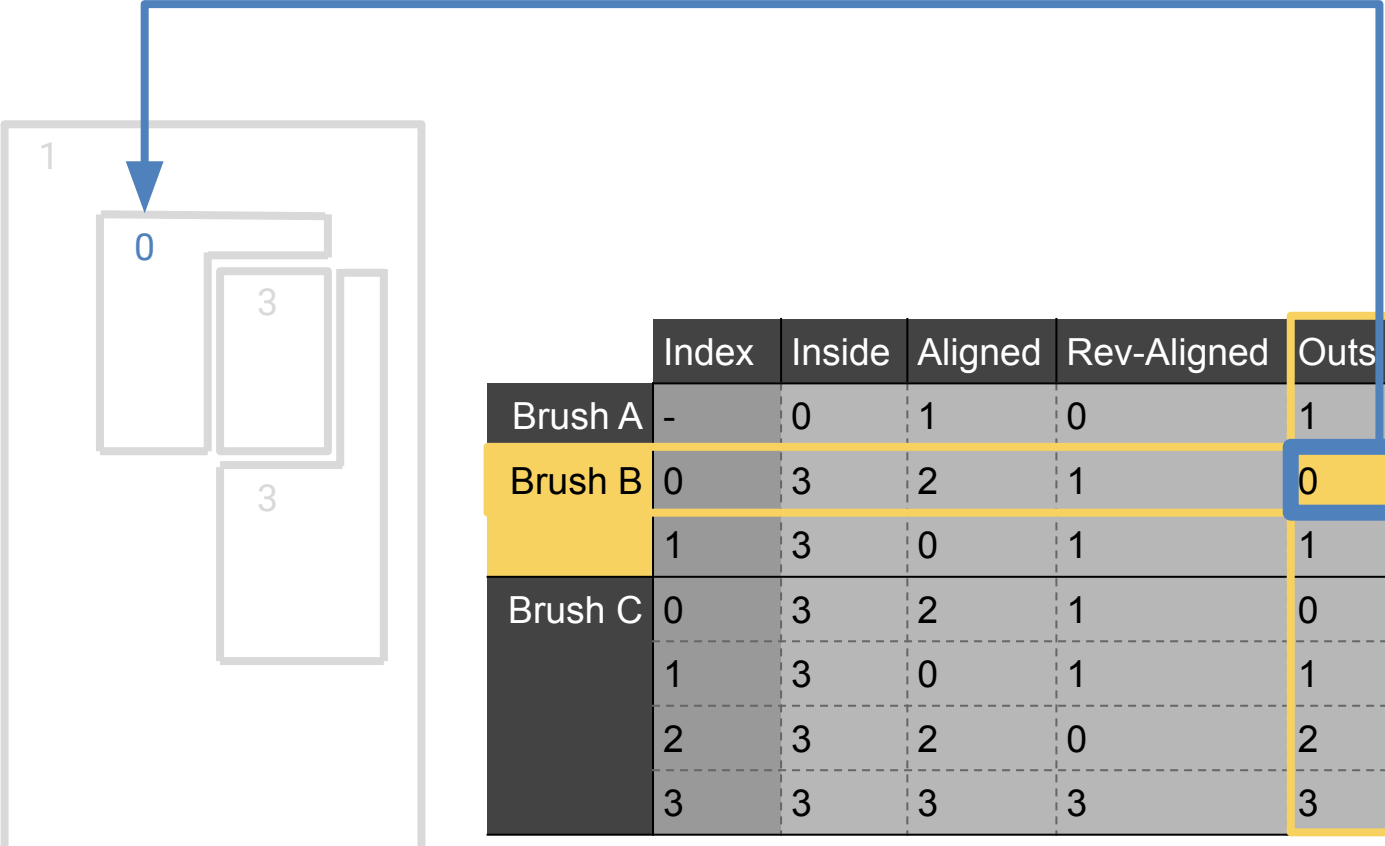
# Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



Determine which polygons to show

- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else

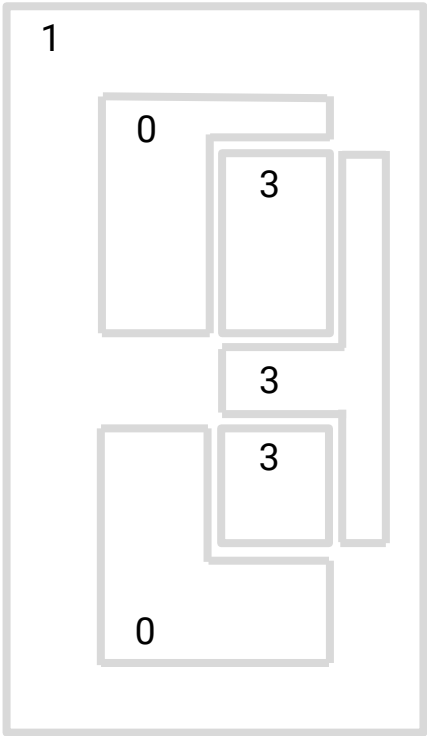


|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |



Determine which polygons to show

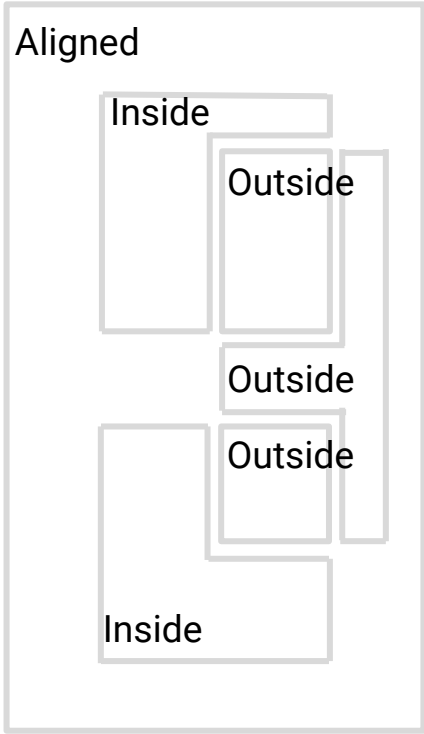
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

Determine which polygons to show

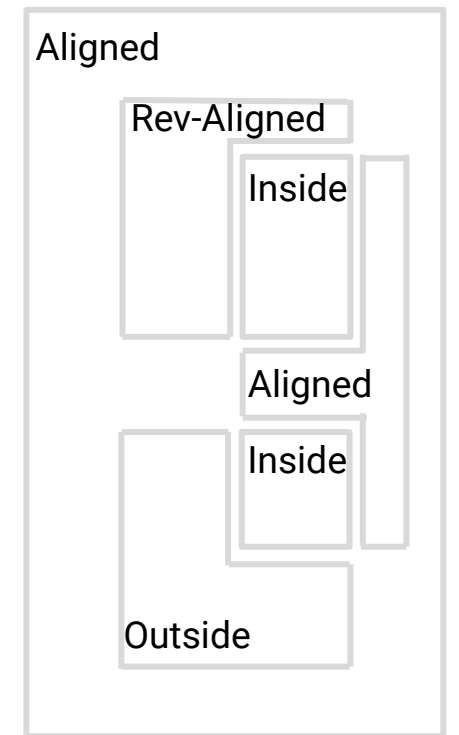
- Use categories on intersection polygons
  - Use interior category (inside polygon)
  - *Outside* for everything else



|         | Index | Inside | Aligned | Rev-Aligned | Outside |
|---------|-------|--------|---------|-------------|---------|
| Brush A | -     | 0      | 1       | 0           | 1       |
| Brush B | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
| Brush C | 0     | 3      | 2       | 1           | 0       |
|         | 1     | 3      | 0       | 1           | 1       |
|         | 2     | 3      | 2       | 0           | 2       |
|         | 3     | 3      | 3       | 3           | 3       |

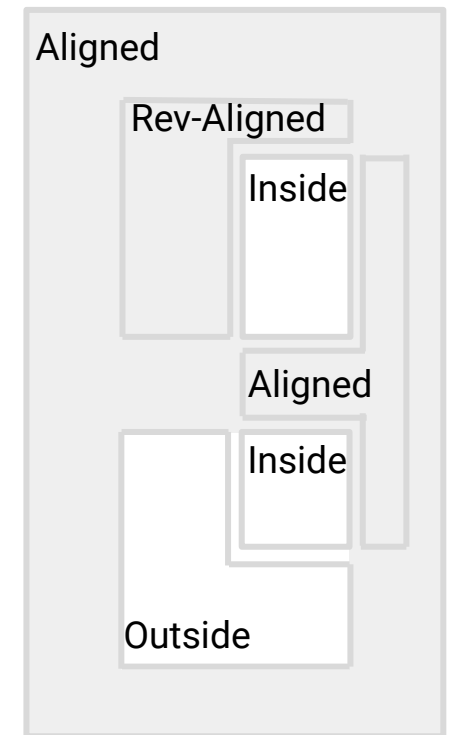
## Creating meshes

- Final polygon category will determine if it's kept or discarded
  - Remember, we only keep (reverse-)aligned polygons



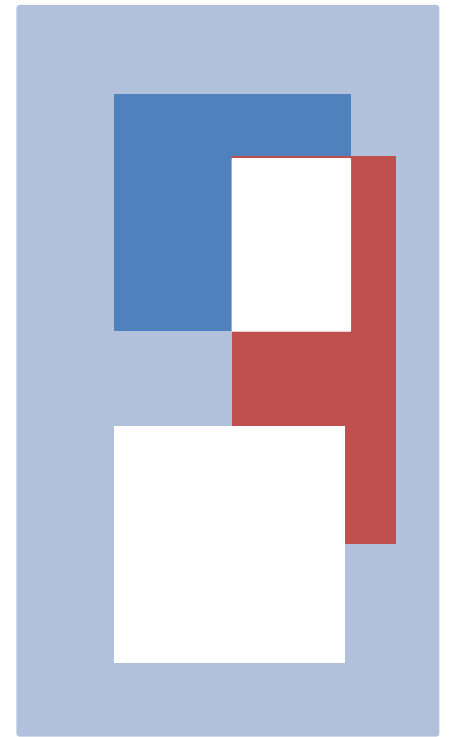
## Creating meshes

- Final polygon category will determine if it's kept or discarded
  - Remember, we only keep (reverse-)aligned polygons



## Creating meshes

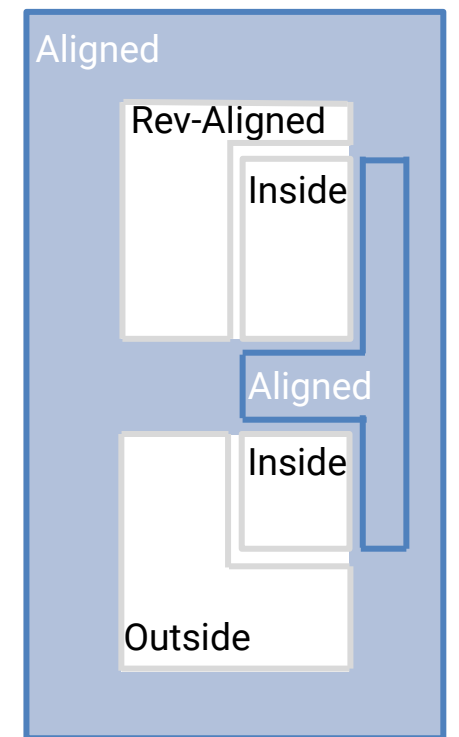
- Final polygon category will determine if it's kept or discarded
  - Remember, we only keep (reverse-)aligned polygons





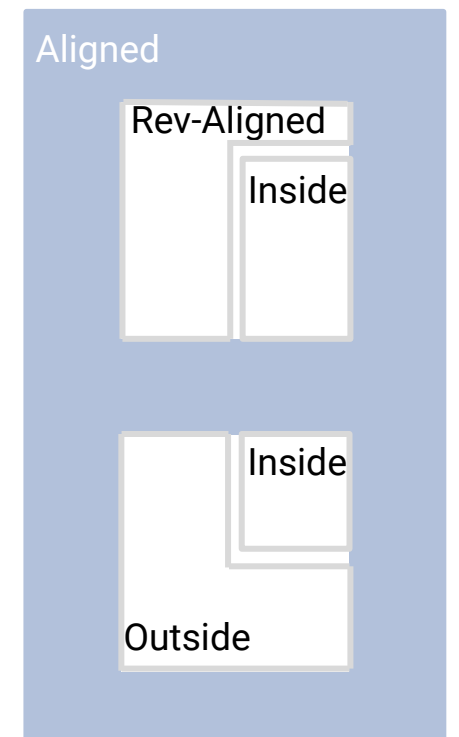
## Creating meshes

- Final polygon category will determine if it's kept or discarded
  - Remember, we only keep (reverse-)aligned polygons
  - We can also combine all polygons that have the same category
    - Remove edges with same indices, but opposite order
    - Identical edges should have their copies removed
    - Combine remaining edges



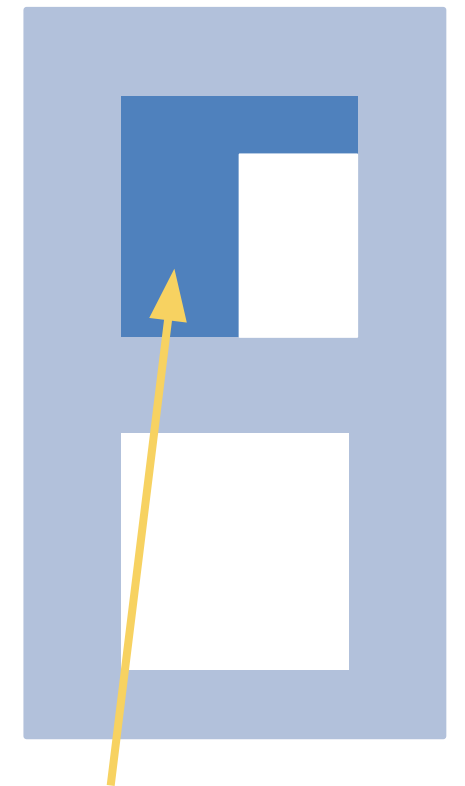
## Creating meshes

- Final polygon category will determine if it's kept or discarded
  - Remember, we only keep (reverse-)aligned polygons
  - We can also combine all polygons that have the same category
    - Remove edges with same indices, but opposite order
    - Identical edges should have their copies removed
    - Combine remaining edges



## Creating meshes

- Final polygon category will determine if it's kept or discarded
  - Remember, we only keep (reverse-)aligned polygons
  - We can also combine all polygons that have the same category
    - Remove edges with same indices, but opposite order
    - Identical edges should have their copies removed
    - Combine remaining edges
- We only triangulate the polygons that we keep / after merging
- All reverse aligned polygons need to be flipped around
  - Reverse vertex index order



**Reverse Aligned** polygons  
need to be **flipped**

## The payoff

- Scalable way of building geometry
- Iterative updates
  - Everything we can do per brush, we can cache per brush!
- Updates can be easily split across multiple cores!

Thank you!

## References

Chisel <https://github.com/RadicalCSG/Chisel.Prototype>

Realtime CSG <https://github.com/LogicalError/realtime-CSG-for-unity/>