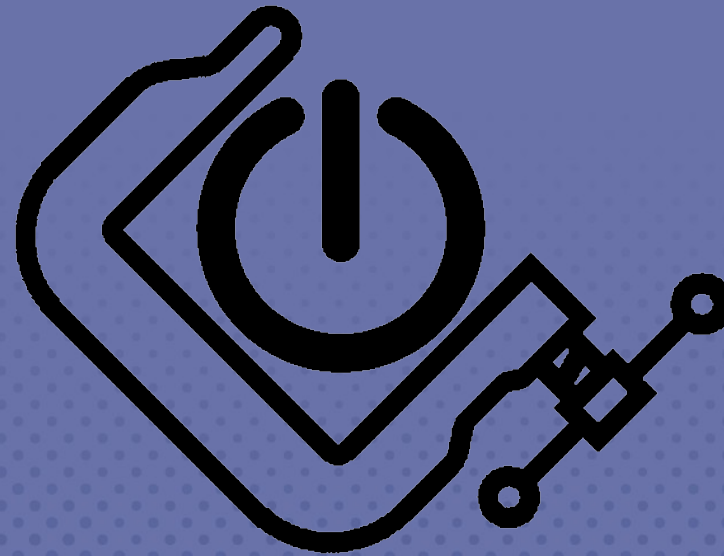


GDC

March 20-24, 2023  
San Francisco, CA

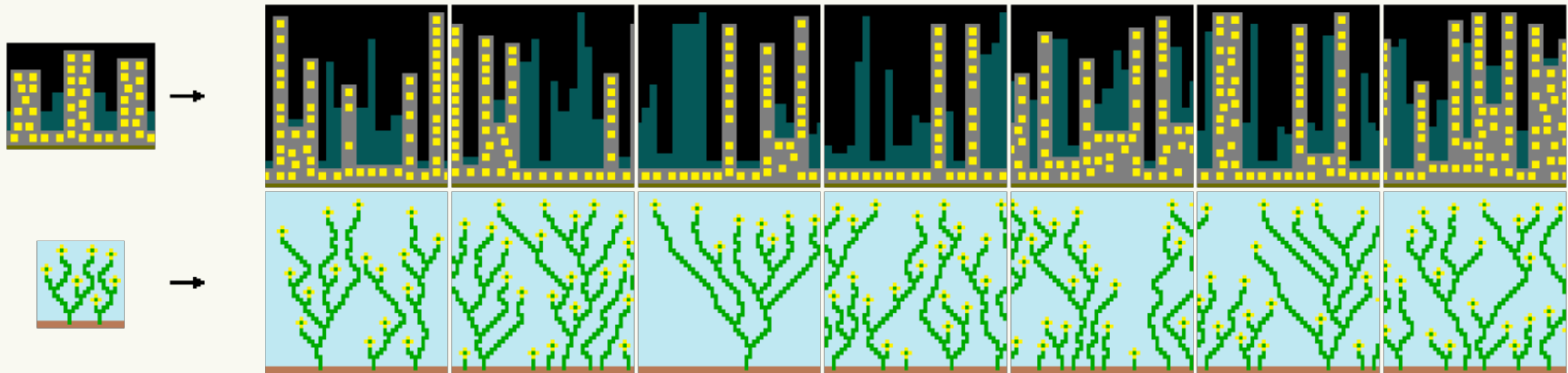
# Beyond WaveFunctionCollapse: Constraint-Based Tile Map Generation and Editing

Seth Cooper  
*Northeastern University*



#GDC23

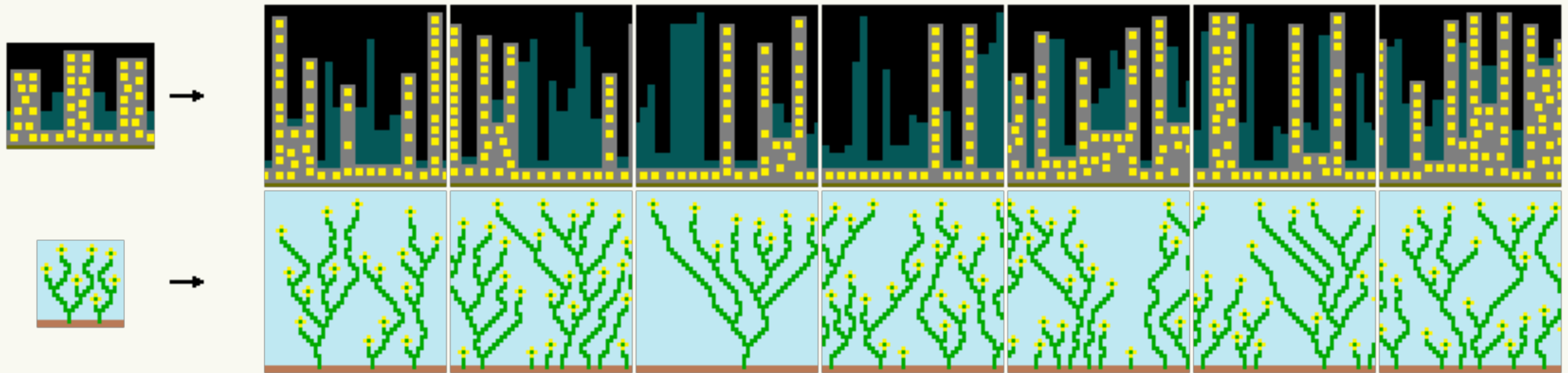
# WaveFunctionCollapse



[Maxim Gumin, <https://github.com/mxgmn/WaveFunctionCollapse>]



# WaveFunctionCollapse



[Maxim Gumin, <https://github.com/mxgmn/WaveFunctionCollapse>]

**Constraint-based Image (and level) generation**

# WaveFunctionCollapse

## Constraints

- Generated images should only contain NxM (e.g. 3x3) patterns from example image (hard)
- The distribution of patterns in generated images should be similar to that in the input (soft)

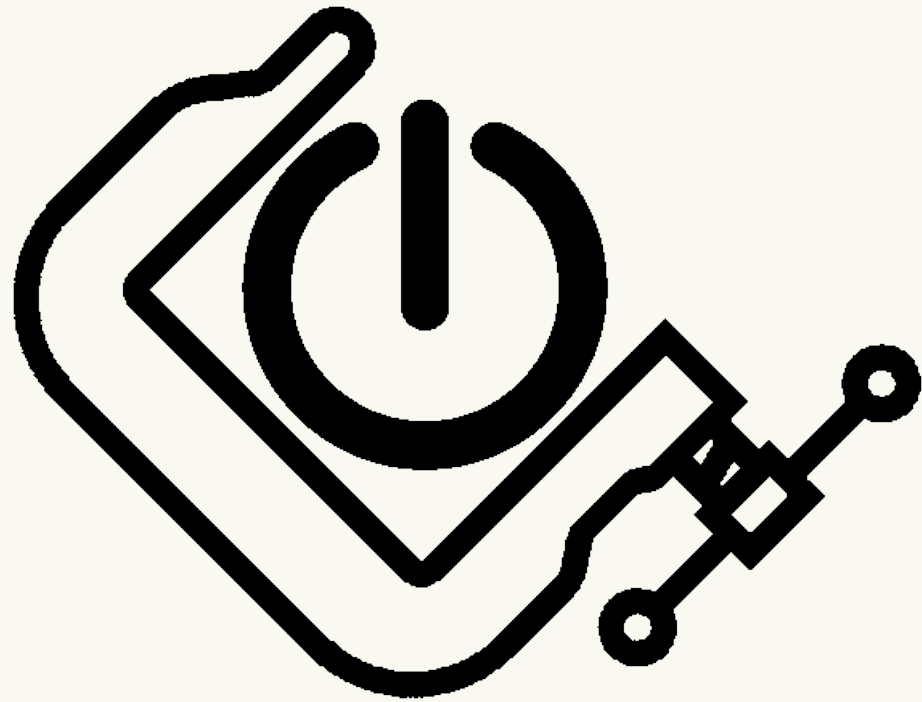
# WaveFunctionCollapse

## Solution Algorithm (roughly)

- Initialize grid so that all patterns can be at all locations
- Repeat:
  - Observation: pick possible pattern to go at a specific location
  - Propagation: update remaining possible patterns at other locations
- Until:
  - Every location has a pattern -> done
  - Some location has no possible patterns -> stuck

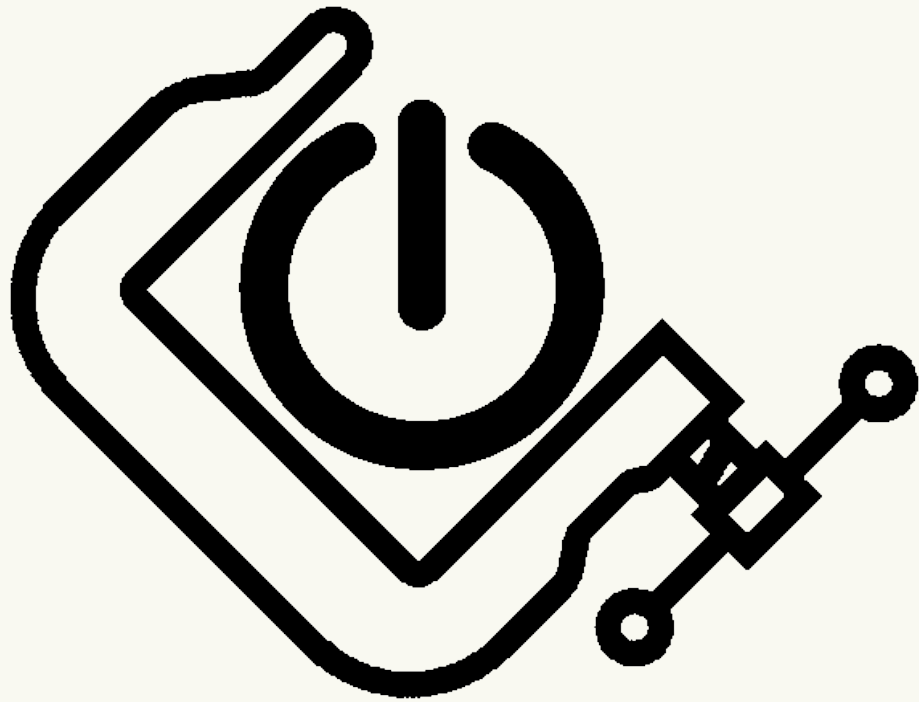


# Constraint-Based Generation



- Express *what* should be in a level (maybe by a few examples) rather than *how* to generate it.
- Could decouple constraints and solver, “plug in” standard constraint solvers.
- “Modular” combination of constraints.

# Constraint-Based Generation



- Surgeon level generation system
- Example levels and applications
- More extensions



# Constraint-Based Generation

## Sturgeon

System for (generally 2D, tile-based) level generation and editing  
via (Boolean) constraint solving



# Constraint-Based Generation

## Sturgeon

System for (generally 2D, tile-based) level generation and editing  
via (Boolean) constraint solving

Set up generic (Boolean) constraint problem

# Constraint-Based Generation

## Sturgeon

System for (generally 2D, tile-based) level generation and editing  
via (Boolean) constraint solving

Set up generic (Boolean) constraint problem

## Give to low-level solver

Takes collection of Boolean variables and constraints  
Returns true/false assignment for variables that satisfies constraints  
(all hard, as many soft as possible)

# Constraint-Based Generation

## Sturgeon

System for (generally 2D, tile-based) level generation and editing  
via (Boolean) constraint solving

Set up generic (Boolean) constraint problem

Give to low-level solver

SAT-style [PySAT]; SMT; Answer Set; portfolio

# Constraint-Based Generation

## Sturgeon

System for (generally 2D, tile-based) level generation and editing  
via (Boolean) constraint solving

Set up generic (Boolean) constraint problem

Give to low-level solver

SAT-style [PySAT]; SMT; Answer Set; portfolio

Process solution into a level



# Constraint-Based Generation

## Sturgeon

System for (generally 2D, tile-based) level generation and editing  
via (Boolean) constraint solving

Set up generic (Boolean) constraint problem

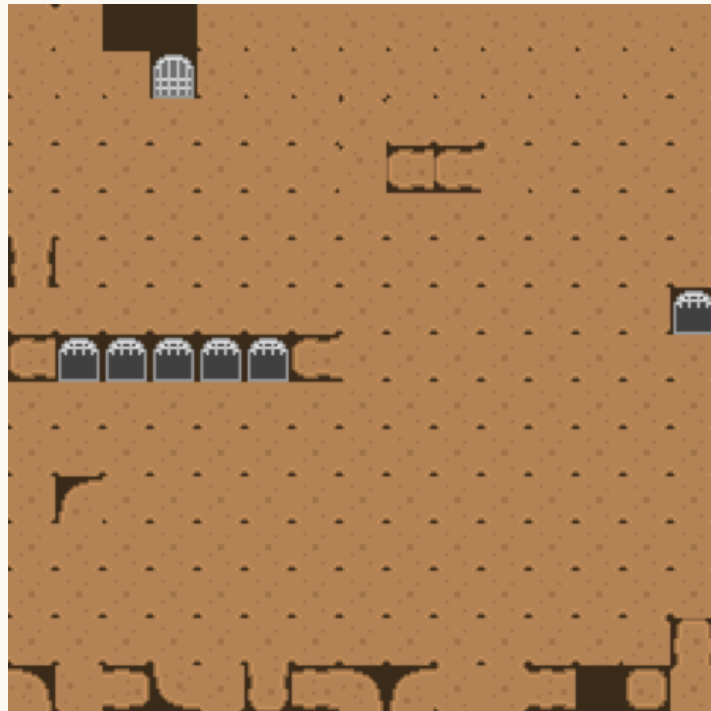
Give to low-level solver

SAT-style [PySAT]; SMT; Answer Set; portfolio

Process solution into a level

*What constraints does Sturgeon use to generate a level?*

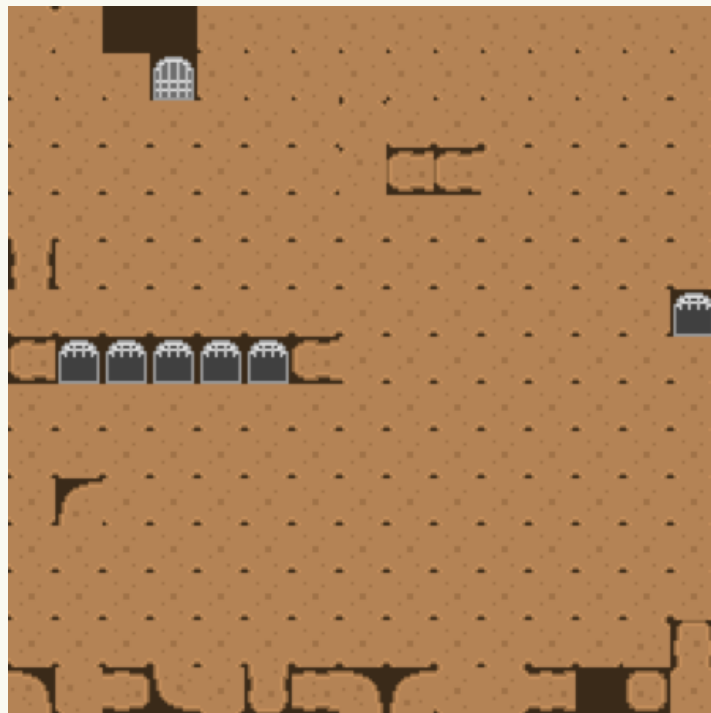
# Constraint-Based Generation



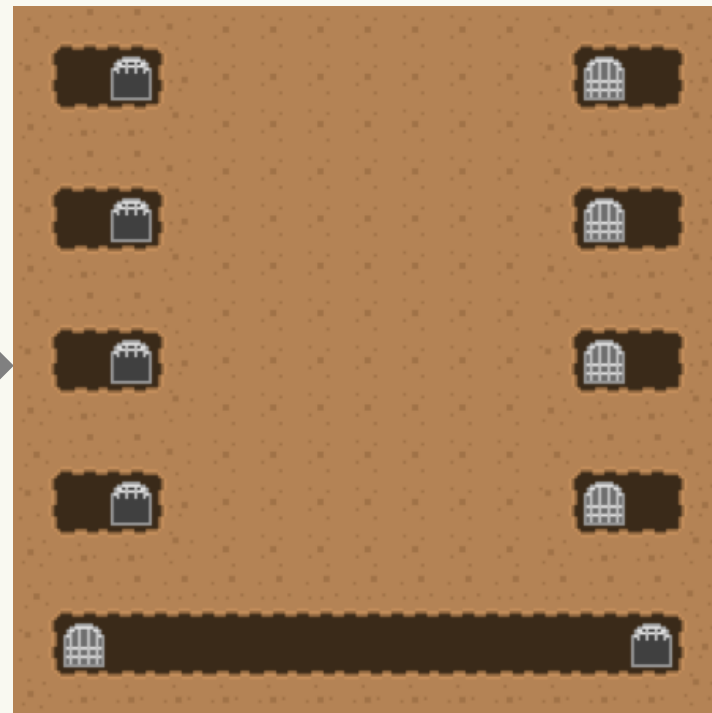
Tile



# Constraint-Based Generation



Tile

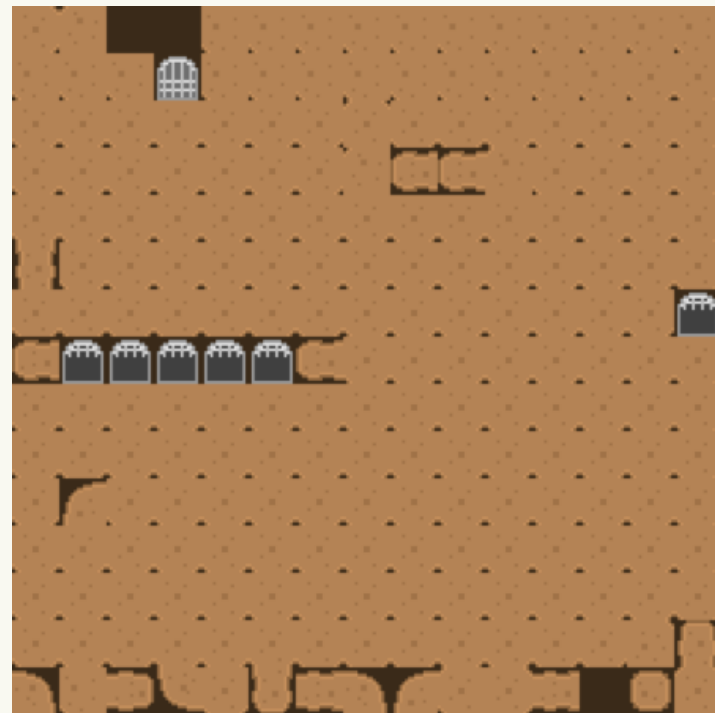


Pattern

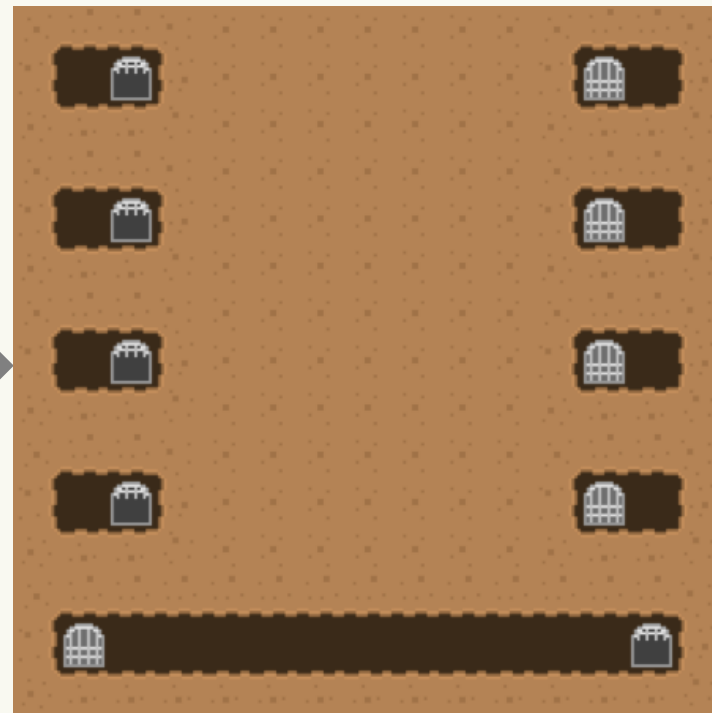
(like WFC solver)



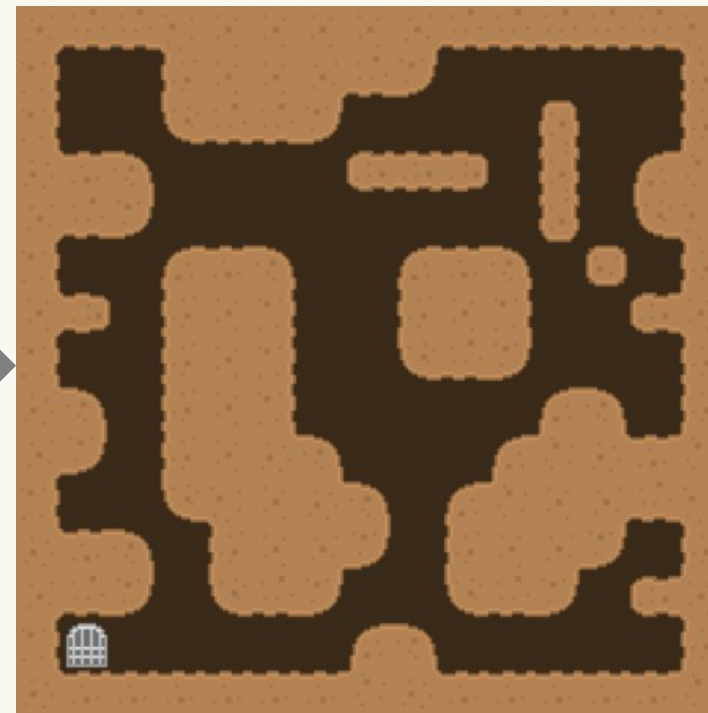
# Constraint-Based Generation



Tile



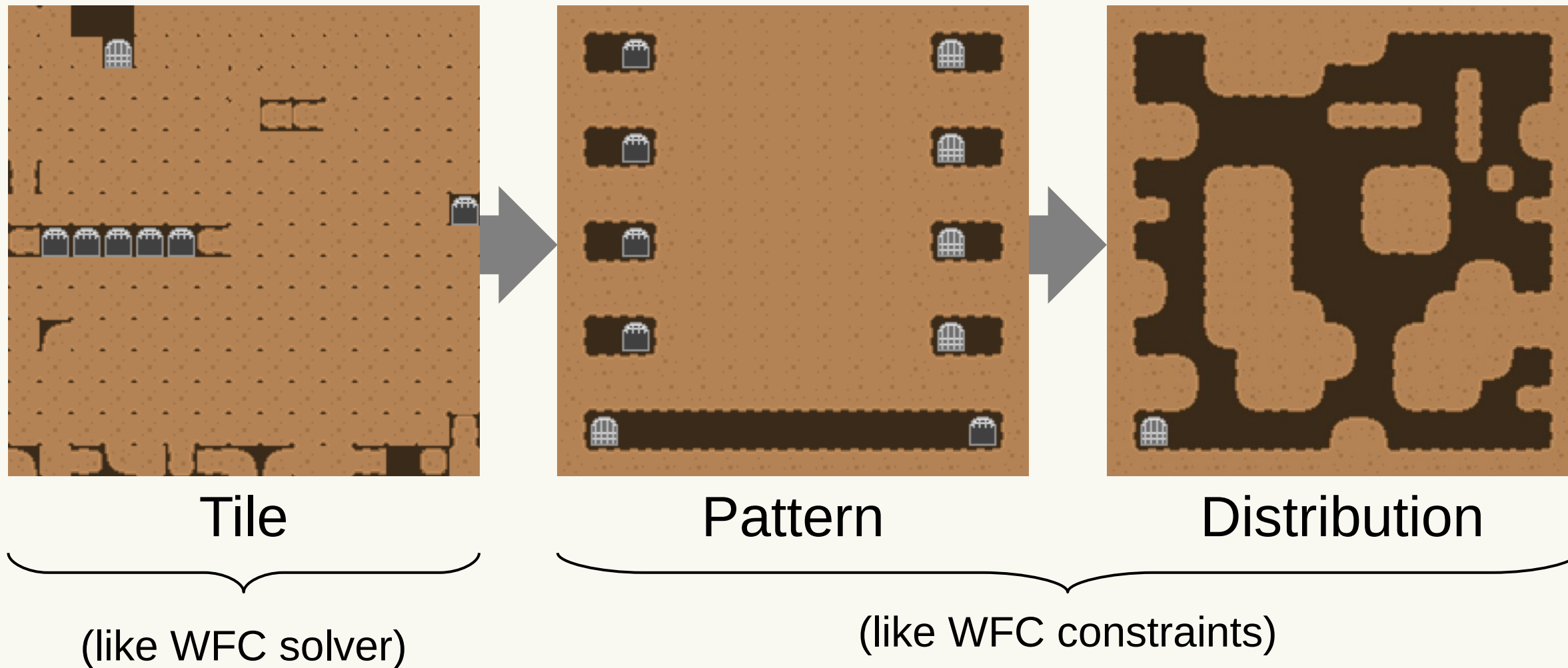
Pattern



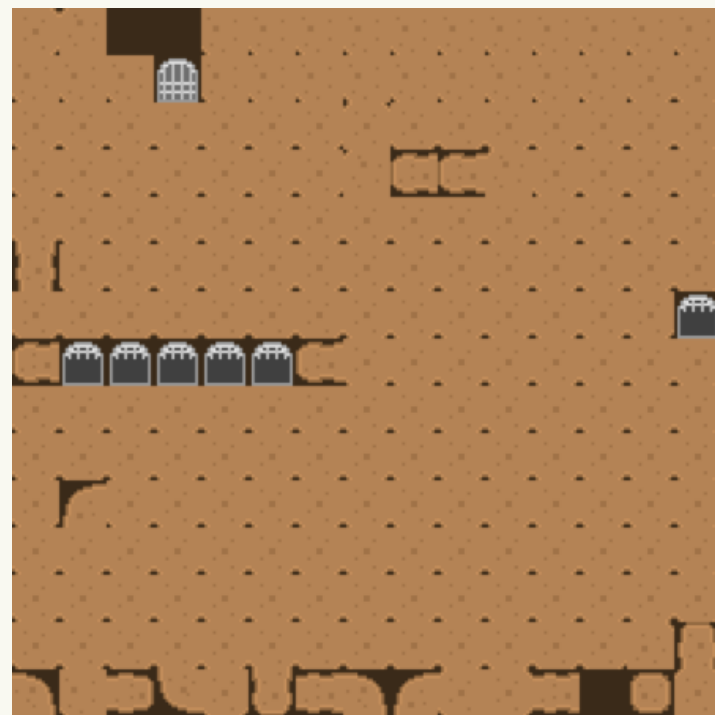
Distribution

(like WFC solver)

# Constraint-Based Generation

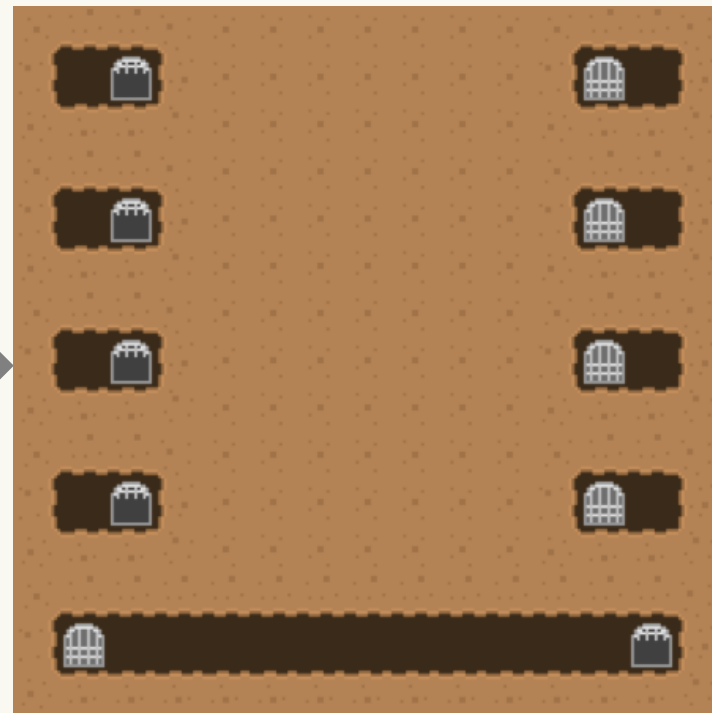


# Constraint-Based Generation

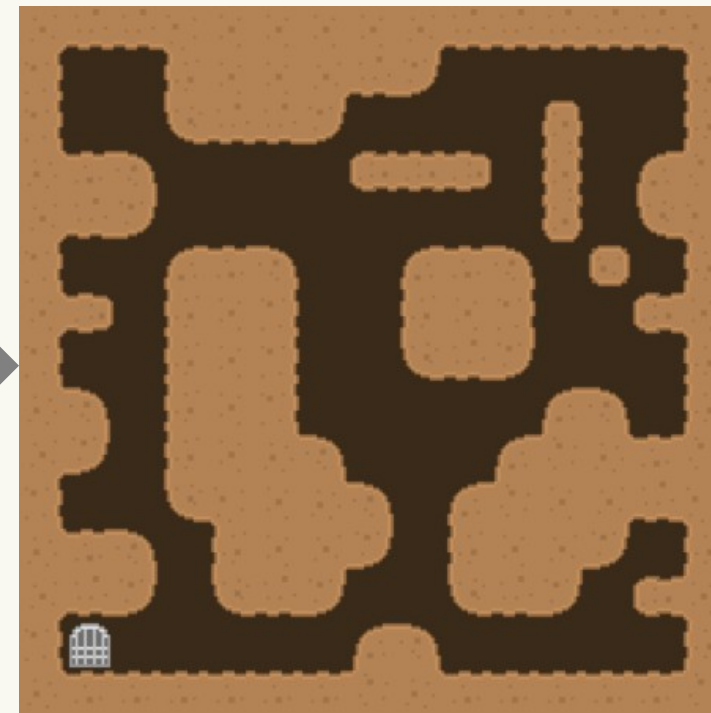


Tile

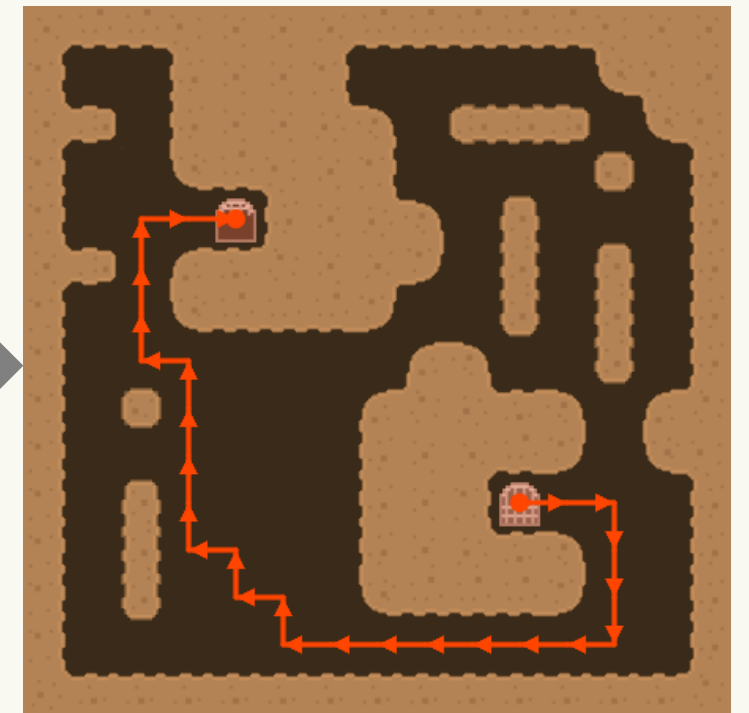
(like WFC solver)



Pattern



Distribution

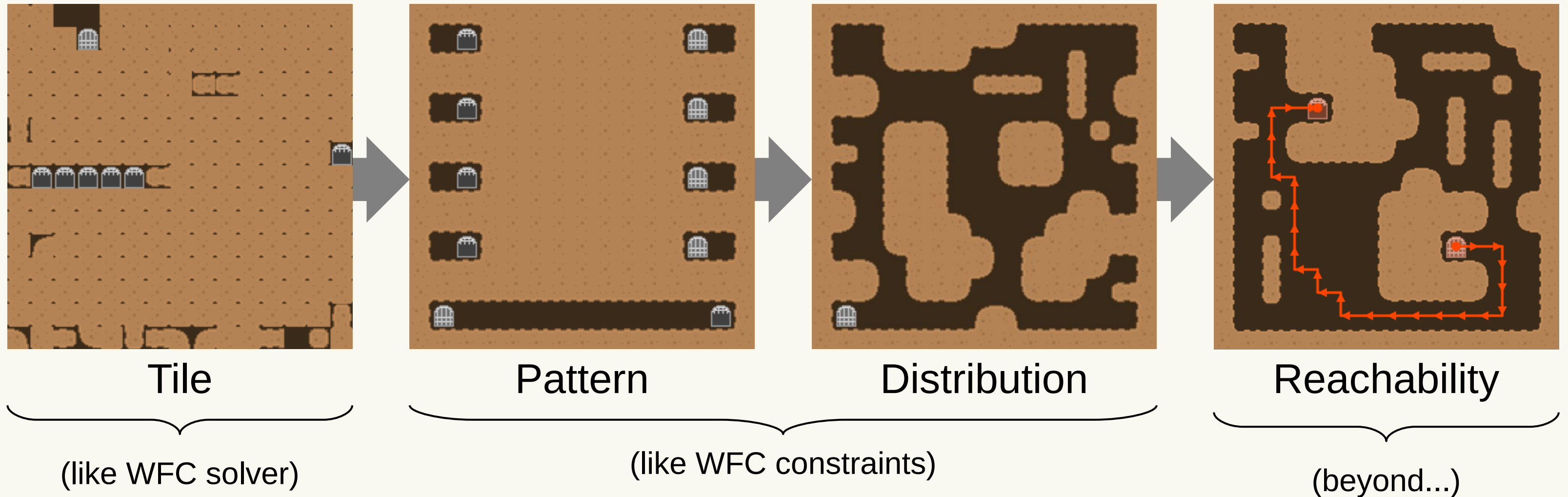


Reachability

(beyond...)



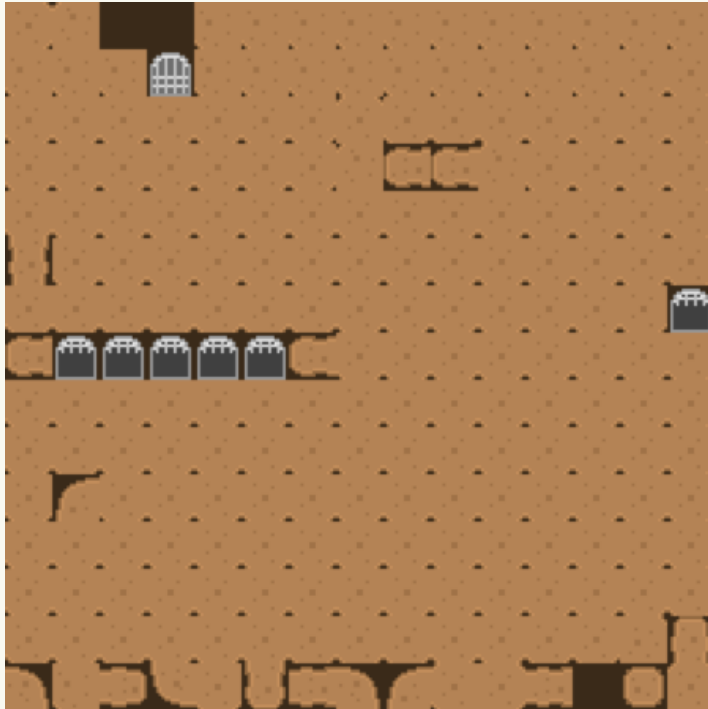
# Constraint-Based Generation



*Take a closer look at how Sturgeon sets up and uses these constraints...*



# Tile Constraints

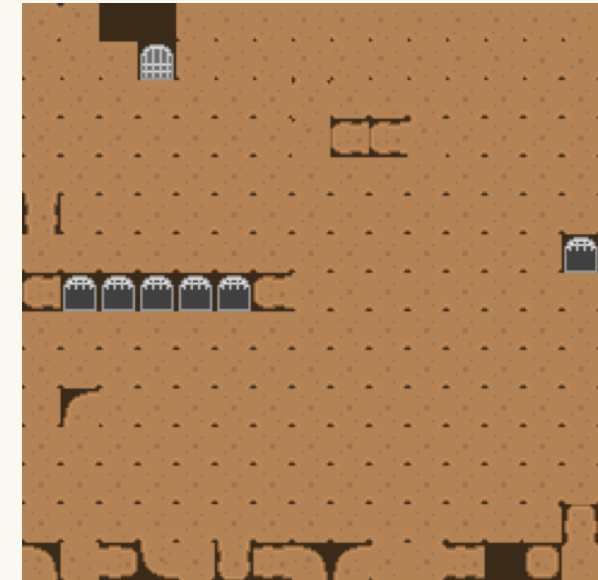


Tile

# Tile Constraints

## Outline

Setup: make a var at each location, for each possible tile there.

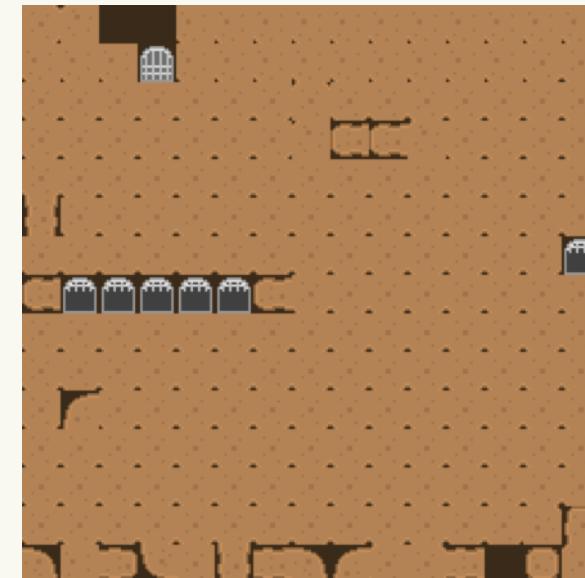


## Interface

# Tile Constraints

## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`



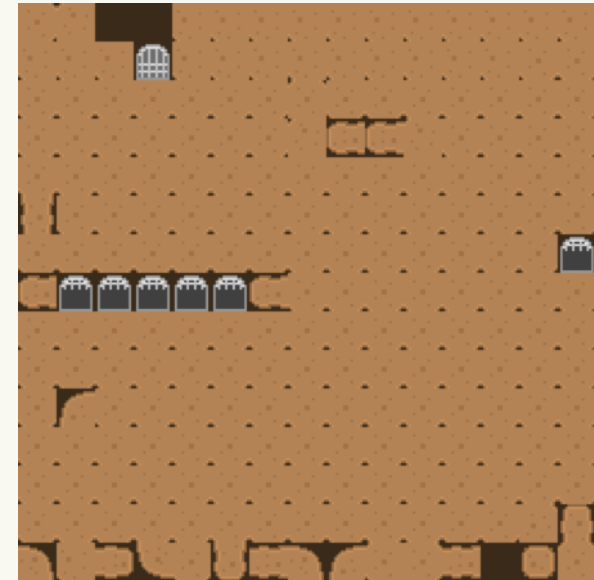
Interface  
`MakeVar()`

# Tile Constraints

## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`

Setup: exactly 1 var can be true at each location.



Interface  
`MakeVar( )`

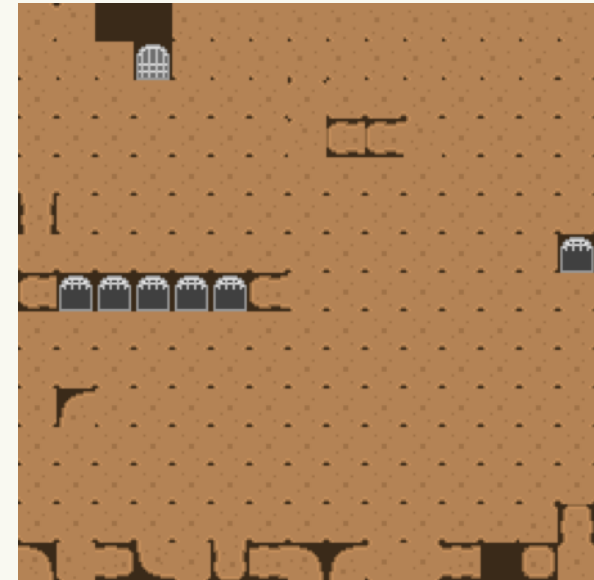


# Tile Constraints

## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`

Setup: exactly 1 var can be true at each location.  
`CnstrCount(tileVarsAtLocation, 1, 1, HARD)`



## Interface

`MakeVar()`

`CnstrCount(...)`

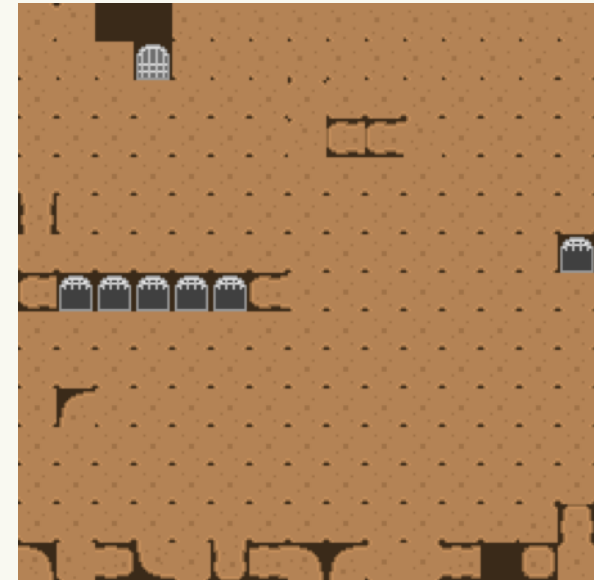
# Tile Constraints

## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`

Setup: exactly 1 var can be true at each location.  
`CnstrCount(tileVarsAtLocation, 1, 1, HARD)`

Find a solution.



Interface  
`MakeVar()`  
`CnstrCount(...)`

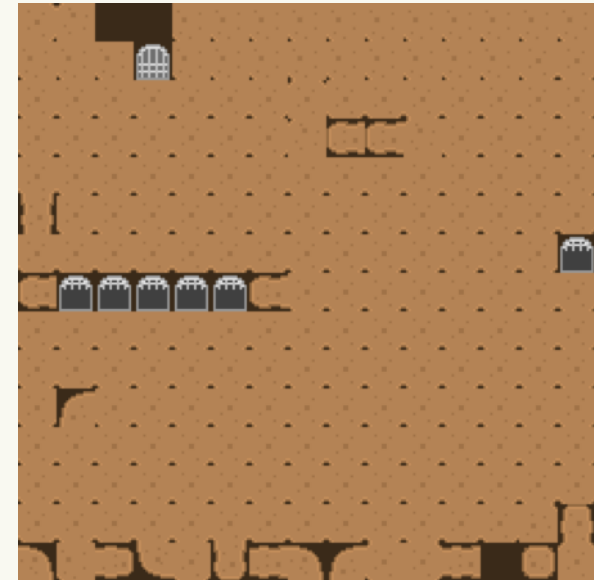
# Tile Constraints

## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`

Setup: exactly 1 var can be true at each location.  
`CnstrCount(tileVarsAtLocation, 1, 1, HARD)`

Find a solution.  
`Solve()`



## Interface

```
MakeVar()  
CnstrCount(...)  
Solve()
```

# Tile Constraints

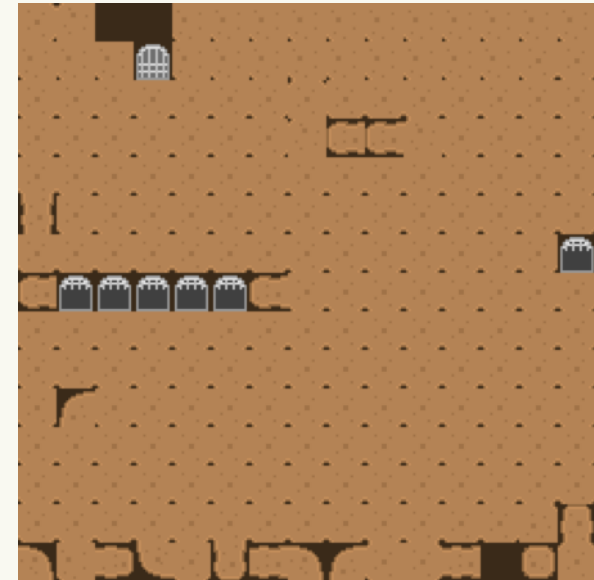
## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`

Setup: exactly 1 var can be true at each location.  
`CnstrCount(tileVarsAtLocation, 1, 1, HARD)`

Find a solution.  
`Solve()`

Process solution: at each location, find the var set to true.



Interface  
`MakeVar()`  
`CnstrCount(...)`  
`Solve()`



# Tile Constraints

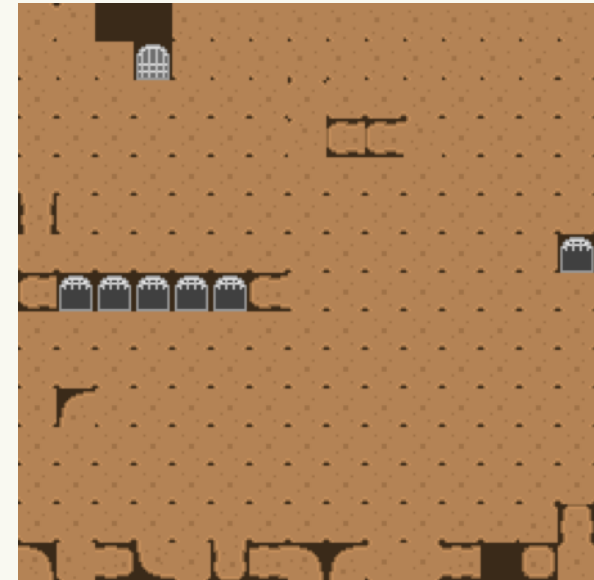
## Outline

Setup: make a var at each location, for each possible tile there.  
`tile = MakeVar()`

Setup: exactly 1 var can be true at each location.  
`CnstrCount(tileVarsAtLocation, 1, 1, HARD)`

Find a solution.  
`Solve()`

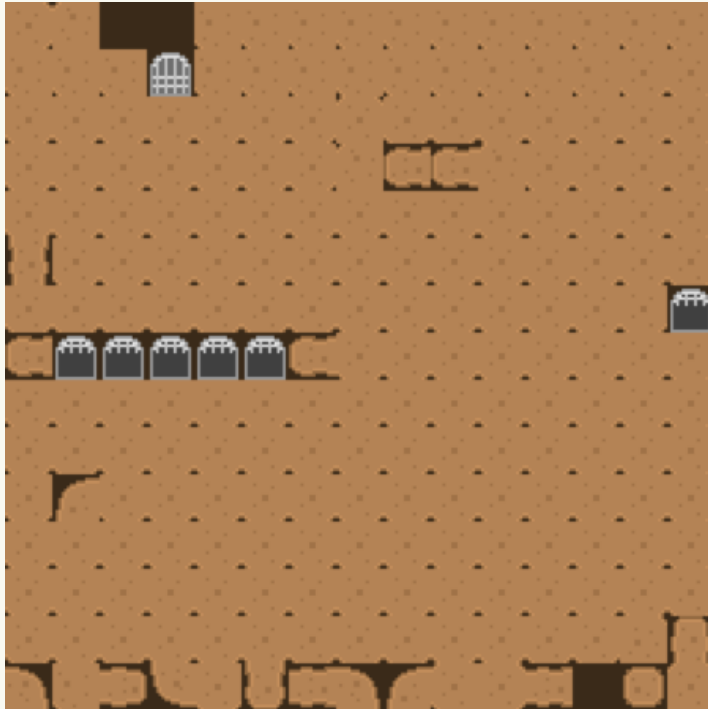
Process solution: at each location, find the var set to true.  
`GetVar(tileVar)`



## Interface

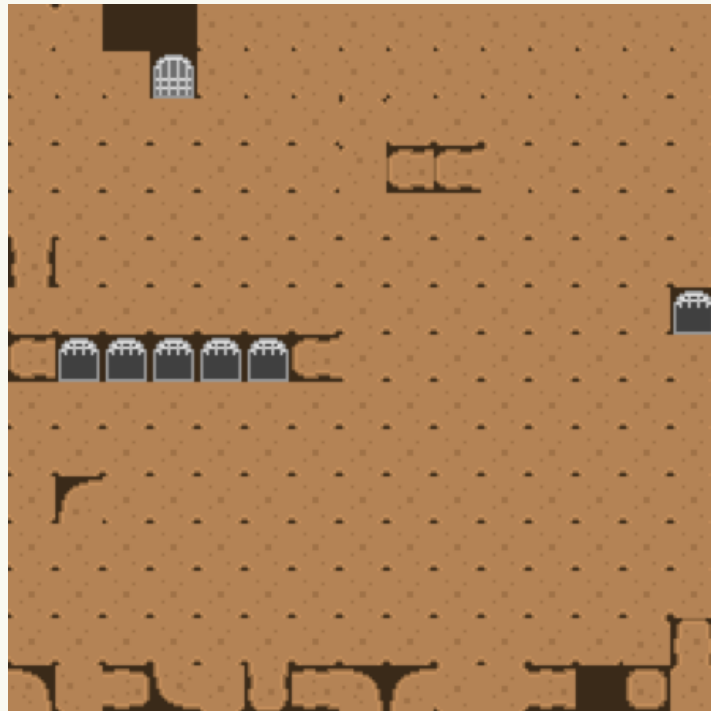
```
MakeVar()  
CnstrCount(...)  
Solve()  
GetVar(...)
```

# Pattern Constraints

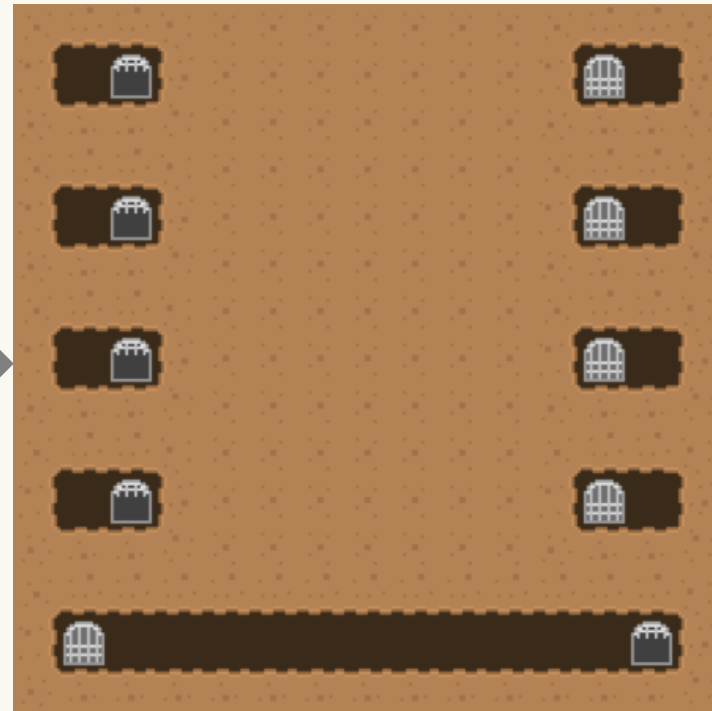


Tile

# Pattern Constraints



Tile

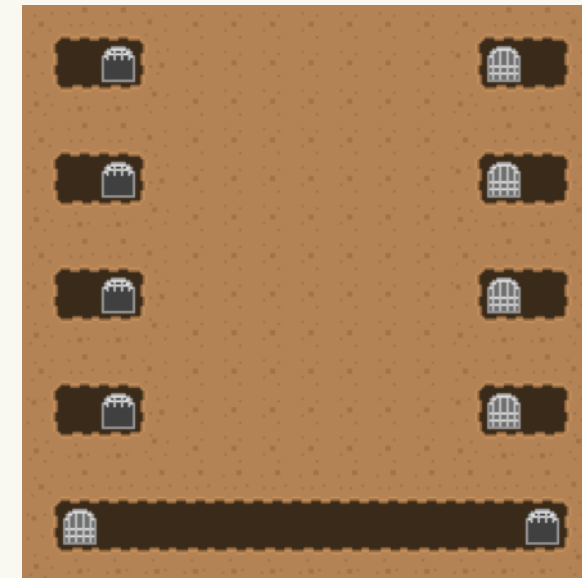


Pattern

# Pattern Constraints

## Outline

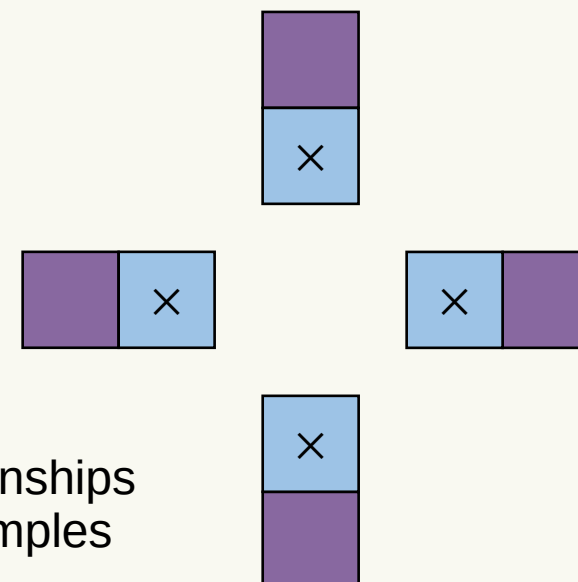
Setup tile constraints.



Setup: at each location, using an example level and pattern template,

Find and process solution.

Pattern template



What local relationships  
to use from examples

Example level





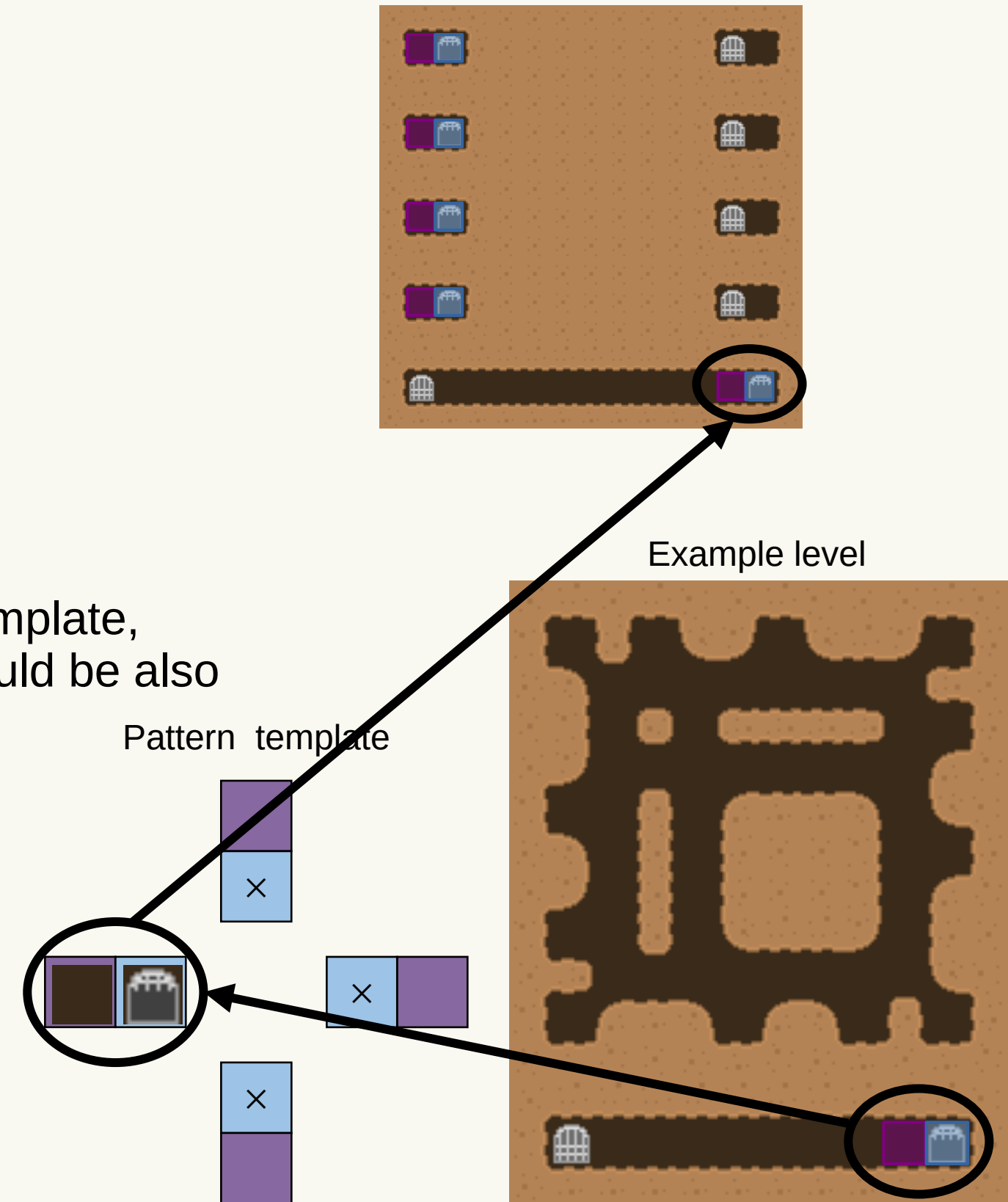
# Pattern Constraints

## Outline

Setup tile constraints.

Setup: at each location, using an example level and pattern template, an **input pattern** there means a relative **output pattern** should be also

Find and process solution.



# Pattern Constraints

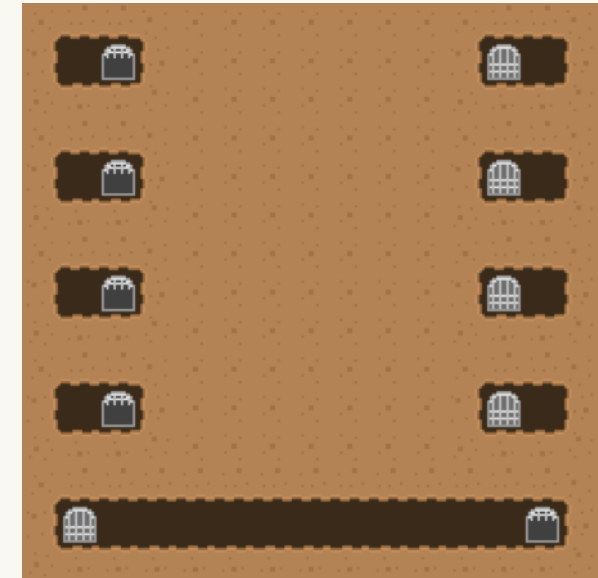
## Outline

Setup tile constraints.

Setup: at each location, using an example level and pattern template,  
an **input pattern** there means a relative **output pattern** should be also

```
CnstrImpliesOr(inPattern,  
              outPatternsSeen, HARD)
```

Find and process solution.



## Interface

**MakeVar()**

**CnstrCount(...)**

**Solve()**

**GetVar(...)**

**CnstrImpliesOr(...)**

# Pattern Constraints

## Outline

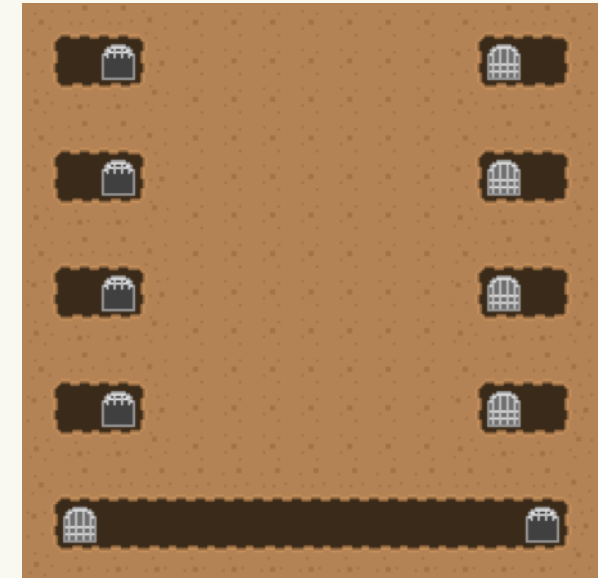
Setup tile constraints.

Setup: individual tile variables can be organized into *patterns* by *templates*.

Setup: at each location, using an example level and pattern template,  
an **input pattern** there means a relative **output pattern** should be also

```
CnstrImpliesOr(inPattern,  
              outPatternsSeen, HARD)
```

Find and process solution.



## Interface

**MakeVar( )**

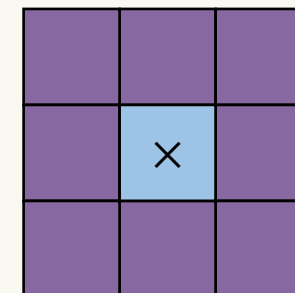
**CnstrCount( . . . )**

**Solve( )**

**GetVar( . . . )**

**CnstrImpliesOr( . . . )**

Pattern template



# Pattern Constraints

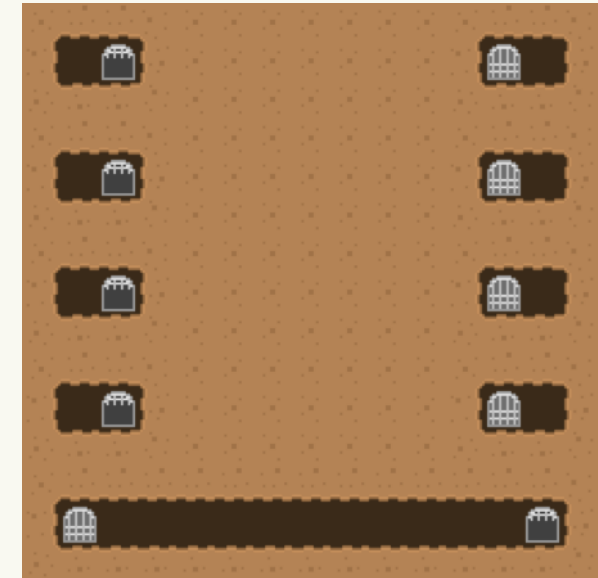
## Outline

Setup tile constraints.

Setup: individual tile variables can be organized into *patterns* by *templates*.  
`pattern = MakeAnd(patternTileVars)`

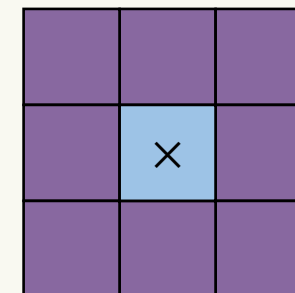
Setup: at each location, using an example level and pattern template,  
an **input pattern** there means a relative **output pattern** should be also  
`CnstrImpliesOr(inPattern,  
                  outPatternsSeen, HARD)`

Find and process solution.



Interface  
`MakeVar( )  
CnstrCount( . . . )  
Solve( )  
GetVar( . . . )  
CnstrImpliesOr( . . . )  
MakeAnd( . . . )`

Pattern template





# Pattern Constraints

## Outline

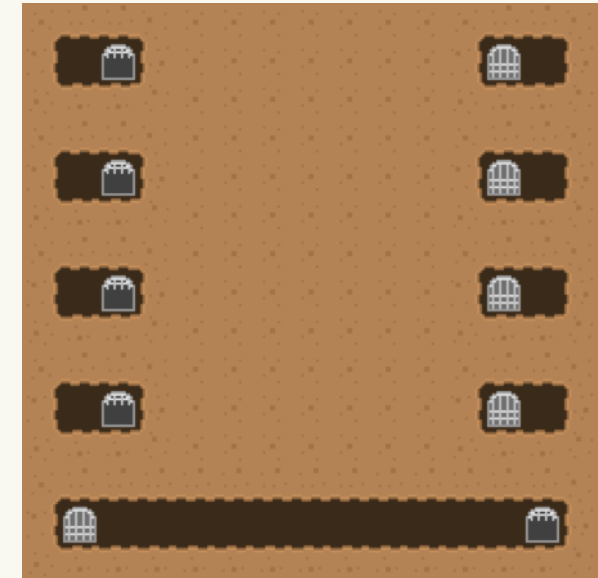
Setup tile constraints.

Setup: individual tile variables can be organized into *patterns* by *templates*.  
`pattern = MakeAnd(patternTileVars)`

Setup: at each location, using an example level and pattern template,  
an **input pattern** there means a relative **output pattern** should be also  
`CnstrImpliesOr(inPattern,  
                  outPatternsSeen, HARD)`

Setup: at each location, using templates, at least one **input pattern** must exist.  
`CnstrCount(inPatternsAtLocation, 1, inf, HARD)`

Find and process solution.



## Interface

`MakeVar( )`

`CnstrCount( . . . )`

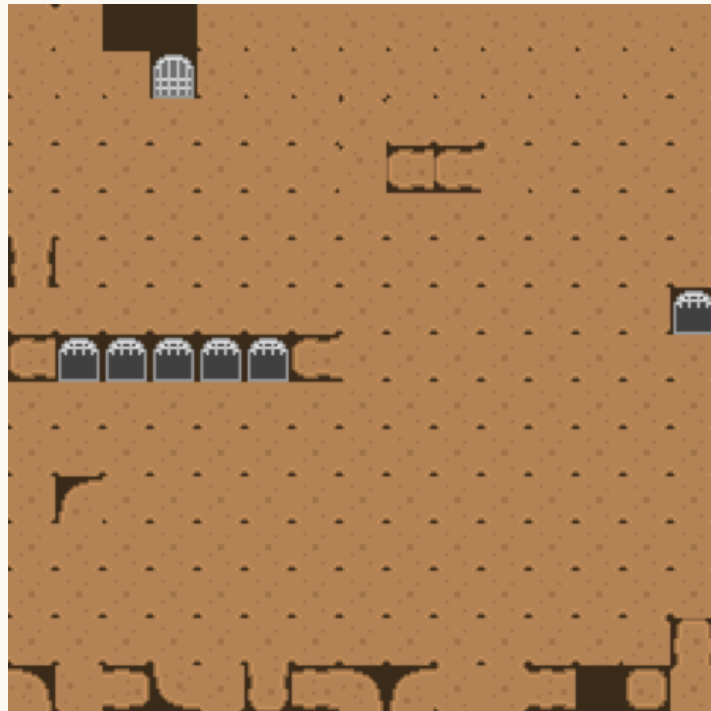
`Solve( )`

`GetVar( . . . )`

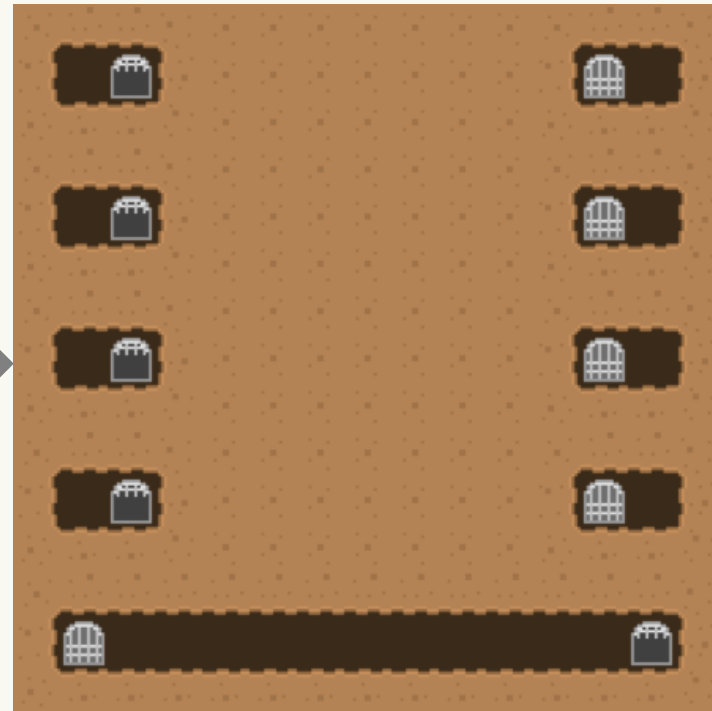
`CnstrImpliesOr( . . . )`

`MakeAnd( . . . )`

# Distribution Constraints

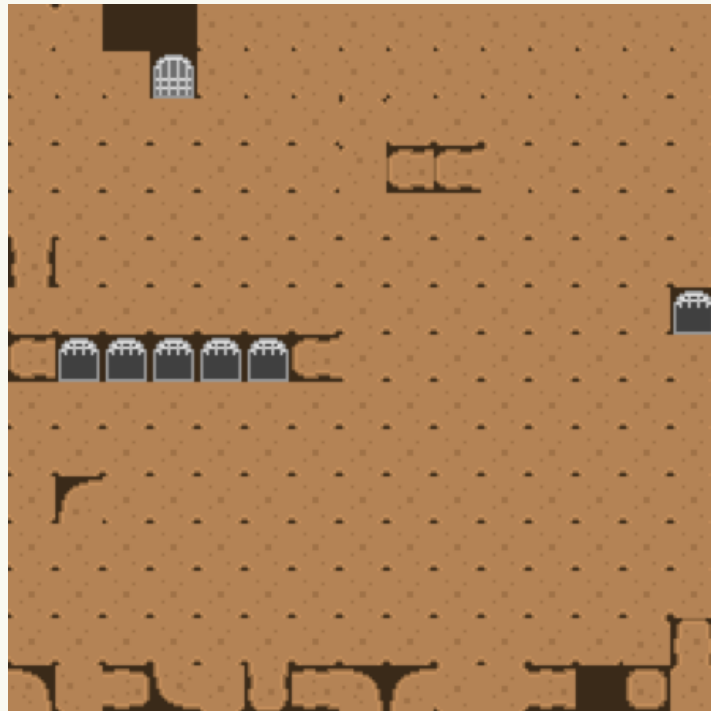


Tile

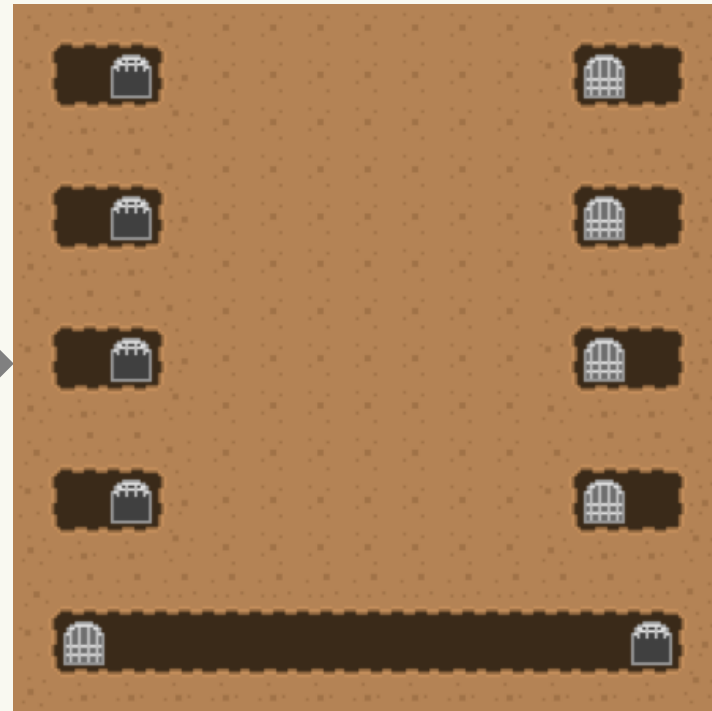


Pattern

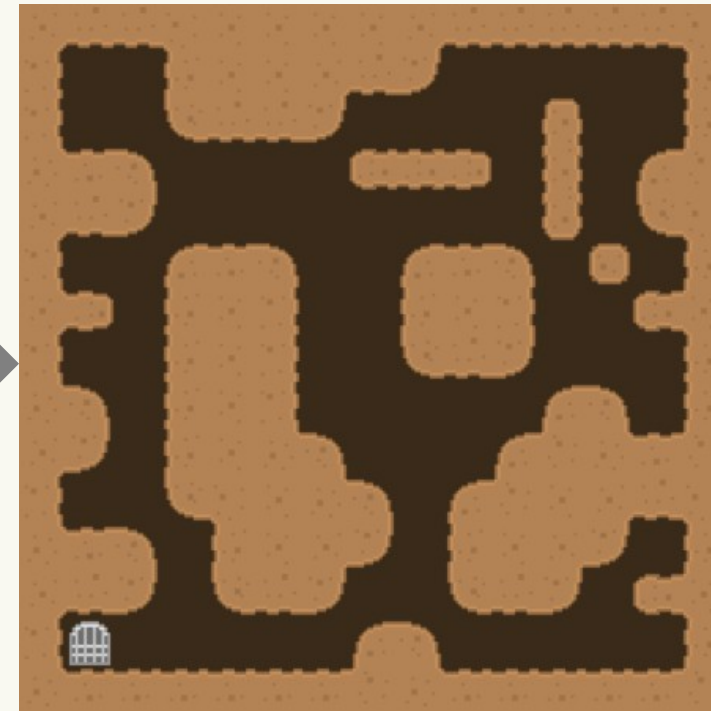
# Distribution Constraints



Tile



Pattern



Distribution

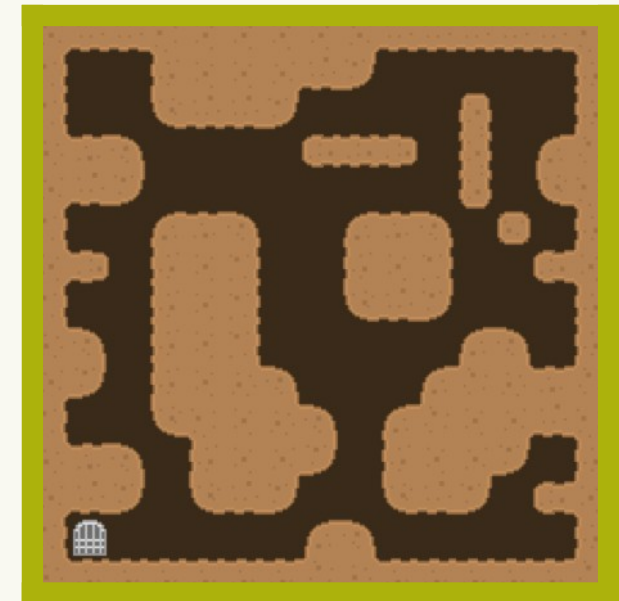
# Distribution Constraints

## Outline

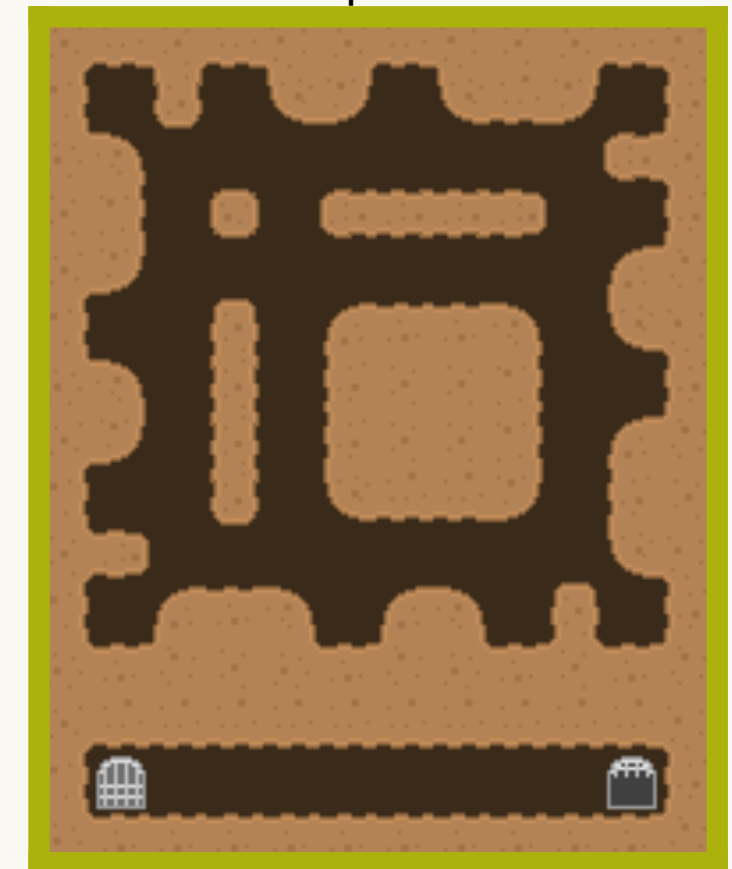
Setup tile and pattern constraints.

Setup: for each corresponding **region**, for each tile type, the counts should be similar to the example.

Find and process solution.



Example level





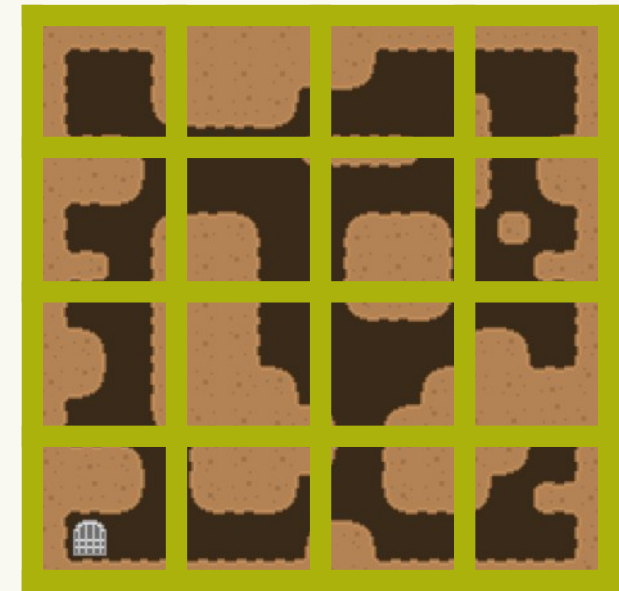
# Distribution Constraints

## Outline

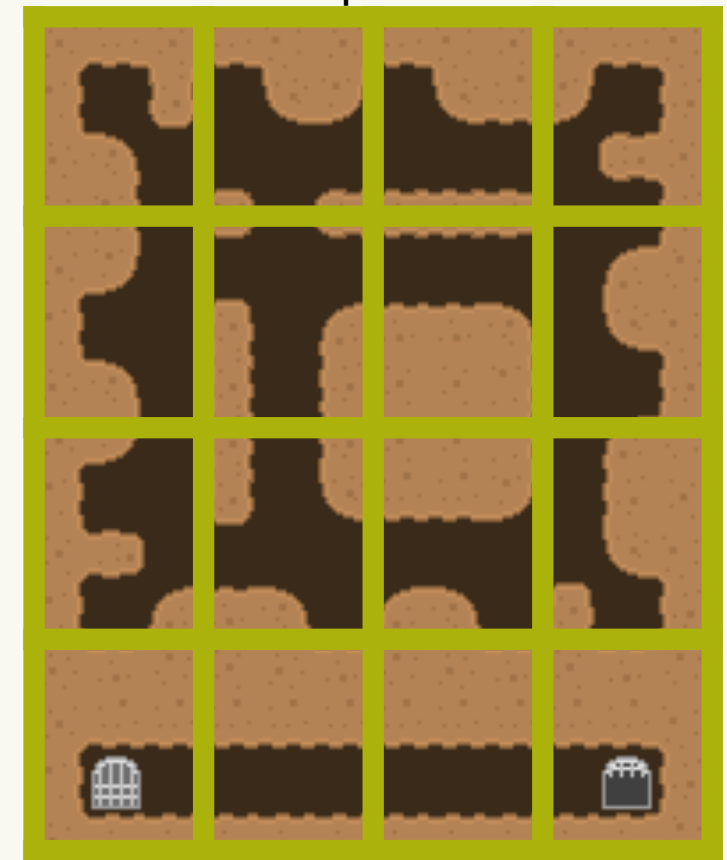
Setup tile and pattern constraints.

Setup: for each corresponding **region**, for each tile type, the counts should be similar to the example.

Find and process solution.



Example level



# Distribution Constraints

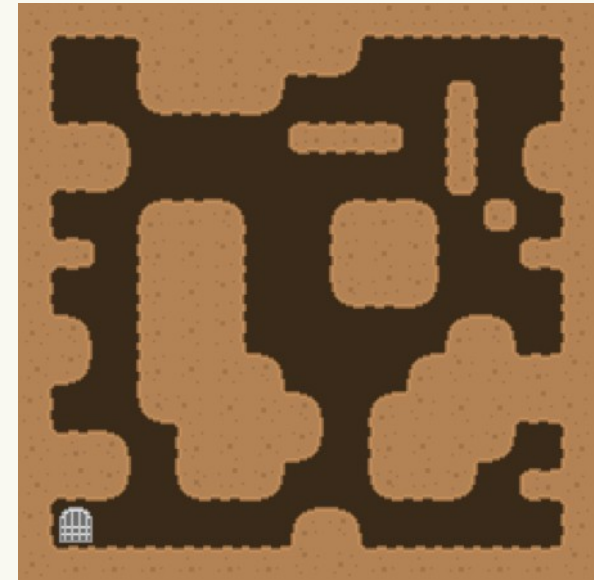
## Outline

Setup tile and pattern constraints.

Setup: for each corresponding **region**, for each tile type, the counts should be similar to the example.

**CnstrCount**(tileVars, lo, hi, SOFT)

Find and process solution.



## Interface

**MakeVar**( )

**CnstrCount**( . . . )

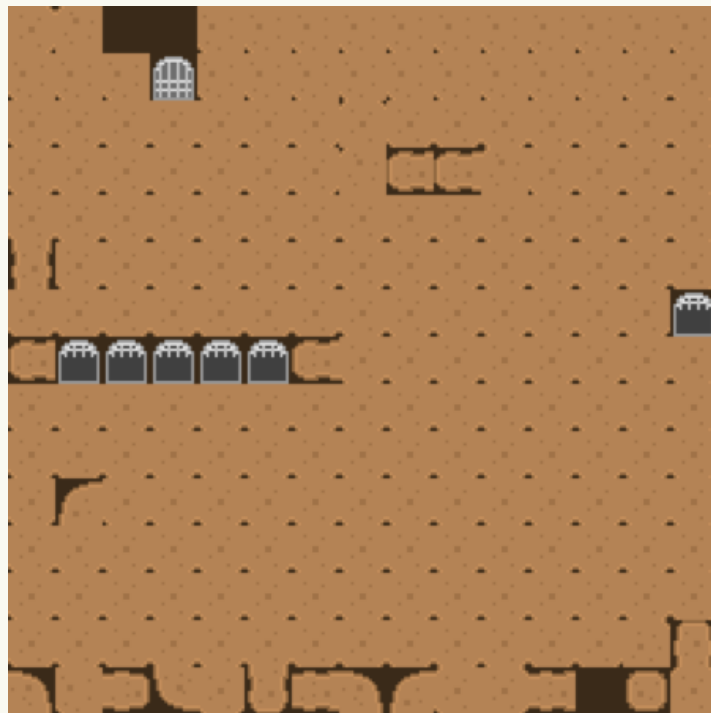
**Solve**( )

**GetVar**( . . . )

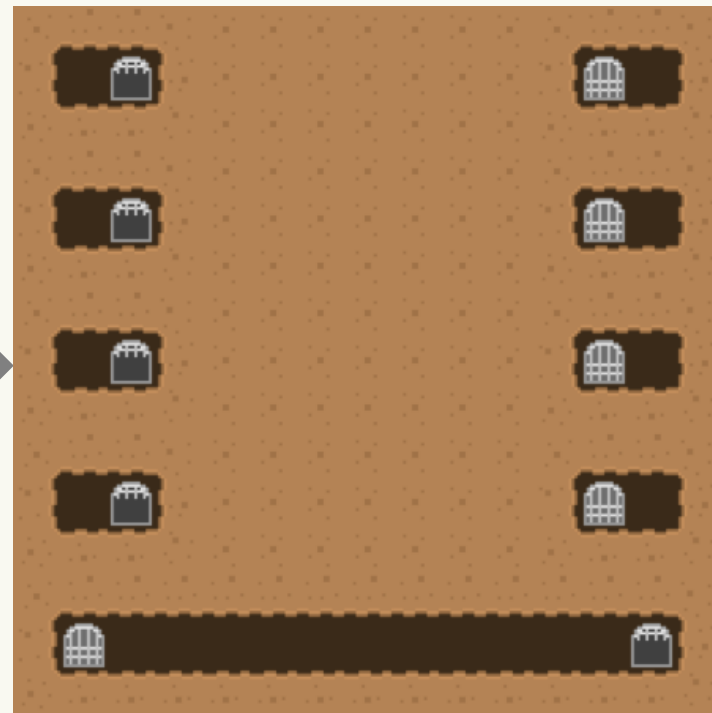
**CnstrImpliesOr**( . . . )

**MakeAnd**( . . . )

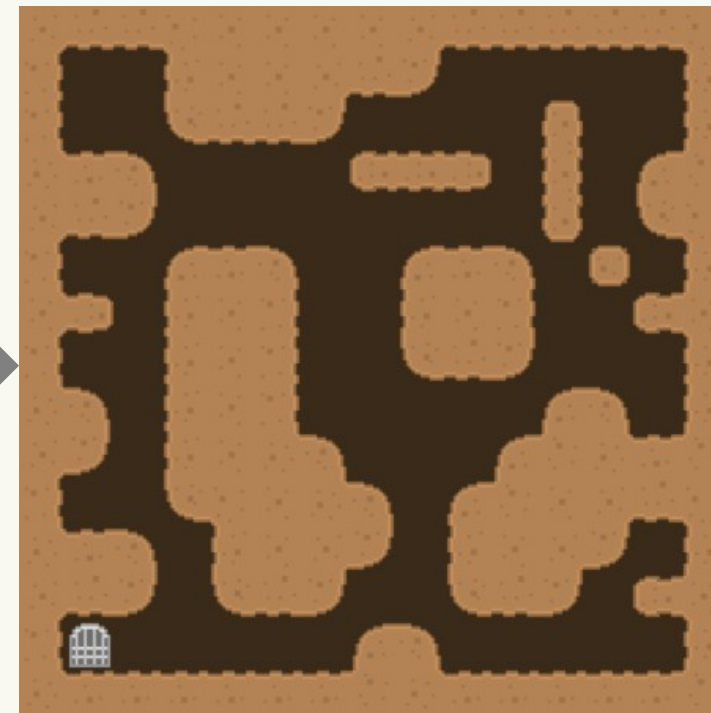
# Constraint-Based Generation



Tile

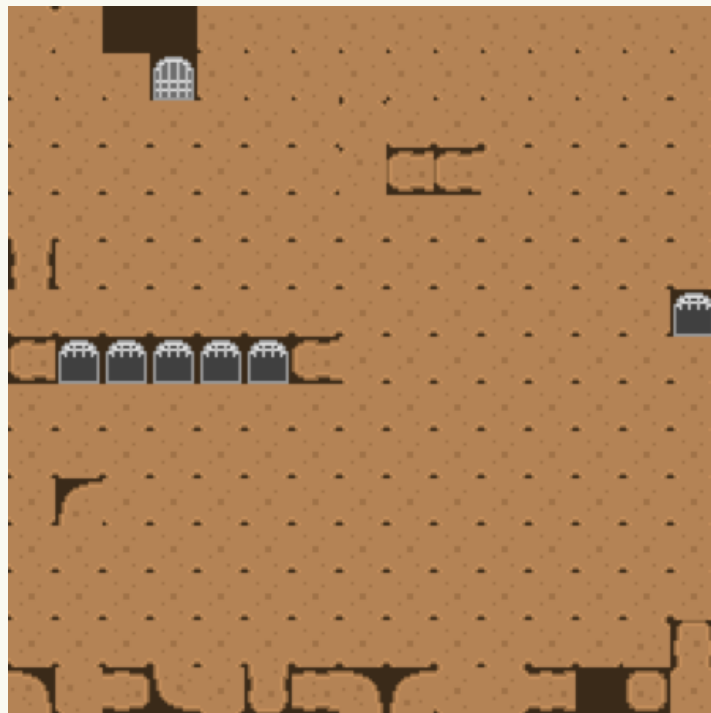


Pattern

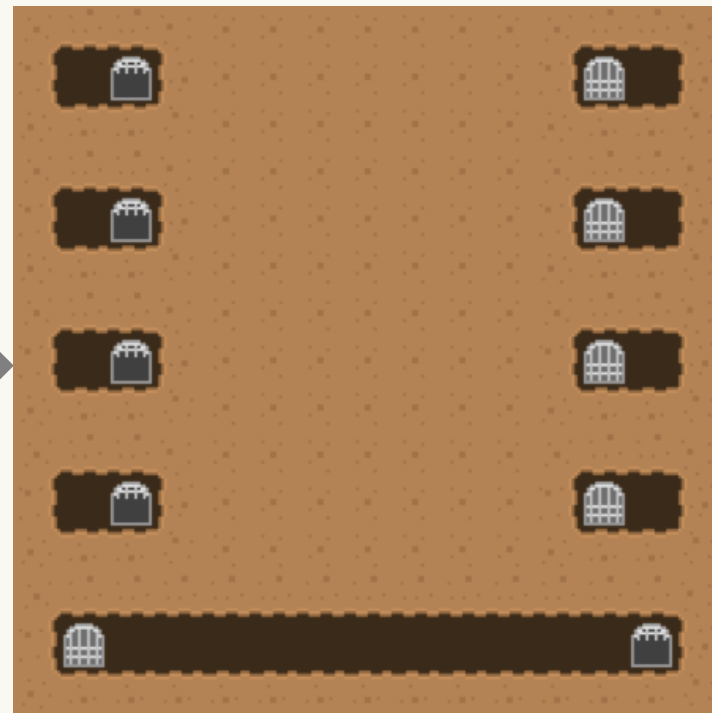


Distribution

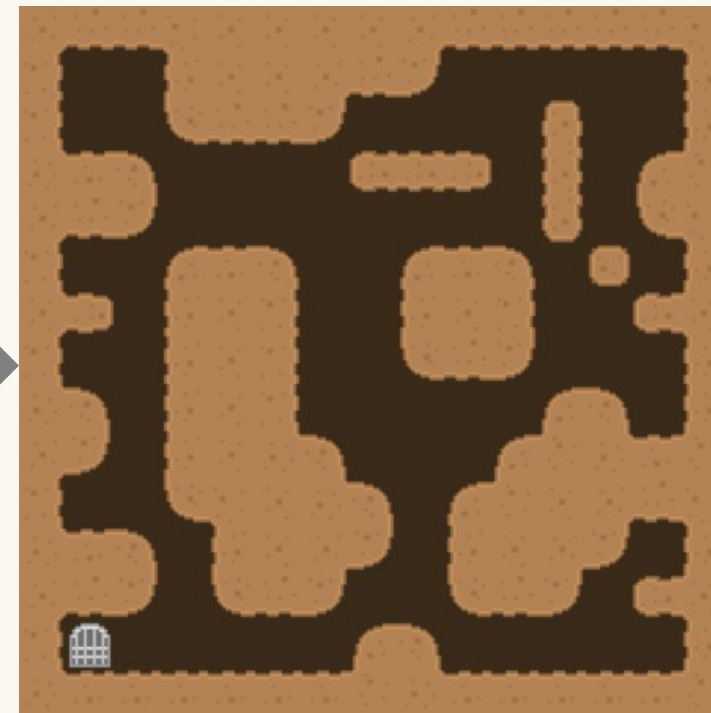
# Constraint-Based Generation



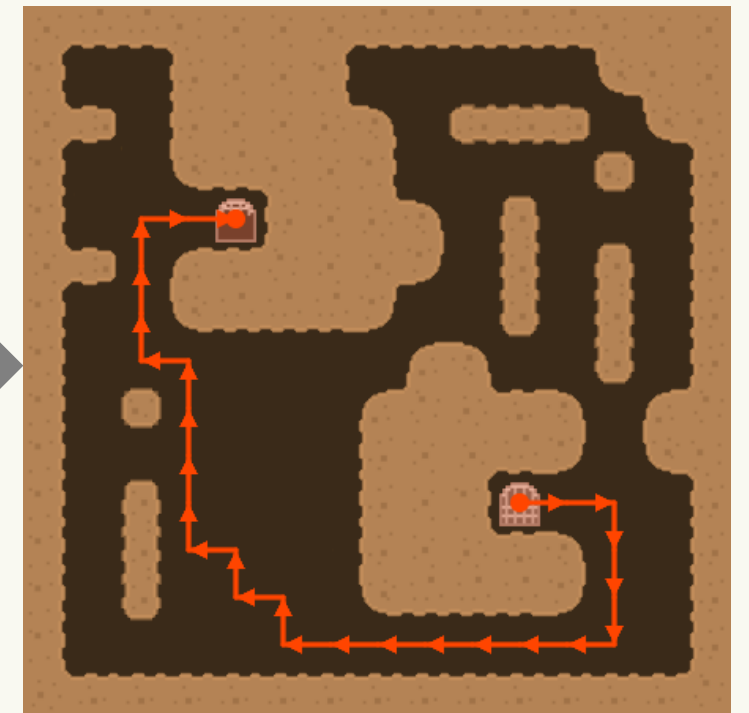
Tile



Pattern



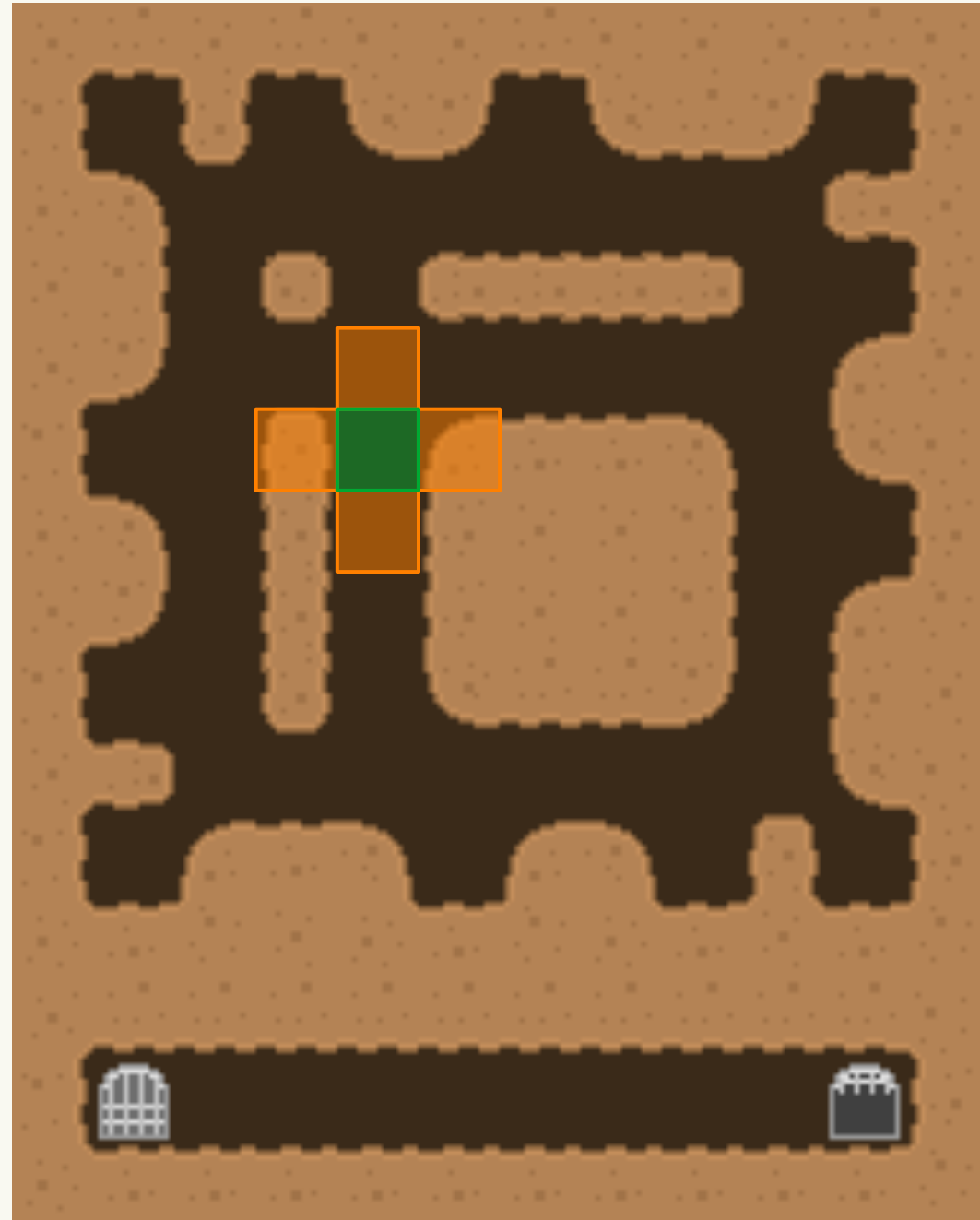
Distribution



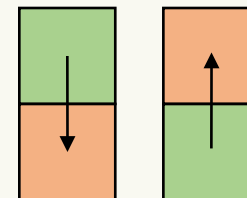
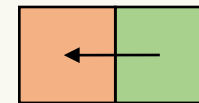
Reachability



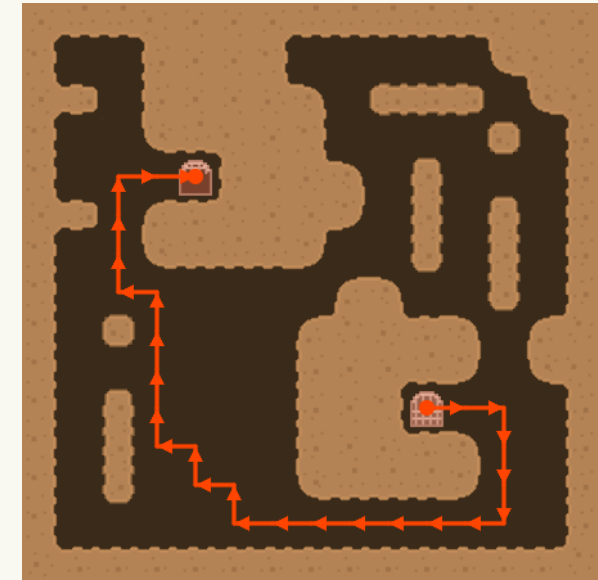
# Reachability Constraints



Reachability template



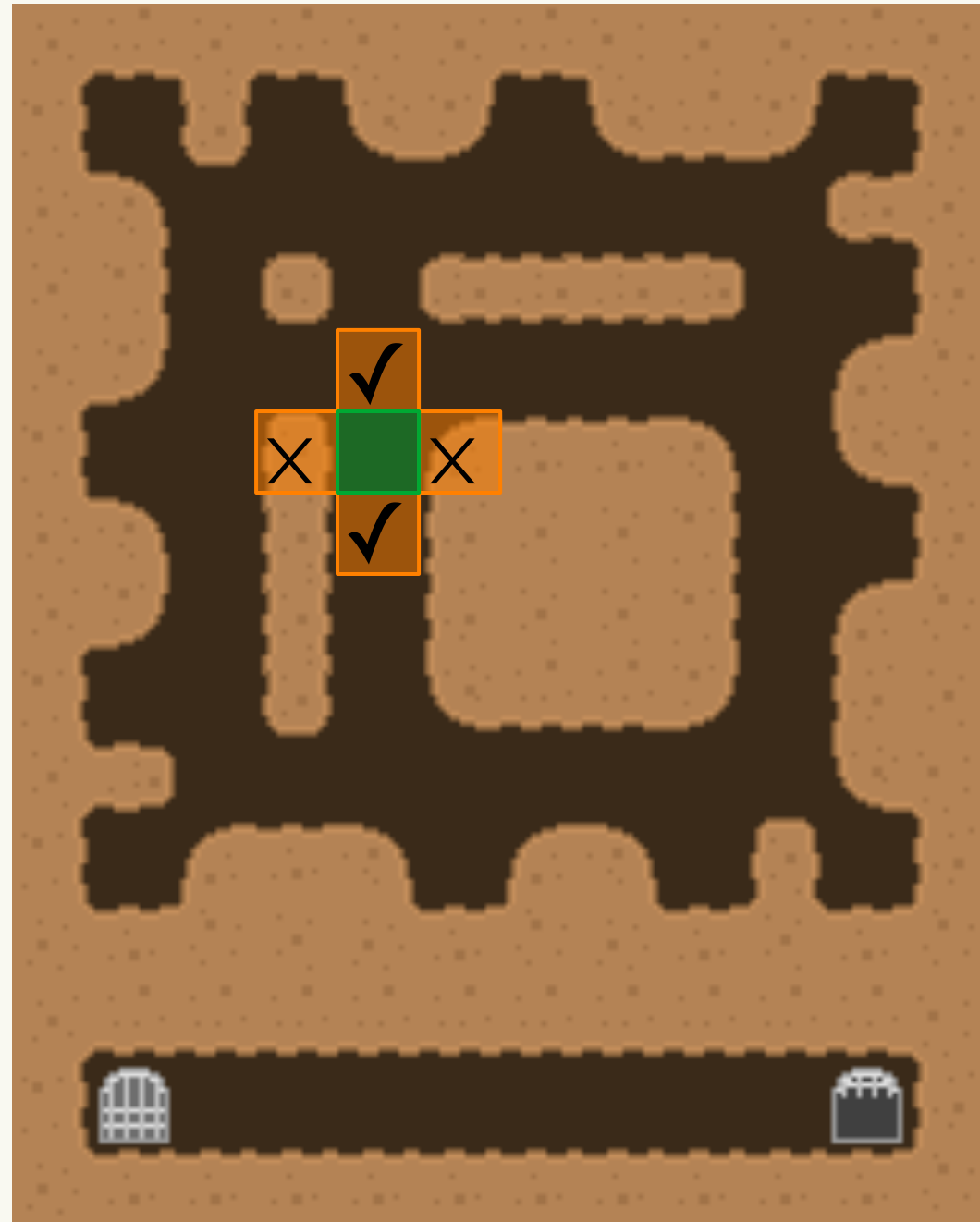
How player can move through level



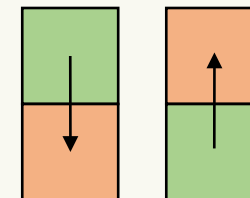
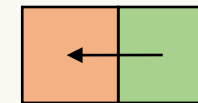
**Interface**  
**MakeVar ( )**  
**CnstrCount ( . . . )**  
**Solve ( )**  
**GetVar ( . . . )**  
**CnstrImpliesOr ( . . . )**  
**MakeAnd ( . . . )**



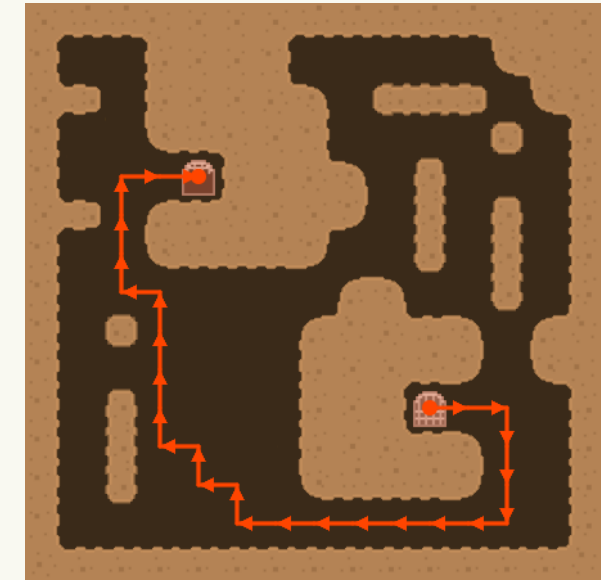
# Reachability Constraints



## Reachability template



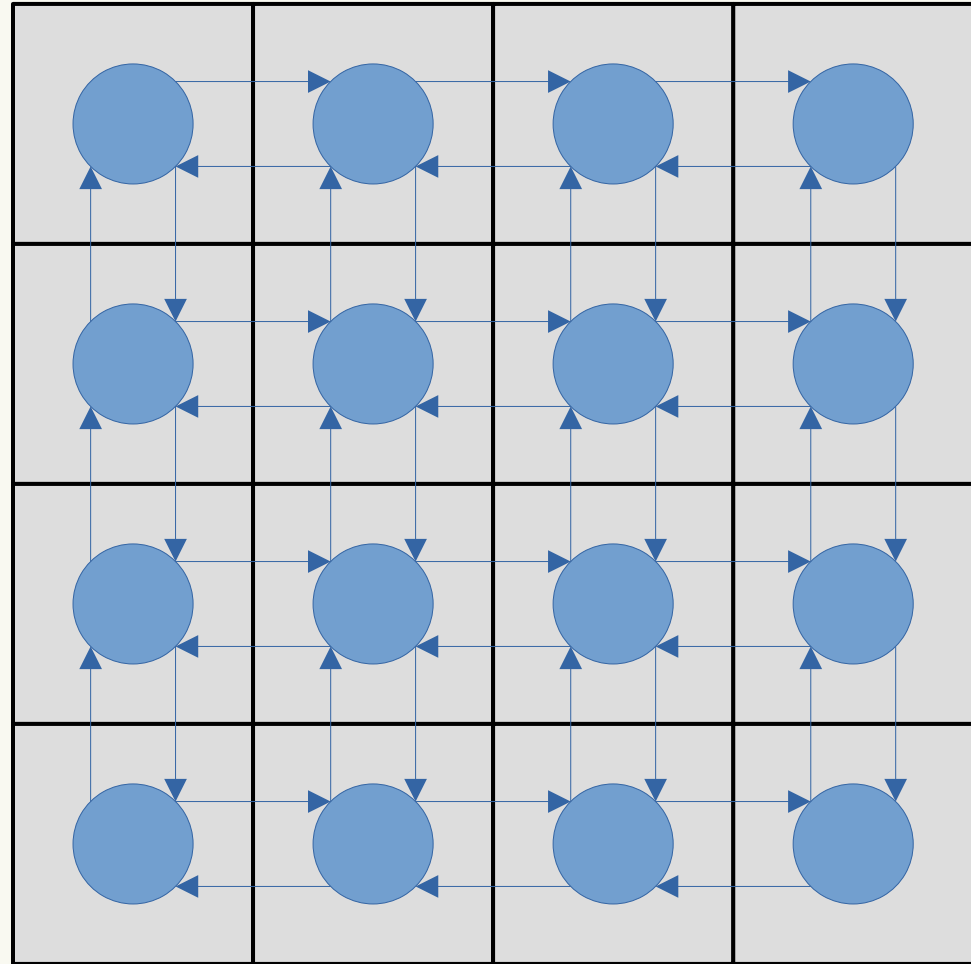
## How player can move through level



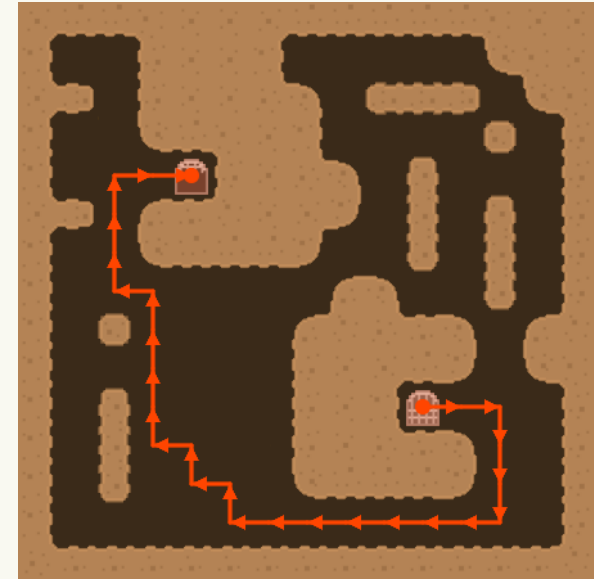
# Interface

- MakeVar()**
- CnstrCount(...)**
- Solve()**
- GetVar(...)**
- CnstrImpliesOr(...)**
- MakeAnd(...)**

# Reachability Constraints



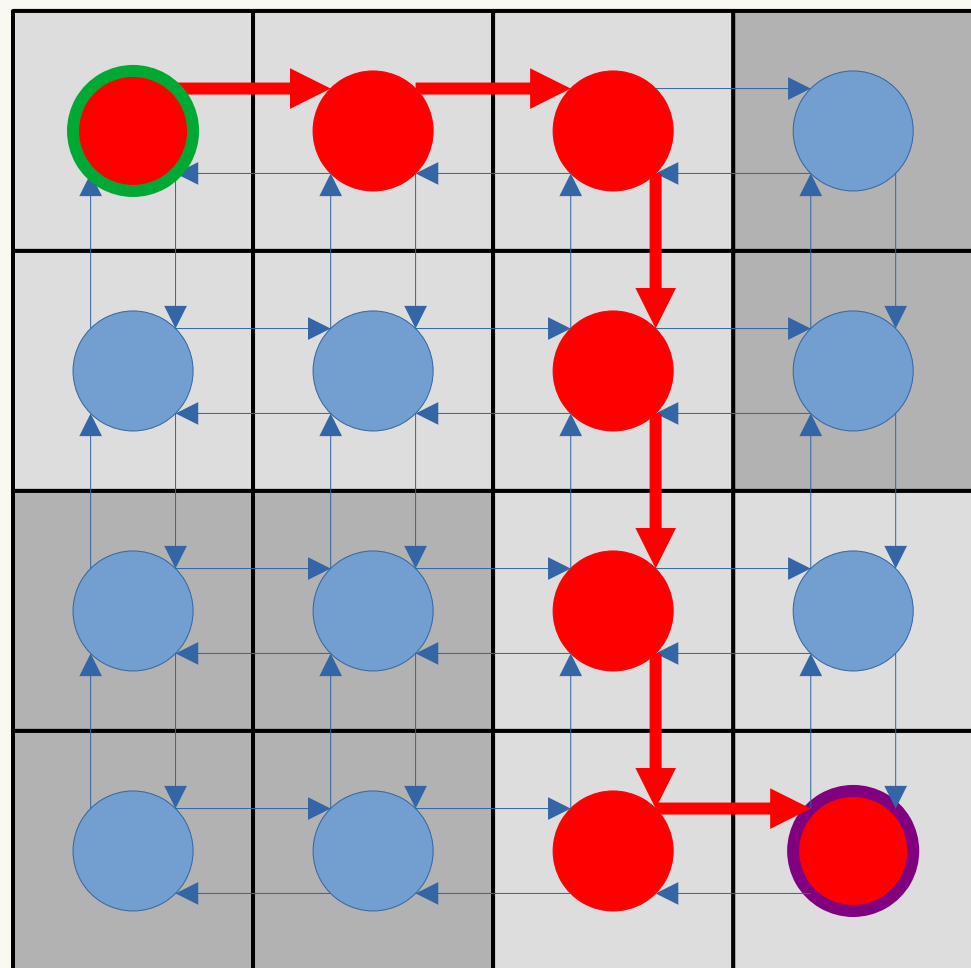
Convert to a graph of *possible* moves, adding variables for nodes and edges being part of the path, and constrain existence of a path in the graph.



# Interface

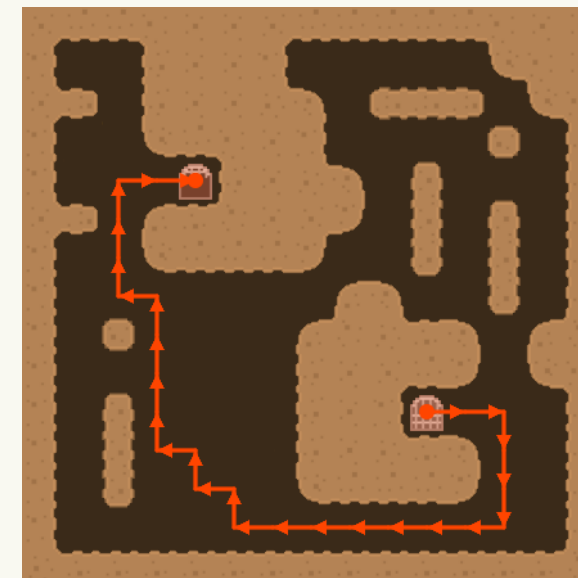
- MakeVar()**
- CnstrCount(...)**
- Solve()**
- GetVar(...)**
- CnstrImpliesOr(...)**
- MakeAnd(...)**

# Reachability Constraints



Convert to a graph of *possible* moves, adding variables for nodes and edges being part of the path, and constrain existence of a path in the graph.

Only requires **a** path, not a short or direct one.



**Interface**  
**MakeVar**( )  
**CnstrCount**( . . . )  
**Solve**( )  
**GetVar**( . . . )  
**CnstrImpliesOr**( . . . )  
**MakeAnd**( . . . )

# Level Generation

## Outline

Setup tile constraints.

Setup pattern constraints.

Setup distribution constraints.

Setup reachability constraints.

Setup any additional custom constraints.

Find solution.

Process solution.

## Interface

**MakeVar( )**

**MakeAnd( . . . )**

**CnstrCount( . . . )**

**CnstrImpliesOr( . . . )**

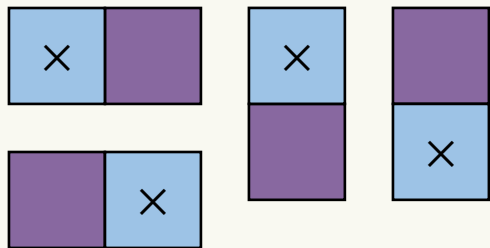
**Solve( )**

**GetVar( . . . )**

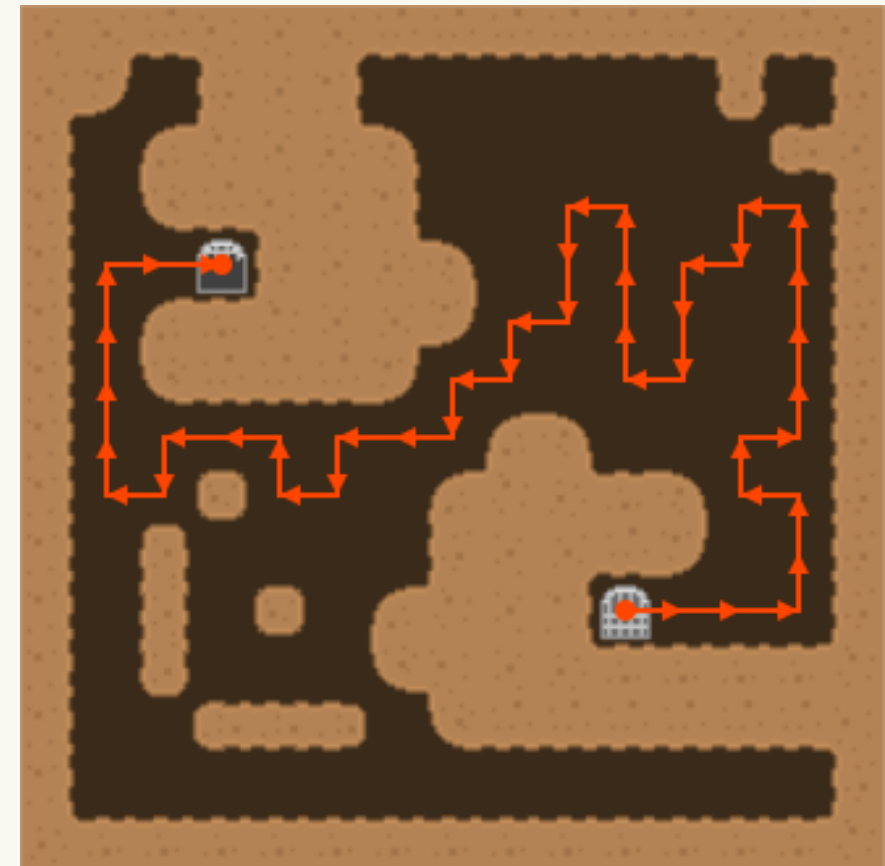
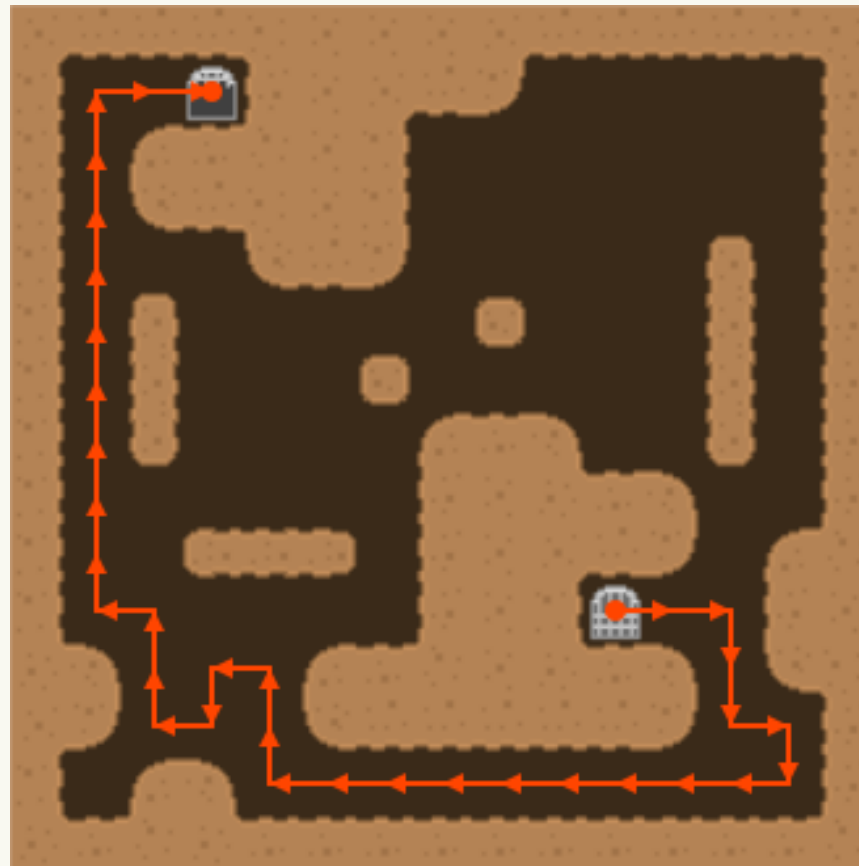
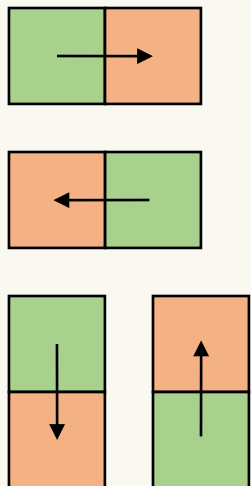
# Level Generation

## Cave

Pattern template



Reachability template

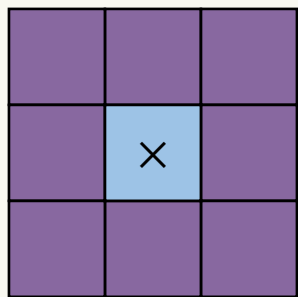




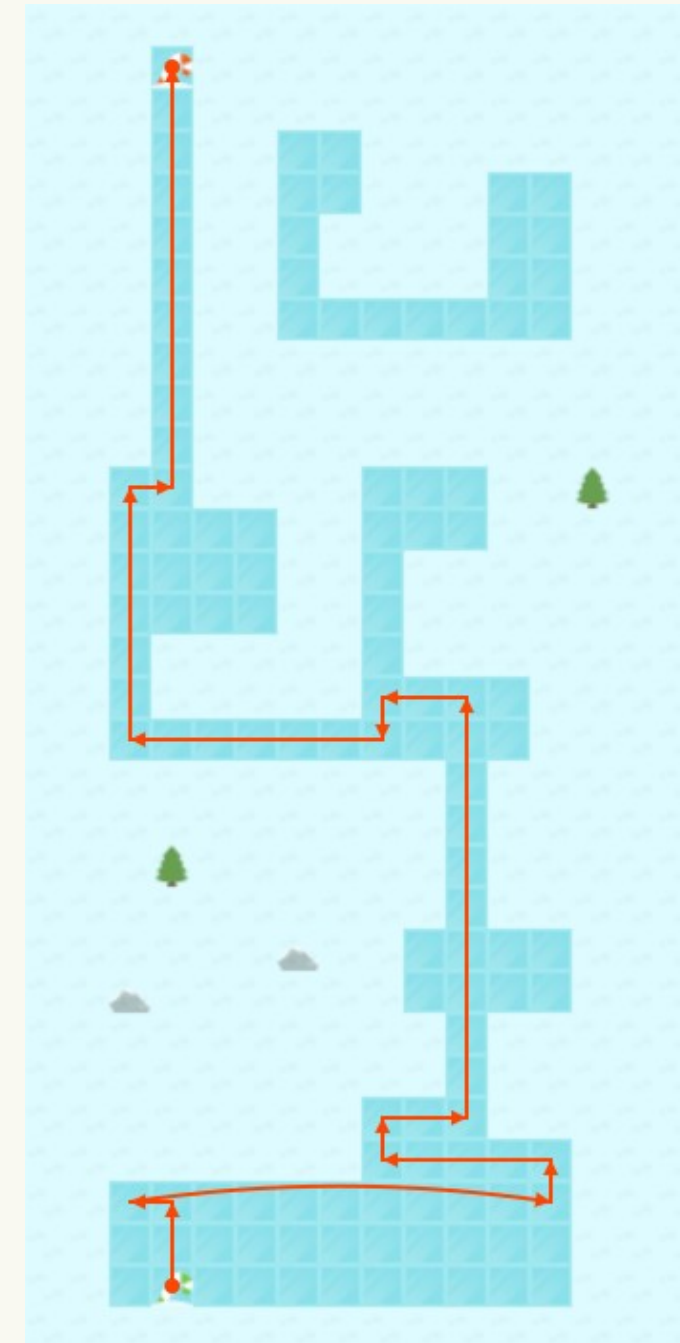
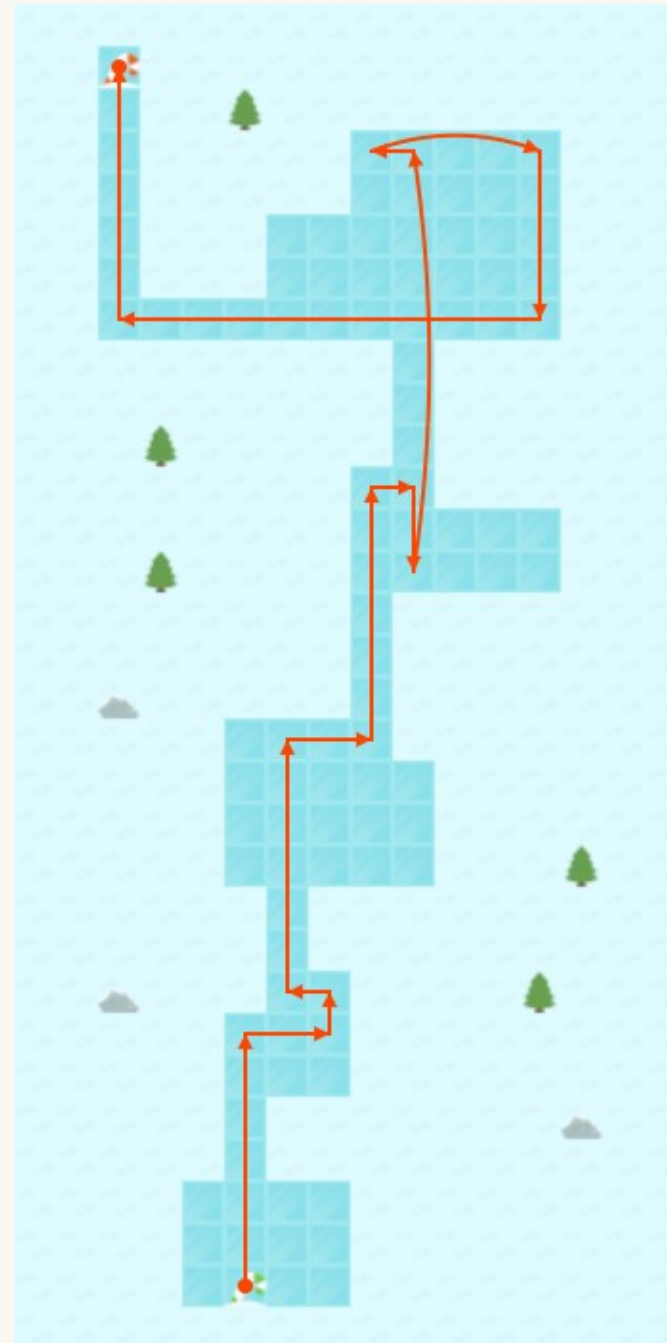
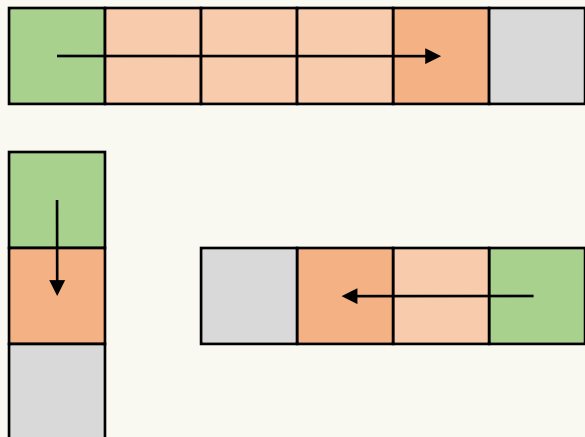
# Level Generation

# Sliding

## Pattern template



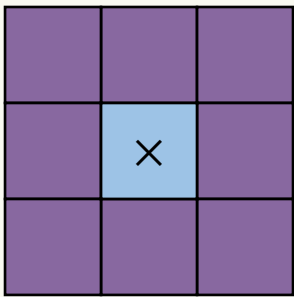
## Reachability template



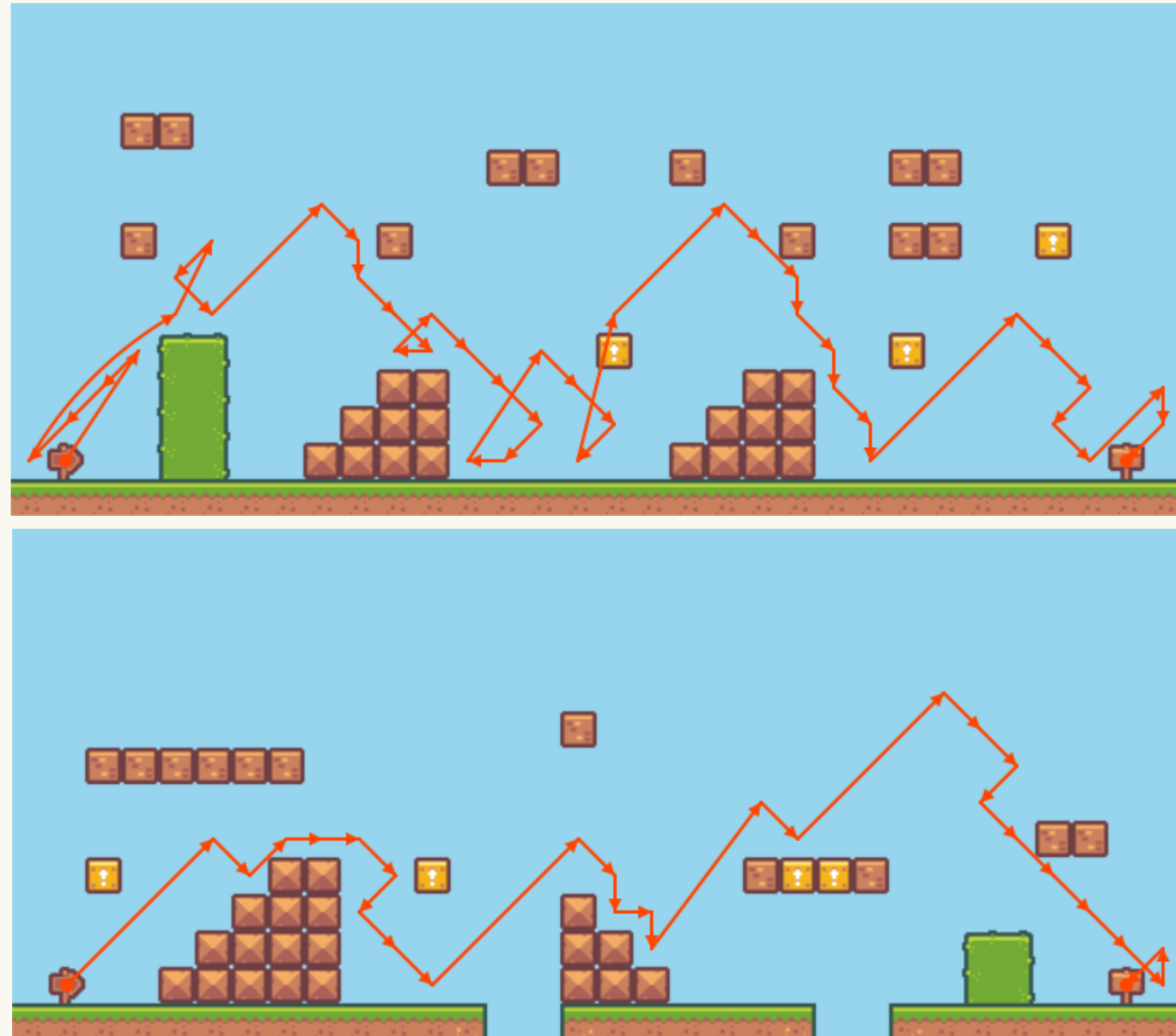
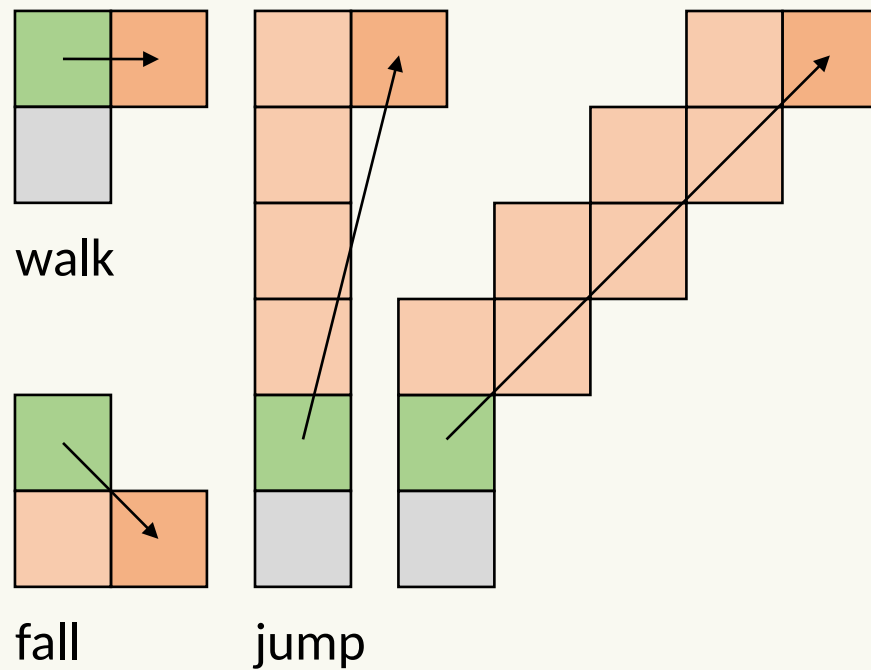
# Level Generation

## Platformer

Pattern template



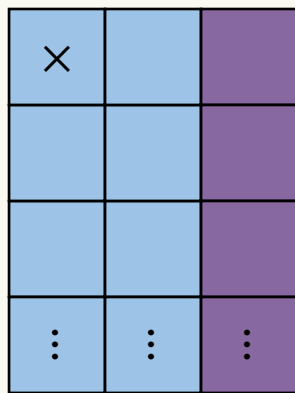
Reachability template



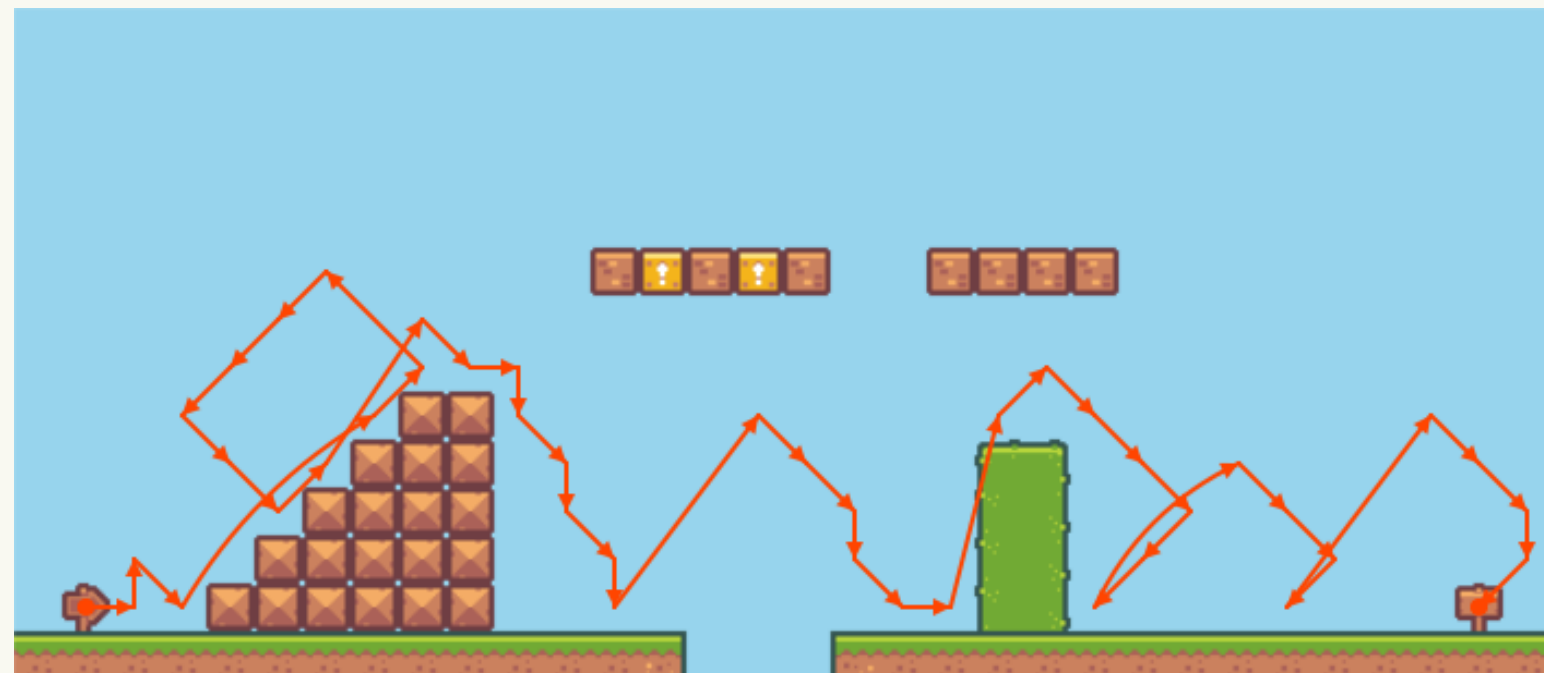
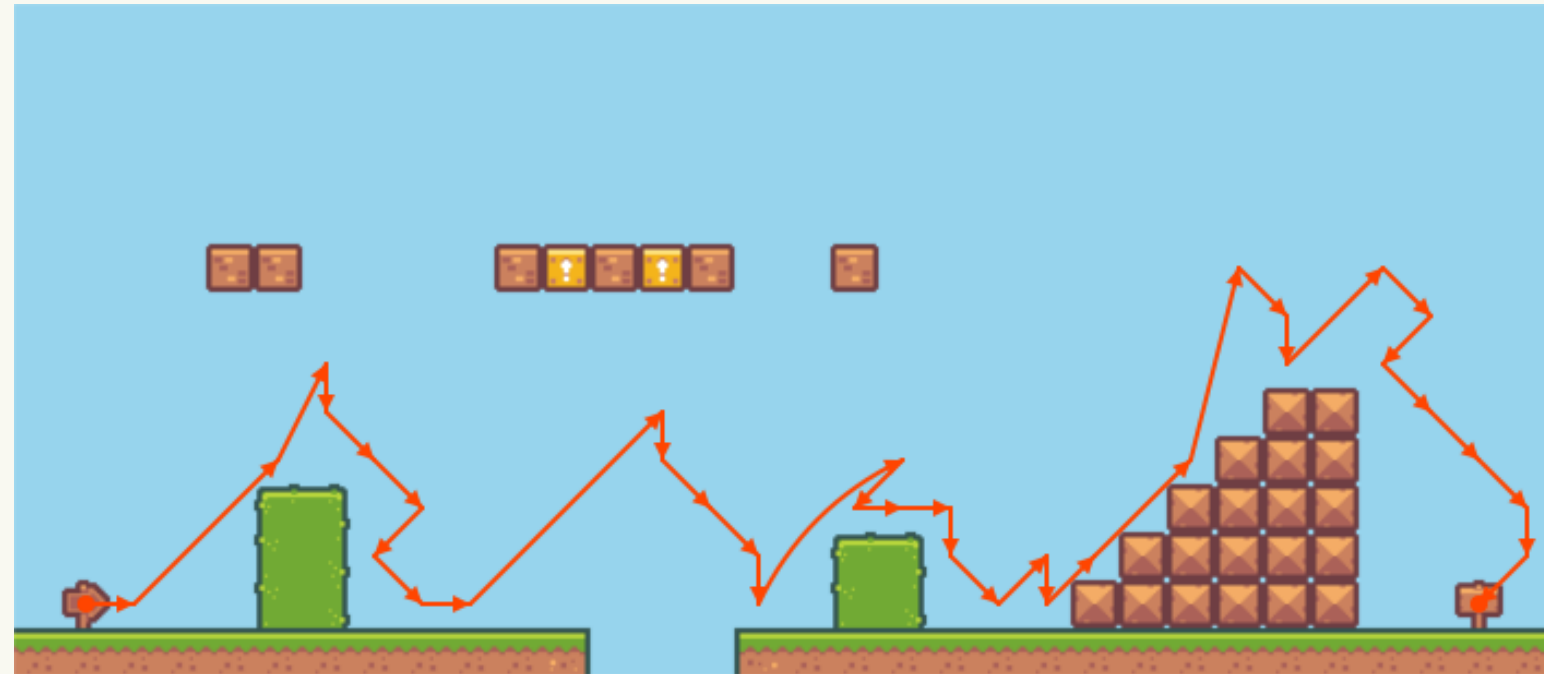
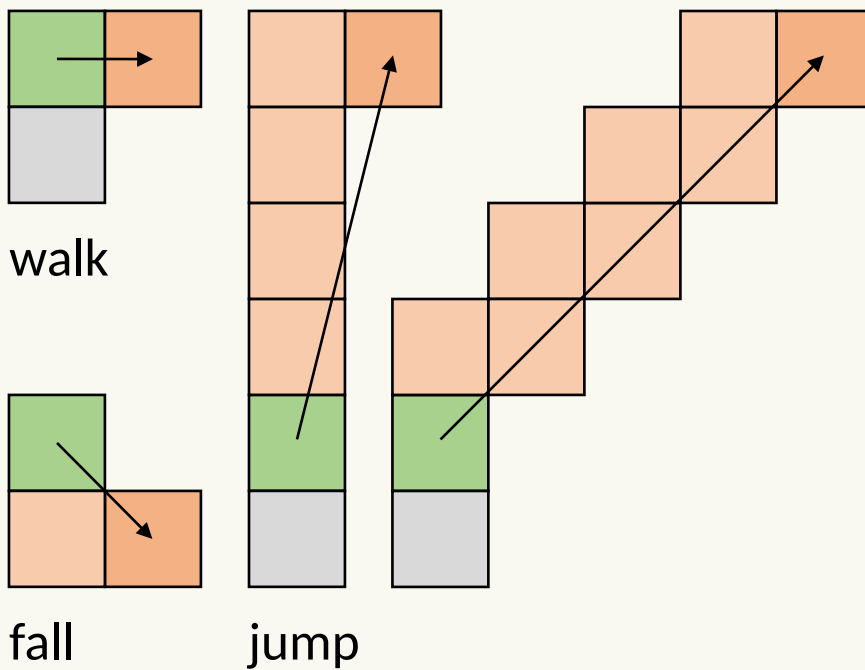
# Level Generation

## Platformer

Pattern template



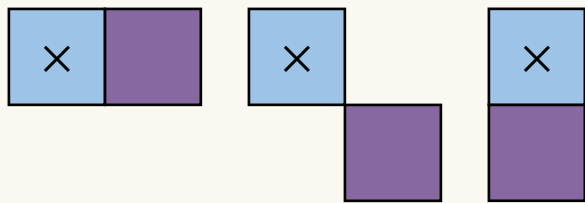
Reachability template



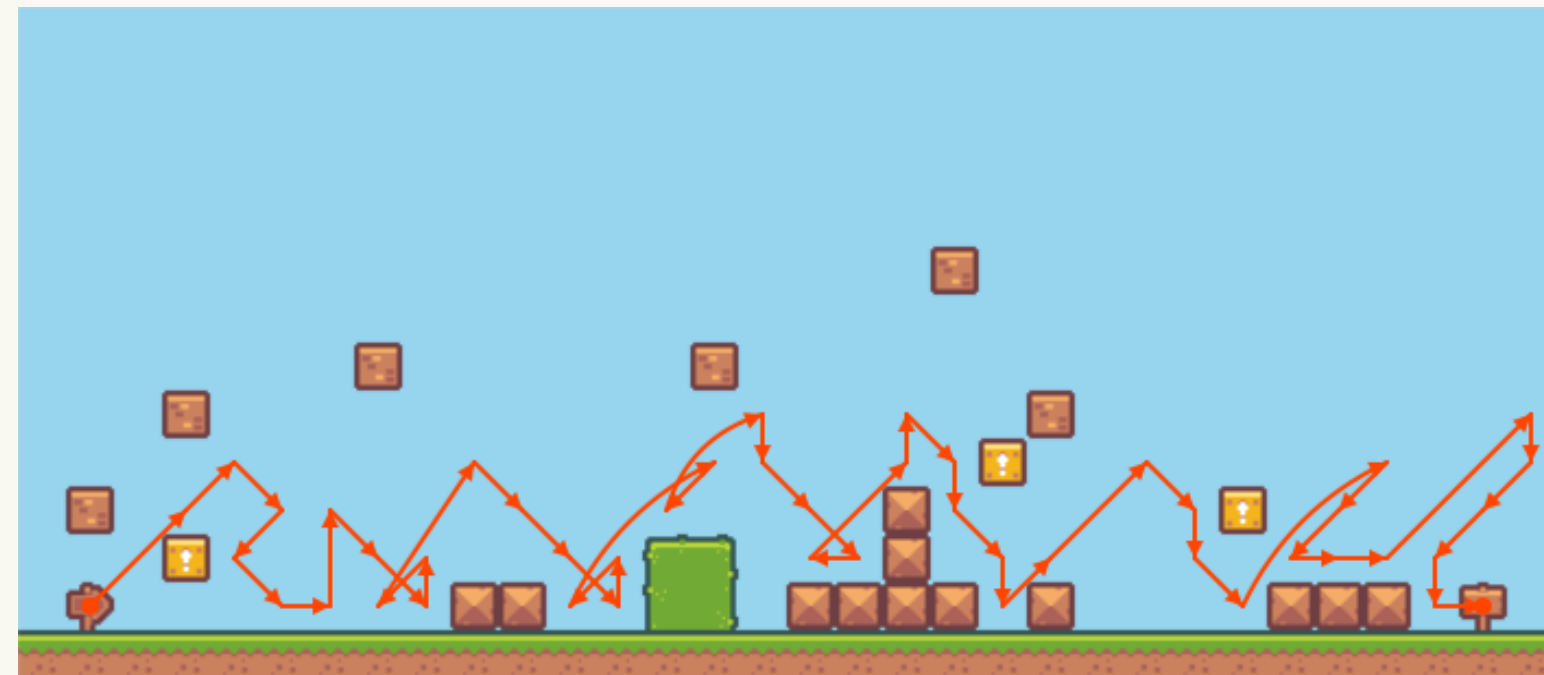
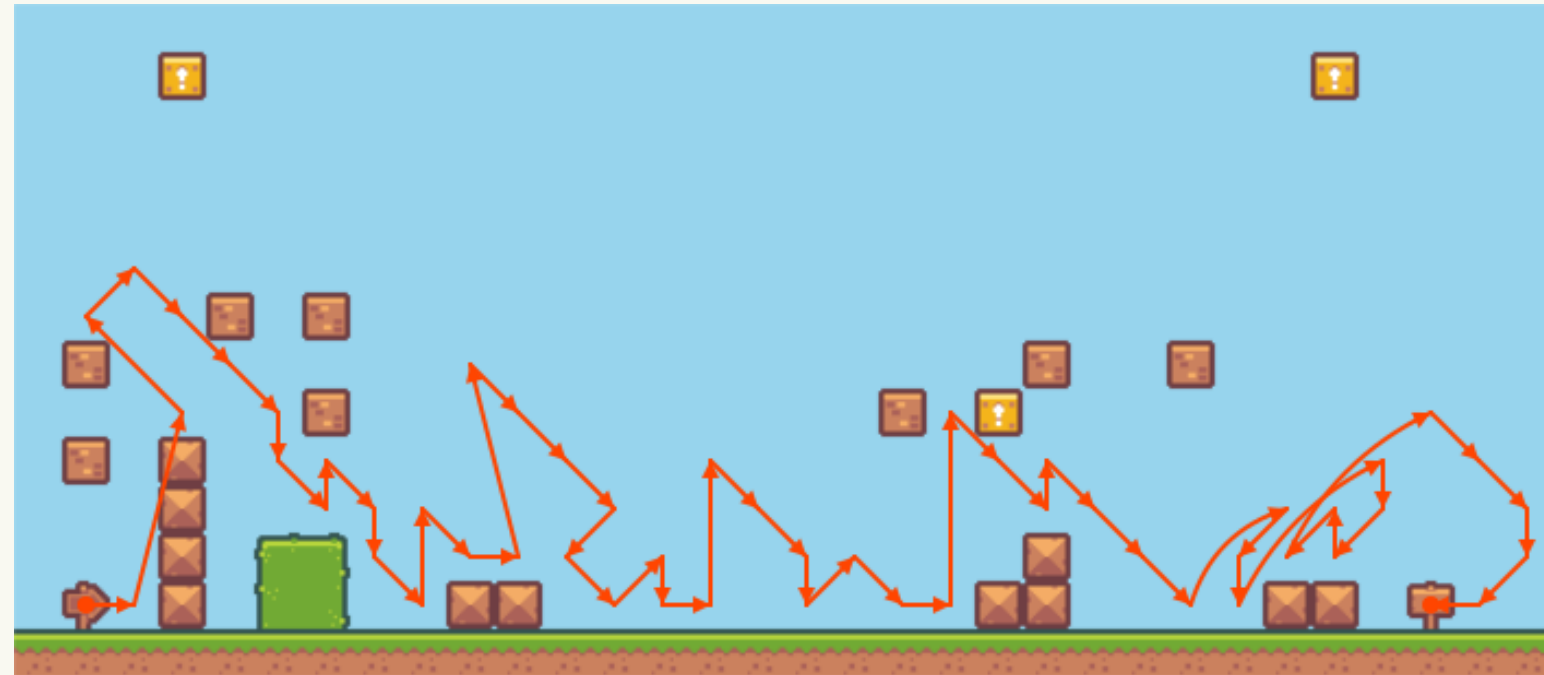
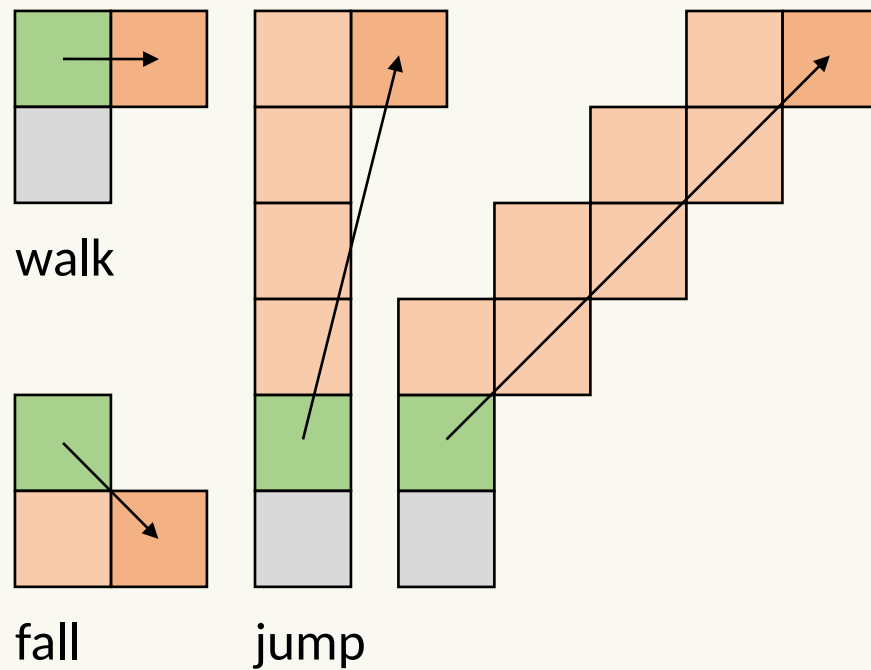
# Level Generation

# Platformer

## Pattern template



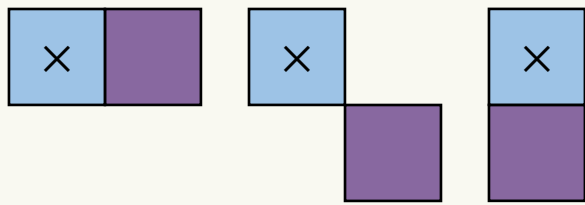
## Reachability template





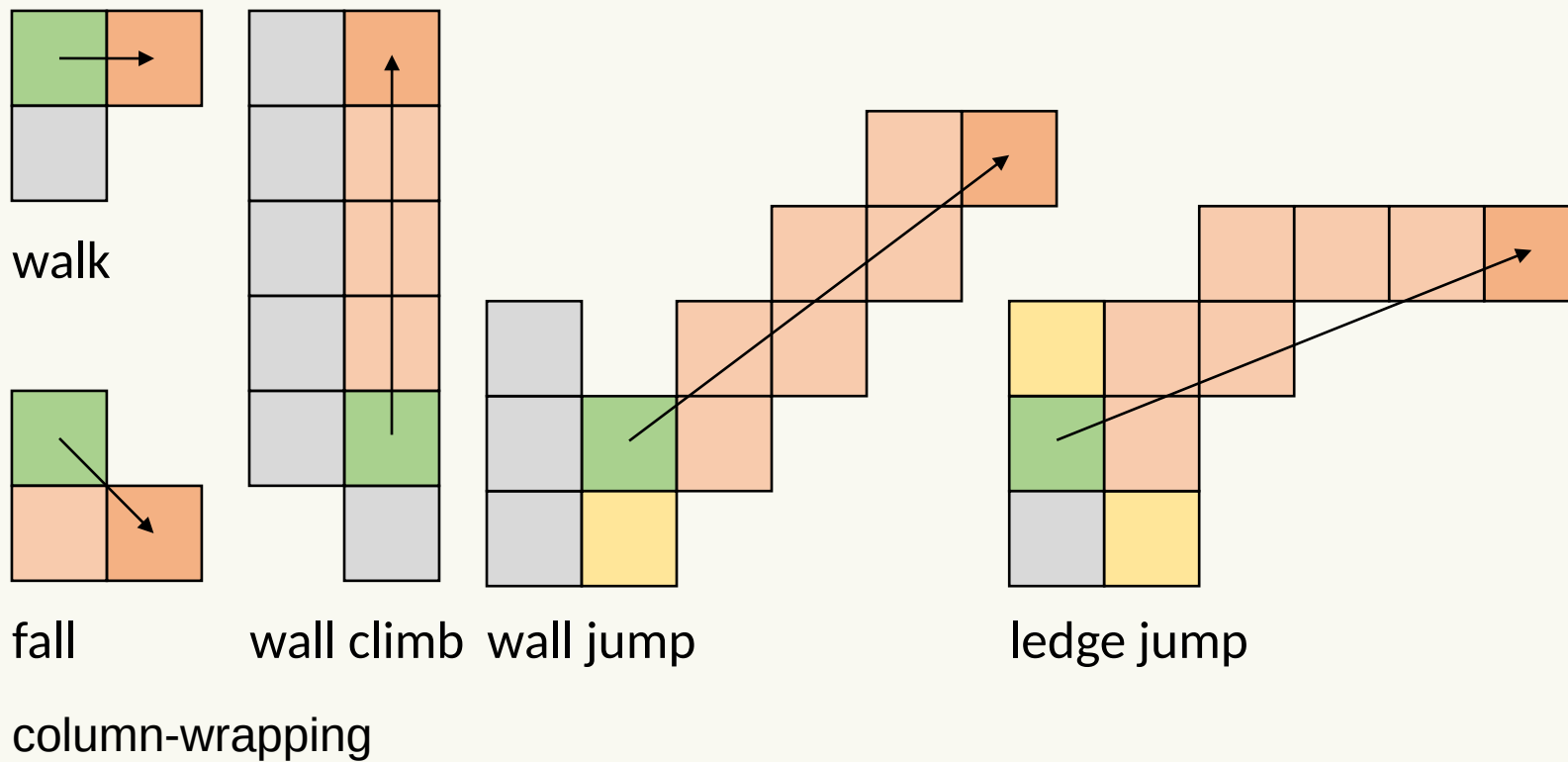
# Level Generation

Pattern template

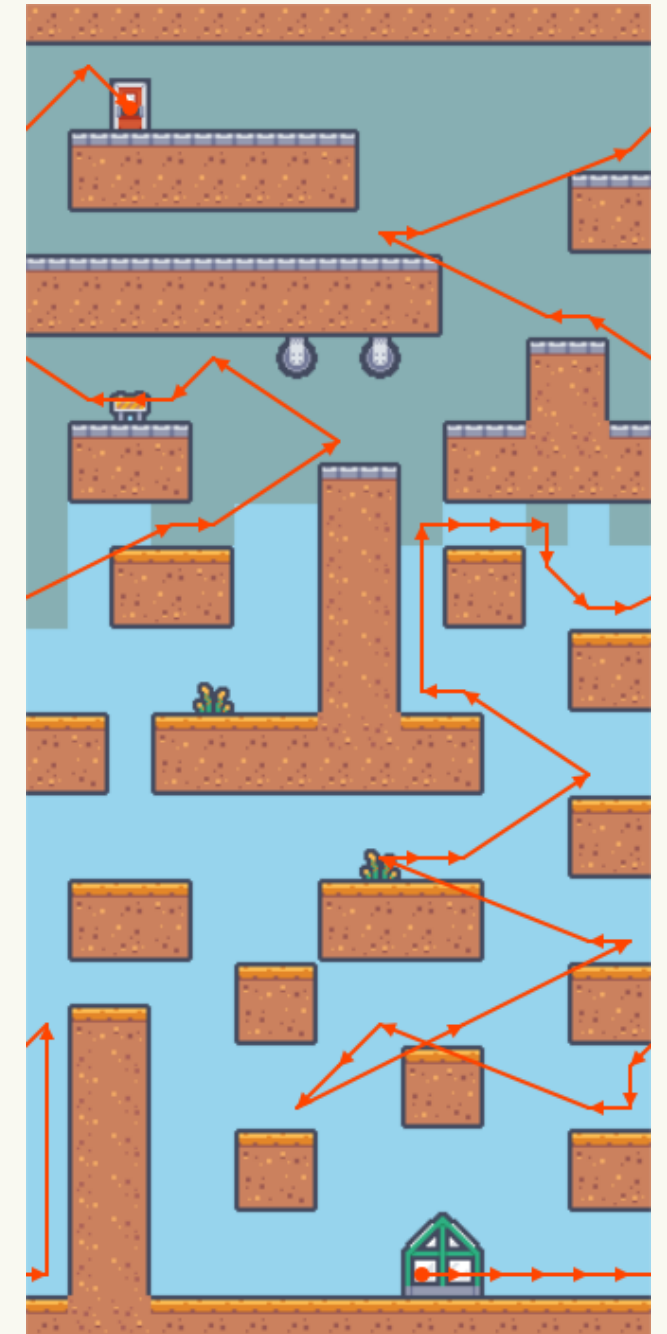
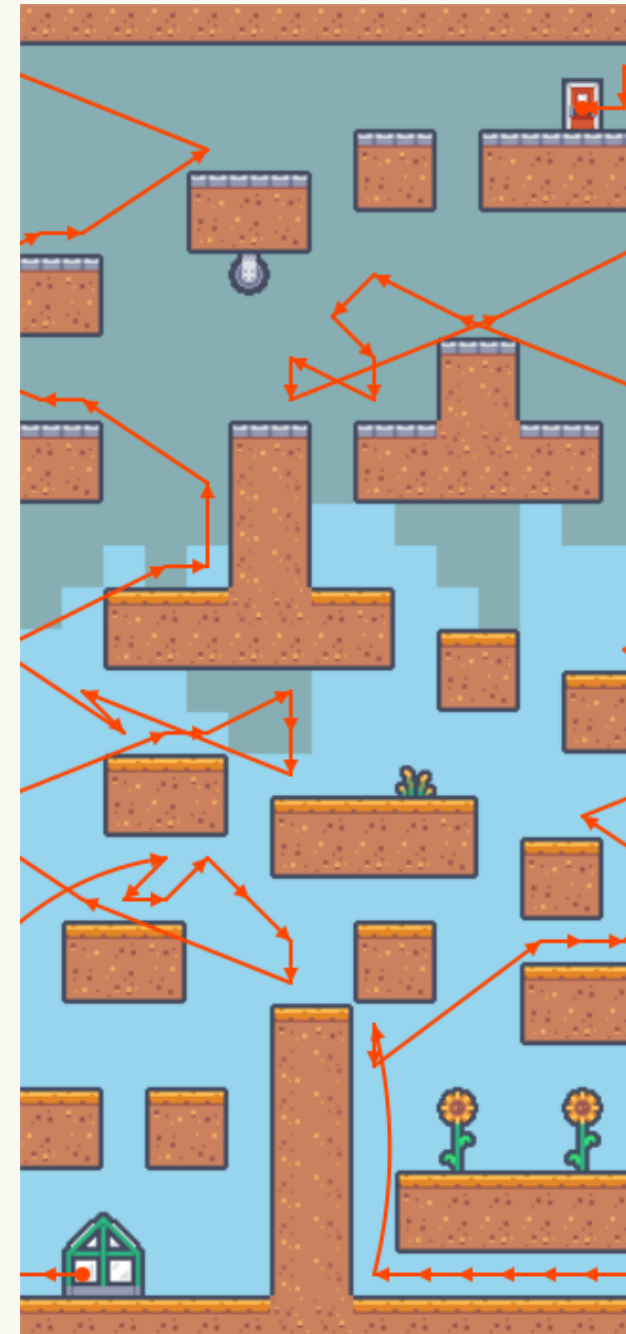


counts regions vertically

Reachability template (e.g. Super Cat Tales)



## Vertical Platformer

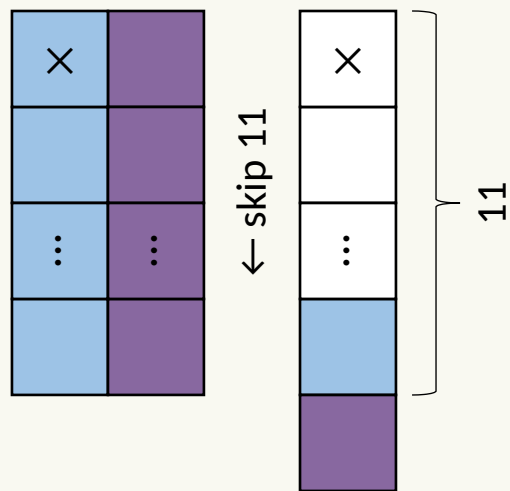




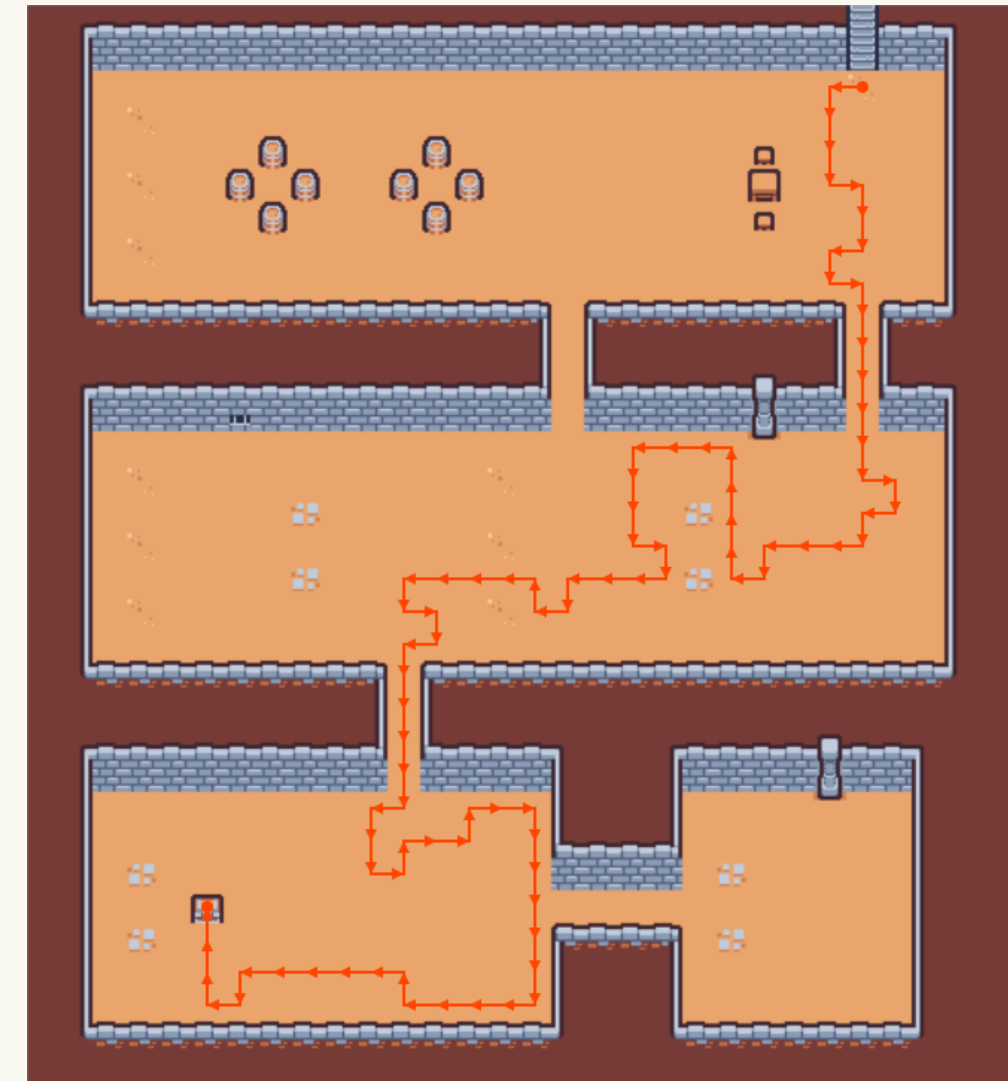
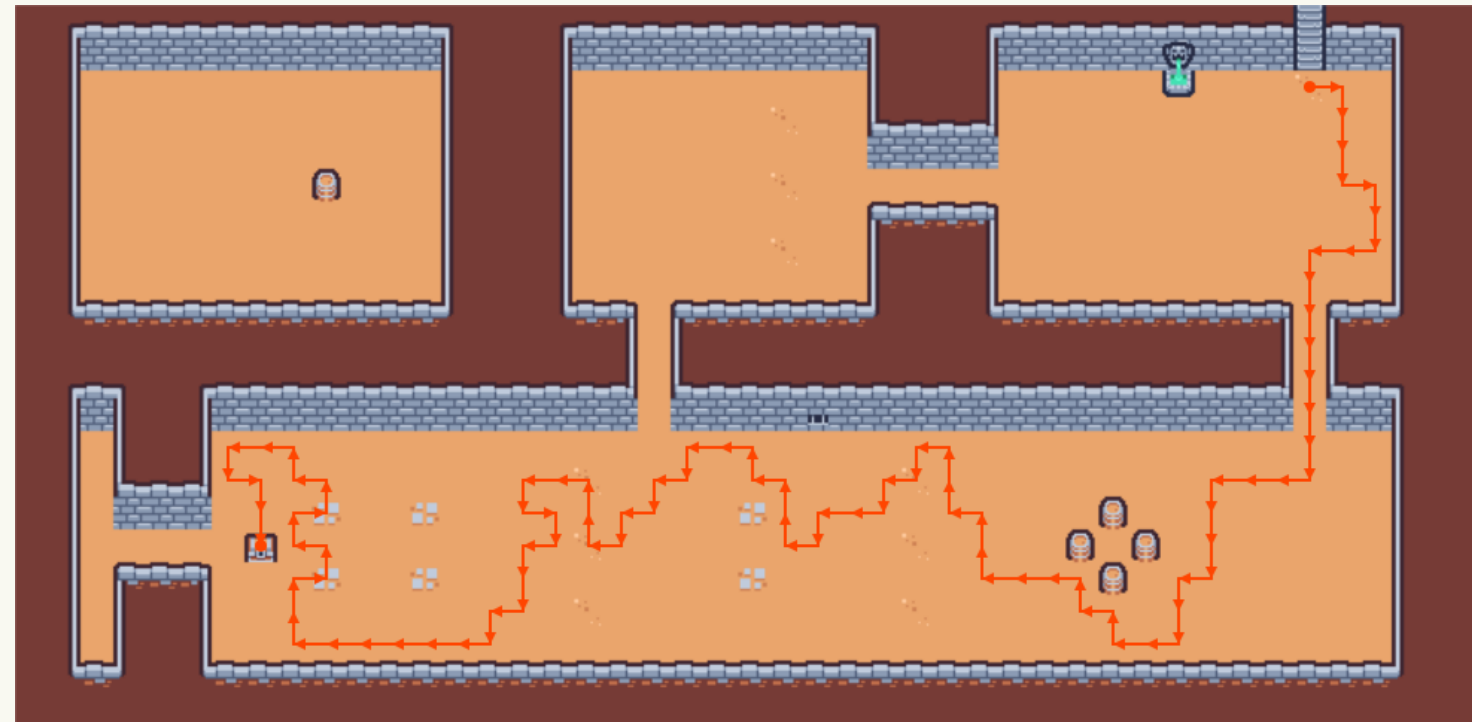
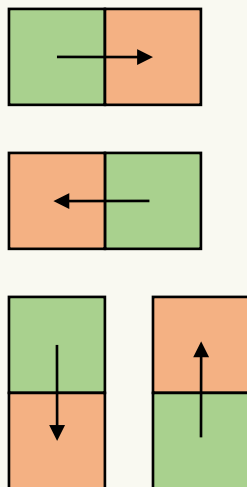
# Level Generation

# Dungeon

## Pattern template



## Reachability template



# Applications

Setup tile constraints.

Setup pattern constraints.

Setup distribution constraints.

Setup reachability constraints.


**Setup any additional custom constraints.**

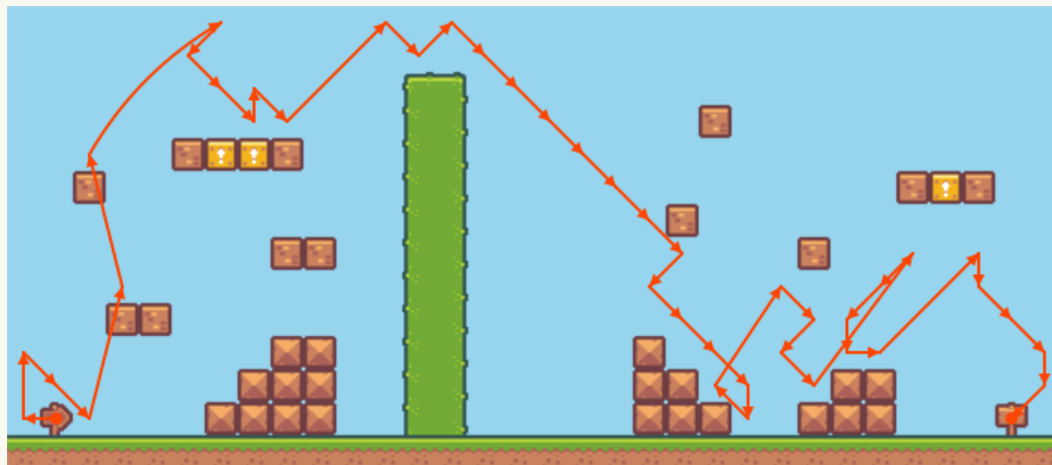
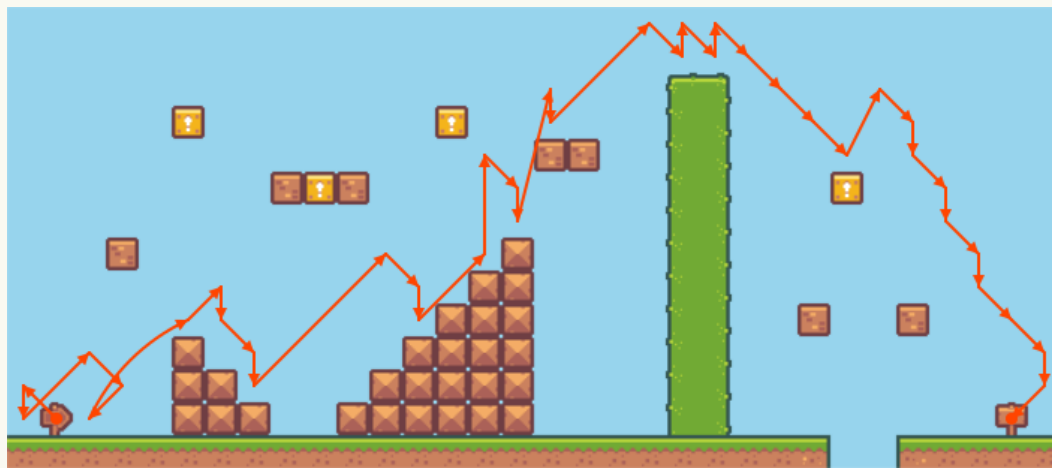
Find solution.

Process solution.

# Applications


## Tile Constraints

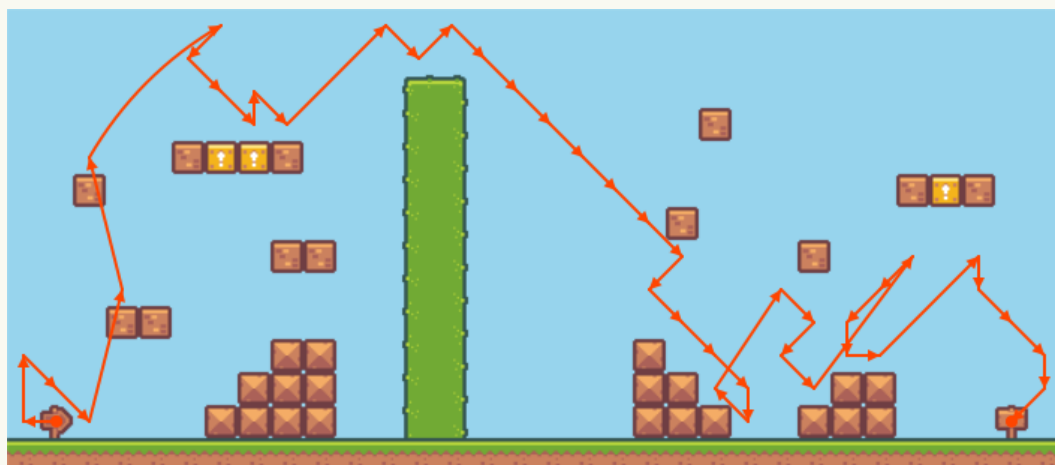
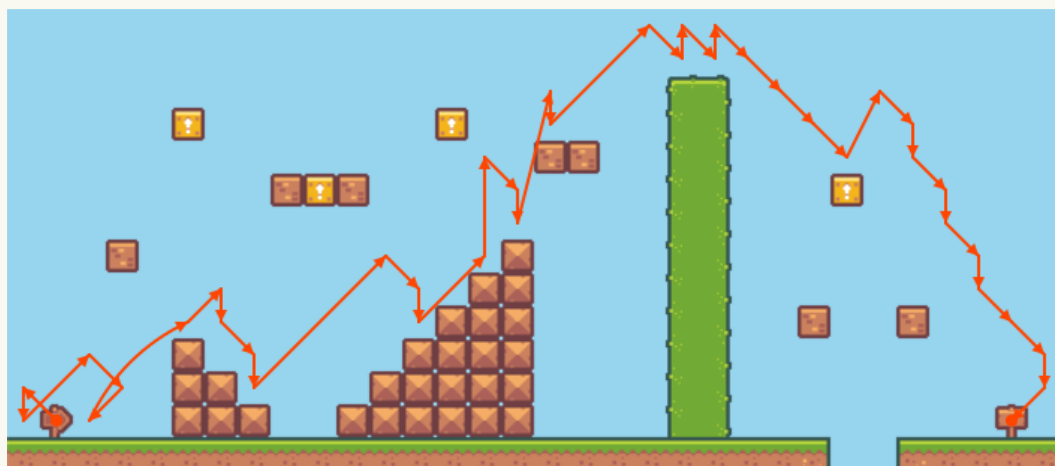
Exactly 1  in 3<sup>rd</sup> from top row



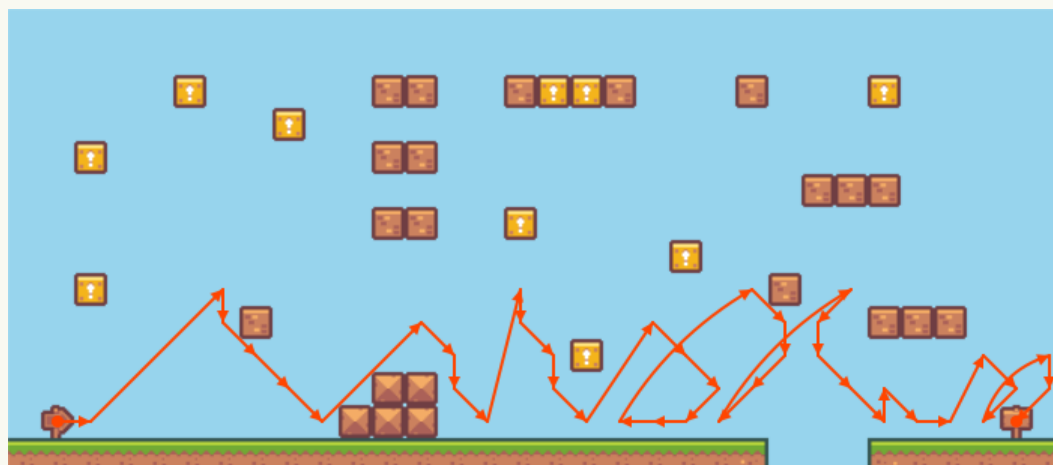
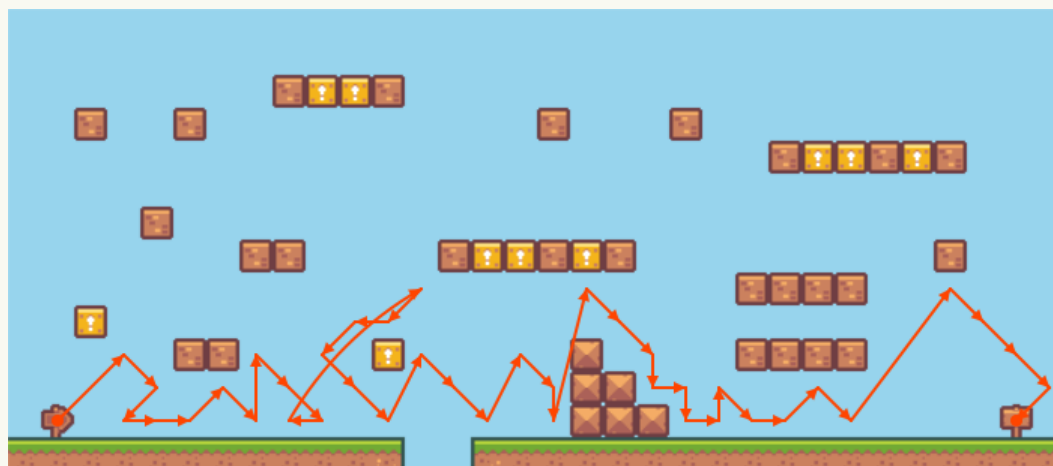
# Applications

## Tile Constraints


Exactly 1  in 3<sup>rd</sup> from top row

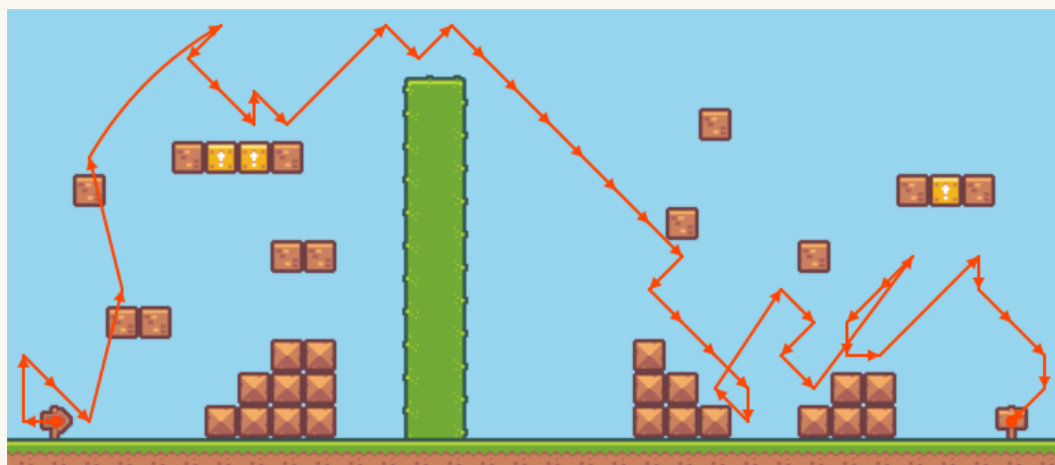
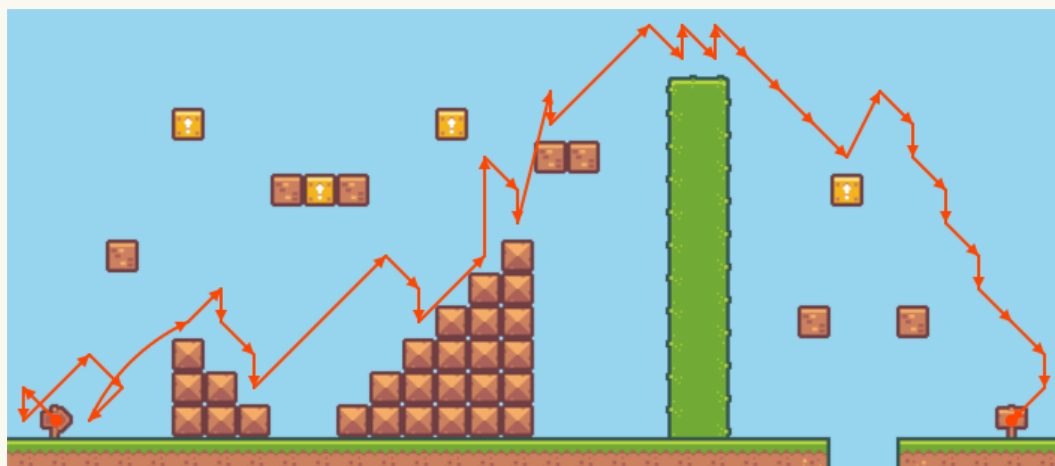


Exactly 10 

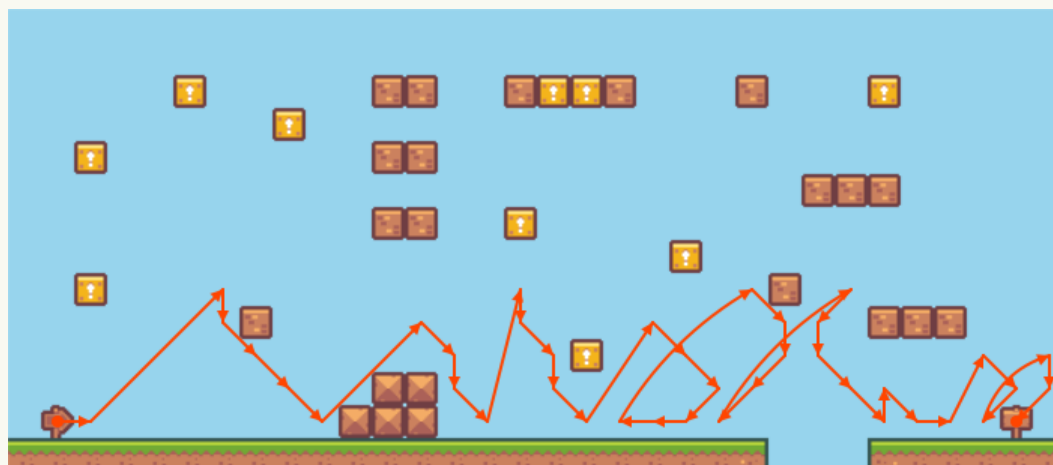
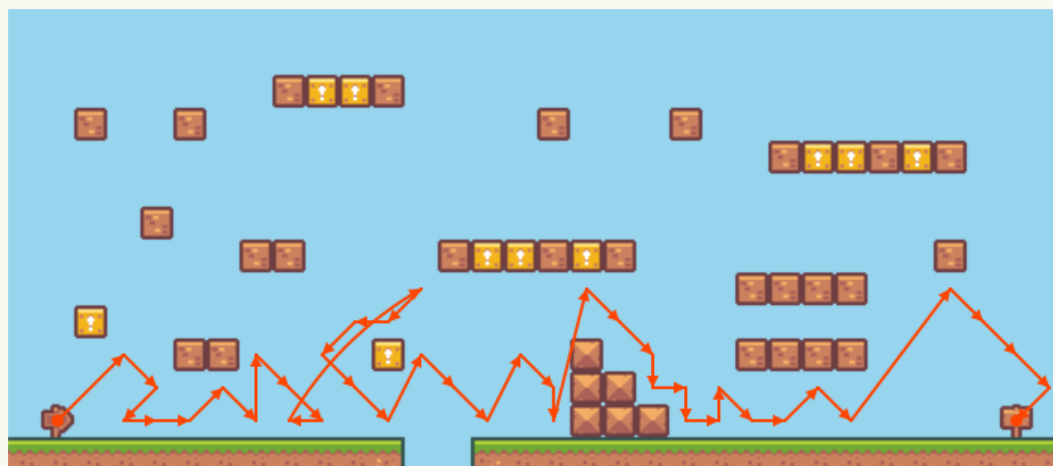


# Applications

Exactly 1  in 3<sup>rd</sup> from top row

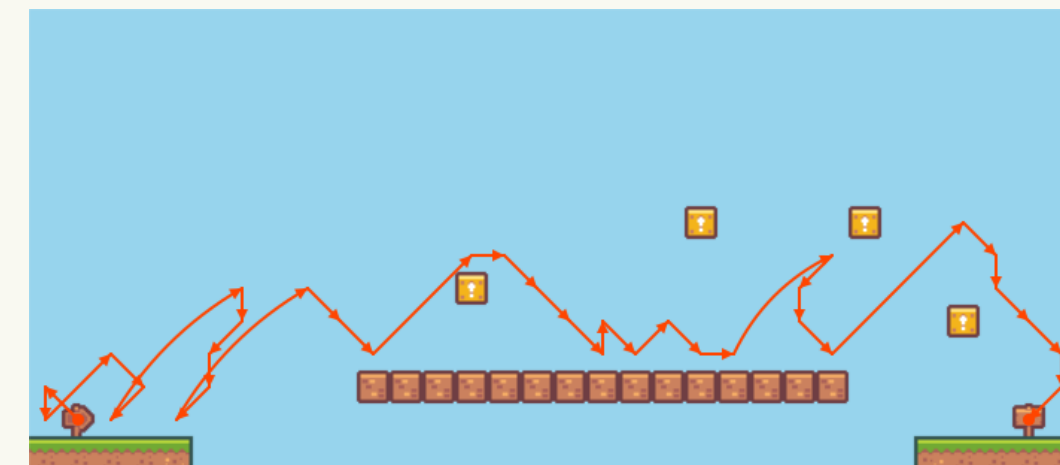
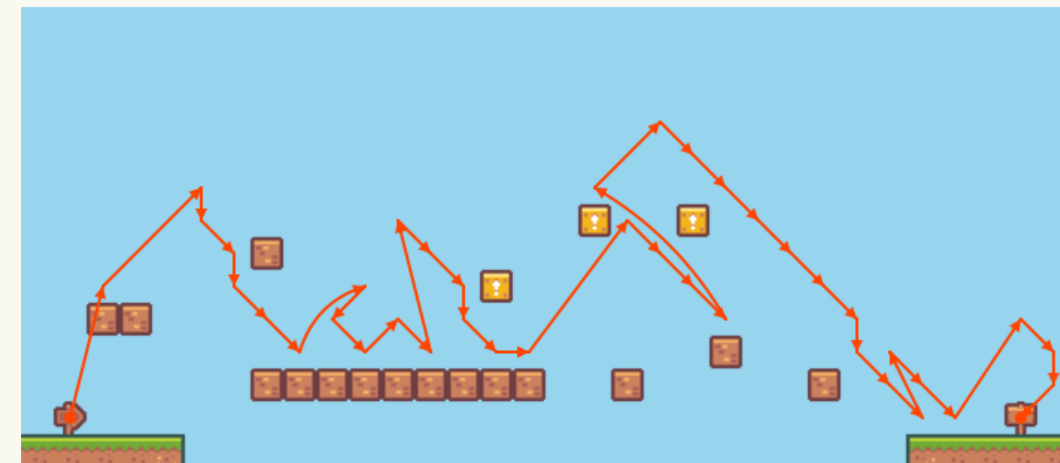


Exactly 10 



# Tile Constraints

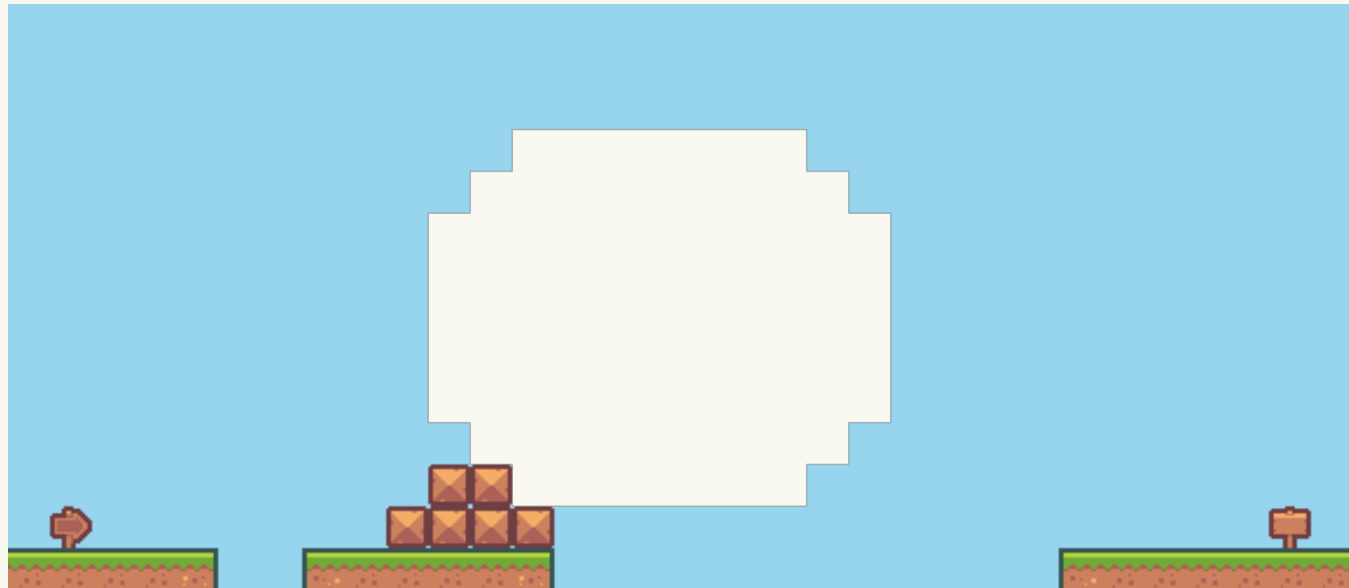
Maximize  in bottom row (soft)





# Applications

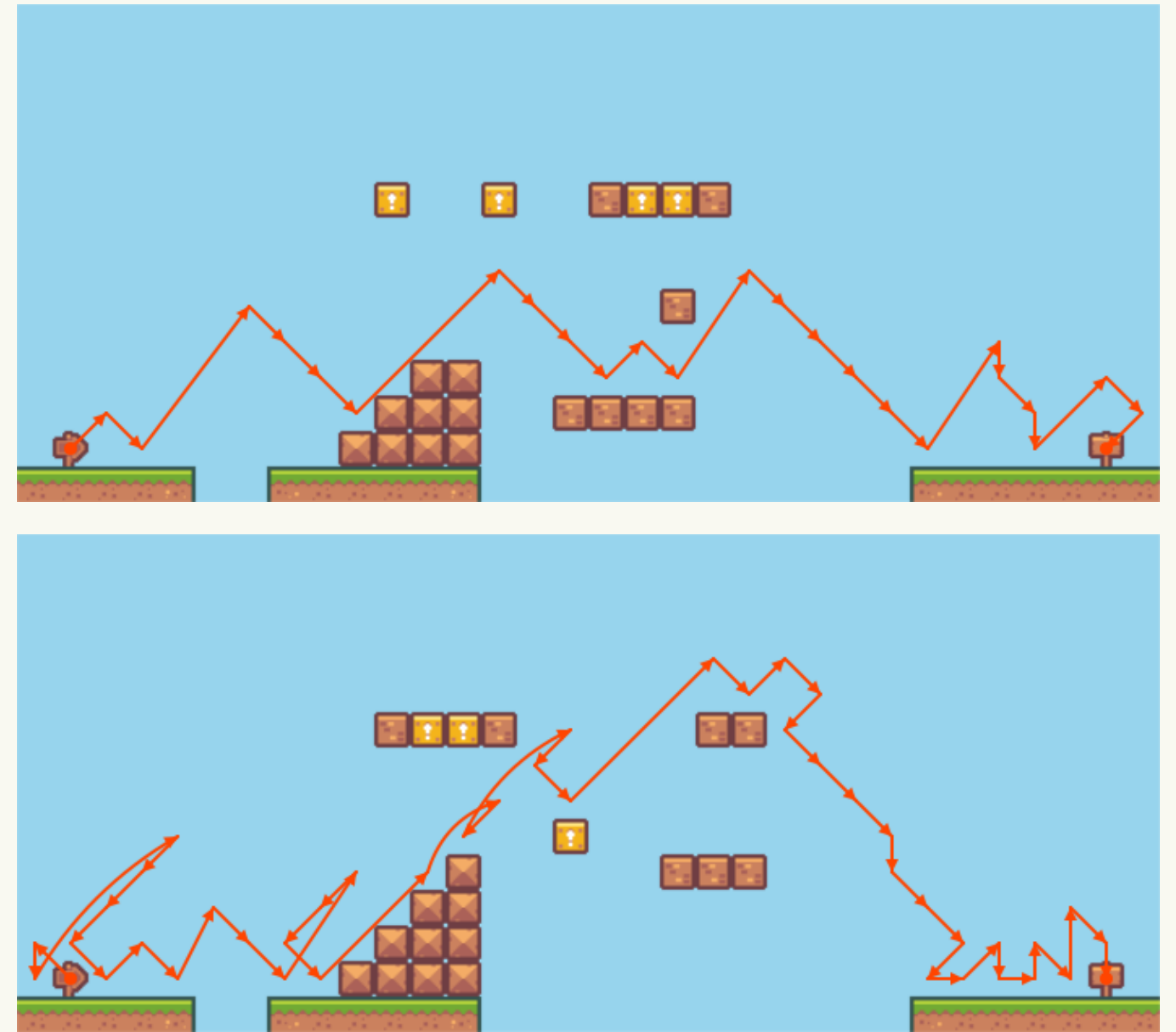
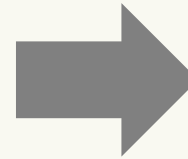
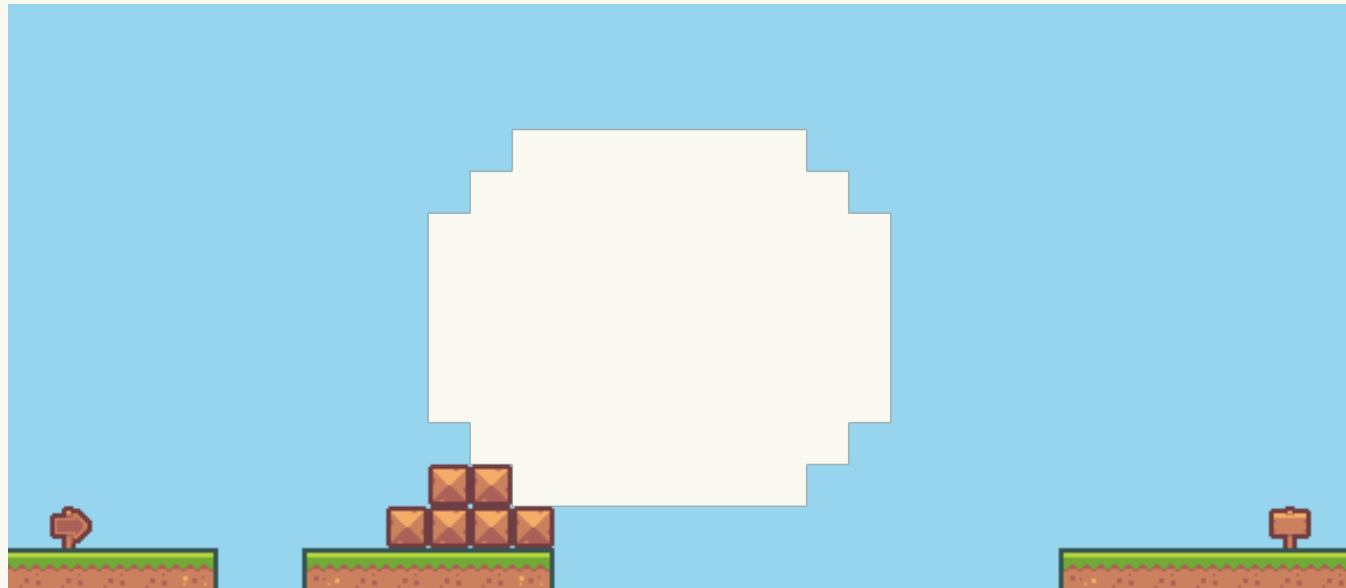
Infilling



Existing tiles & reachability hard; patterns soft

# Applications

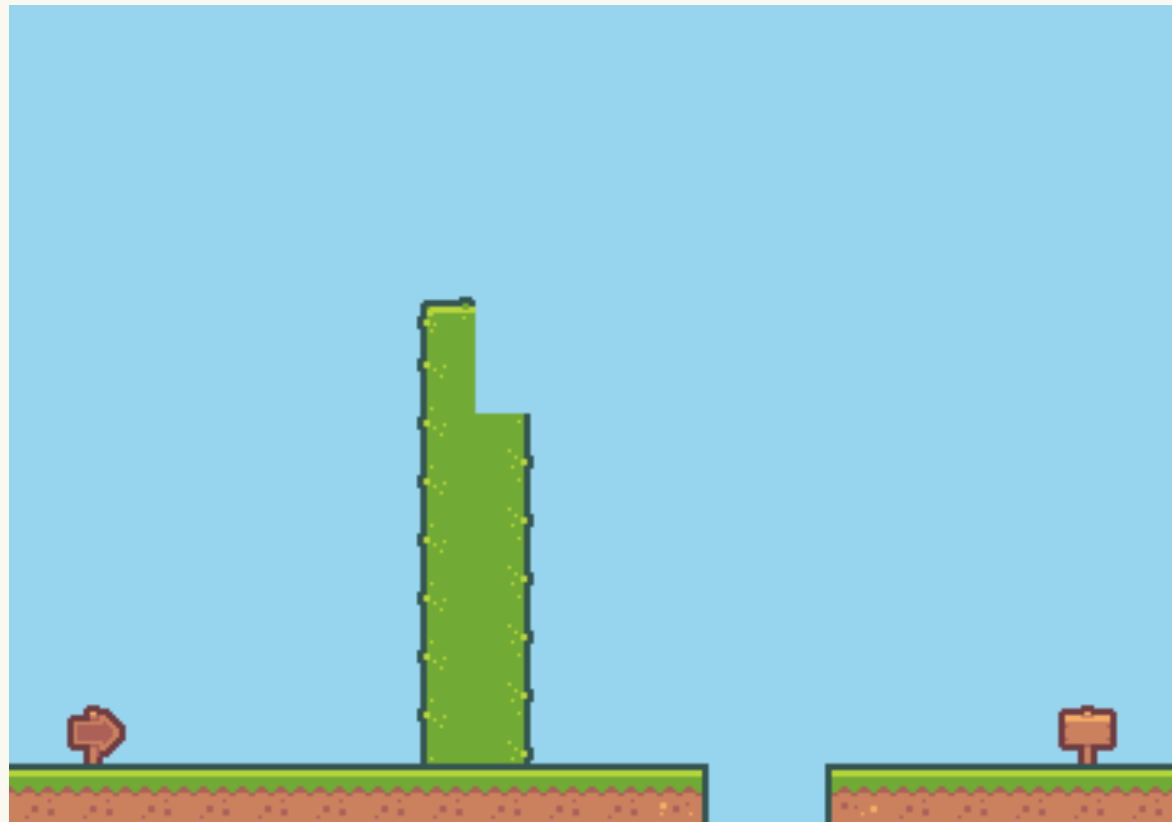
## Infilling



Existing tiles & reachability hard; patterns soft

# Applications

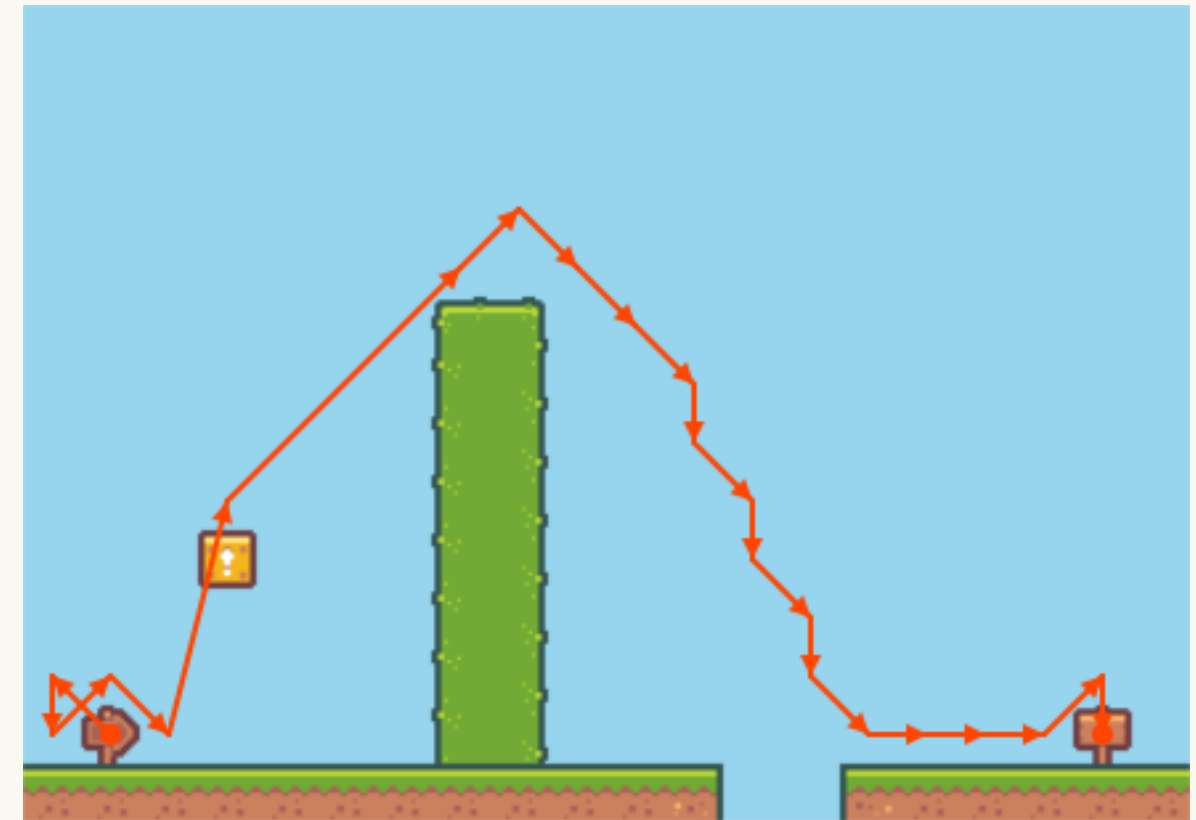
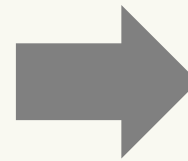
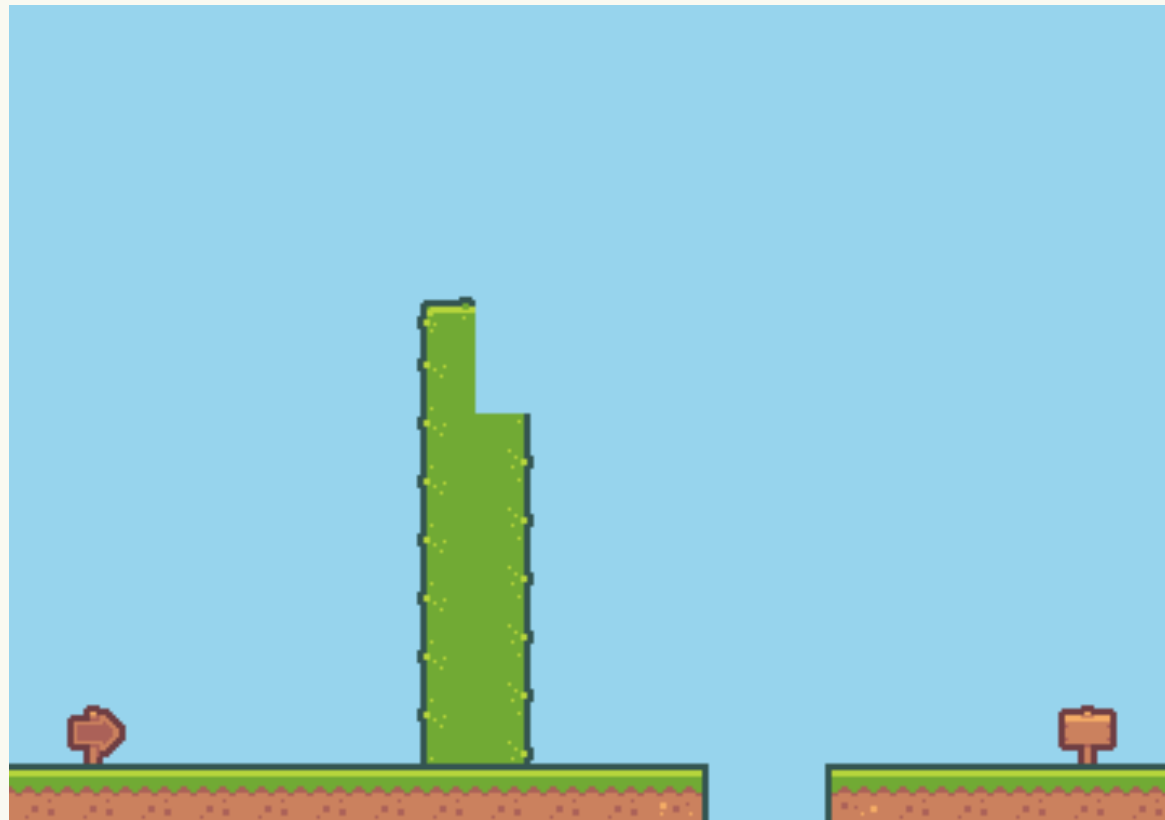
Repair



Patterns & reachability hard; existing tiles soft

# Applications

Repair

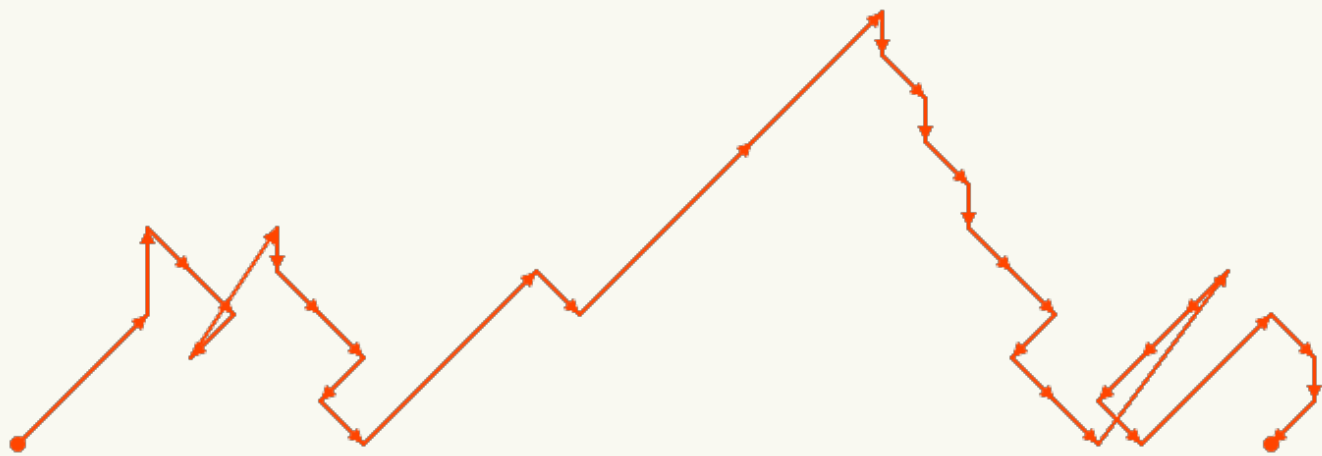


Patterns & reachability hard; existing tiles soft



# Applications

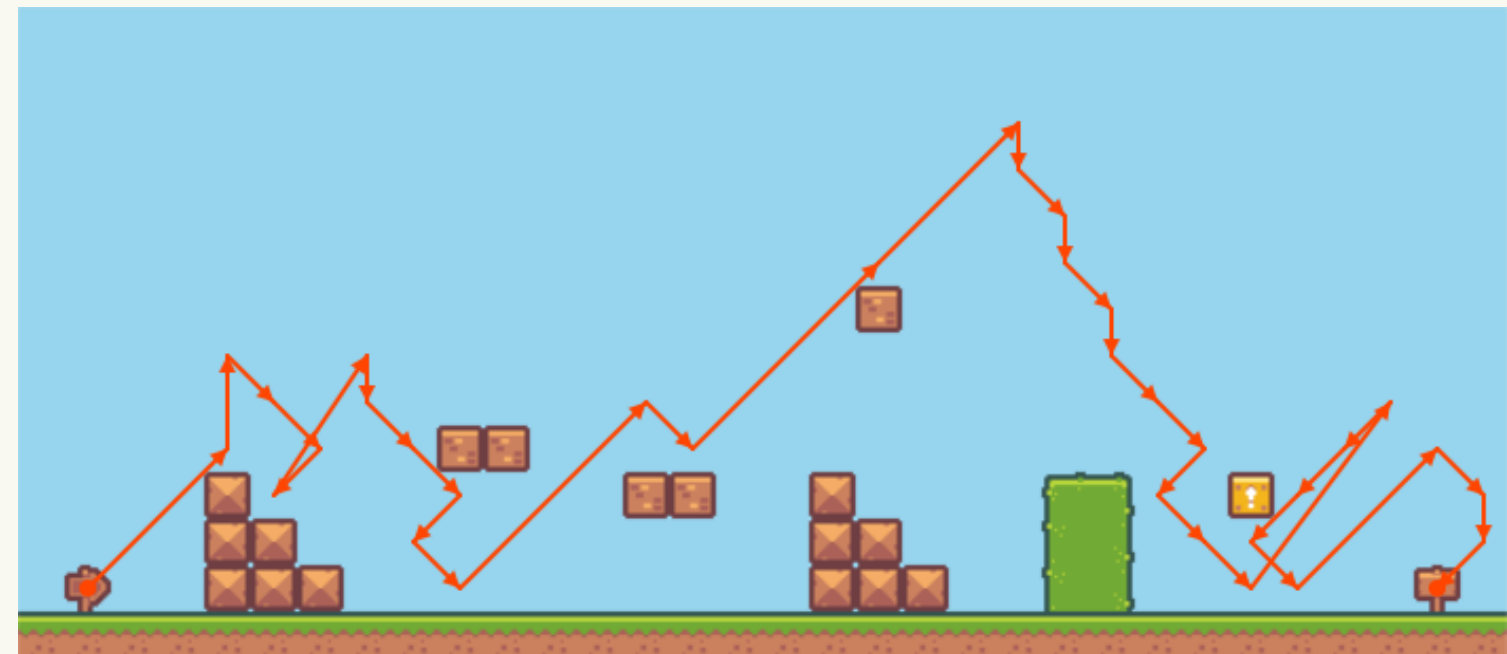
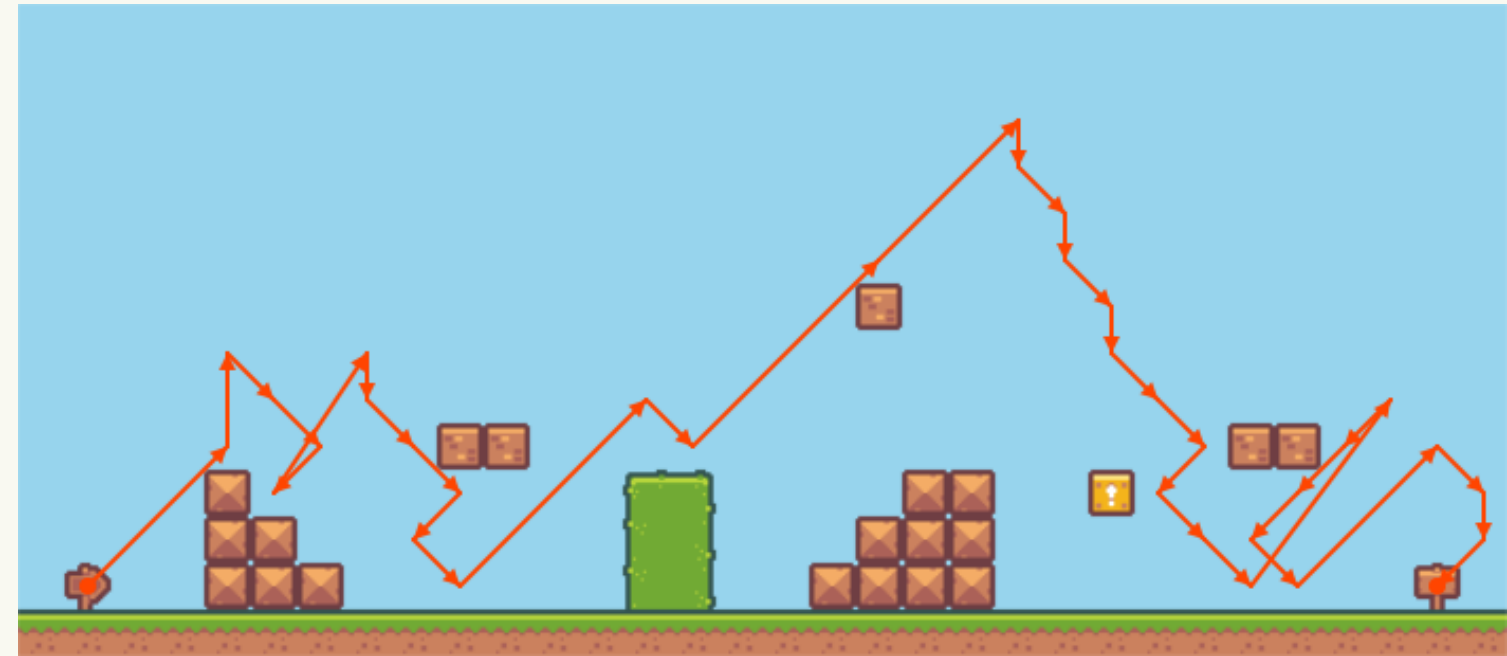
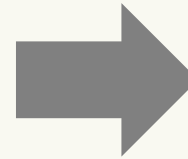
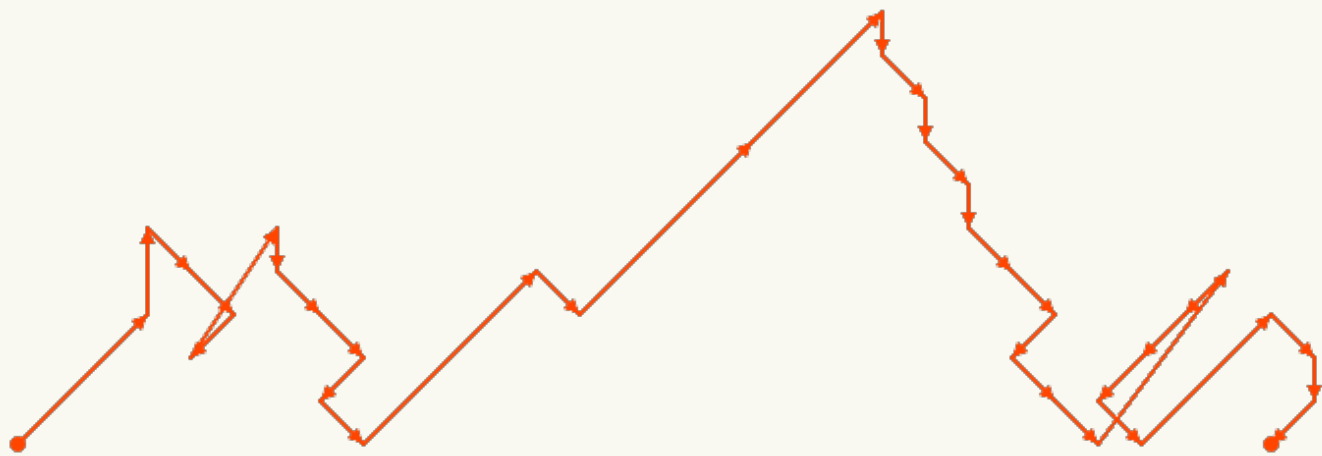
# Path to Level



## Generate level with desired “solution”

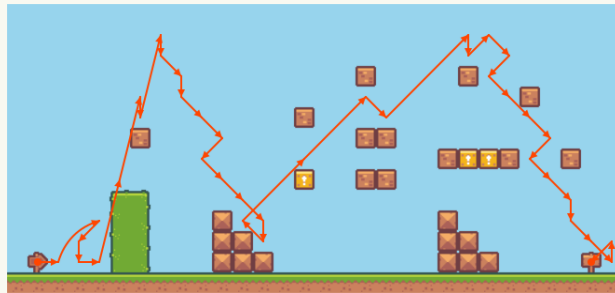
# Applications

# Path to Level



## Generate level with desired “solution”

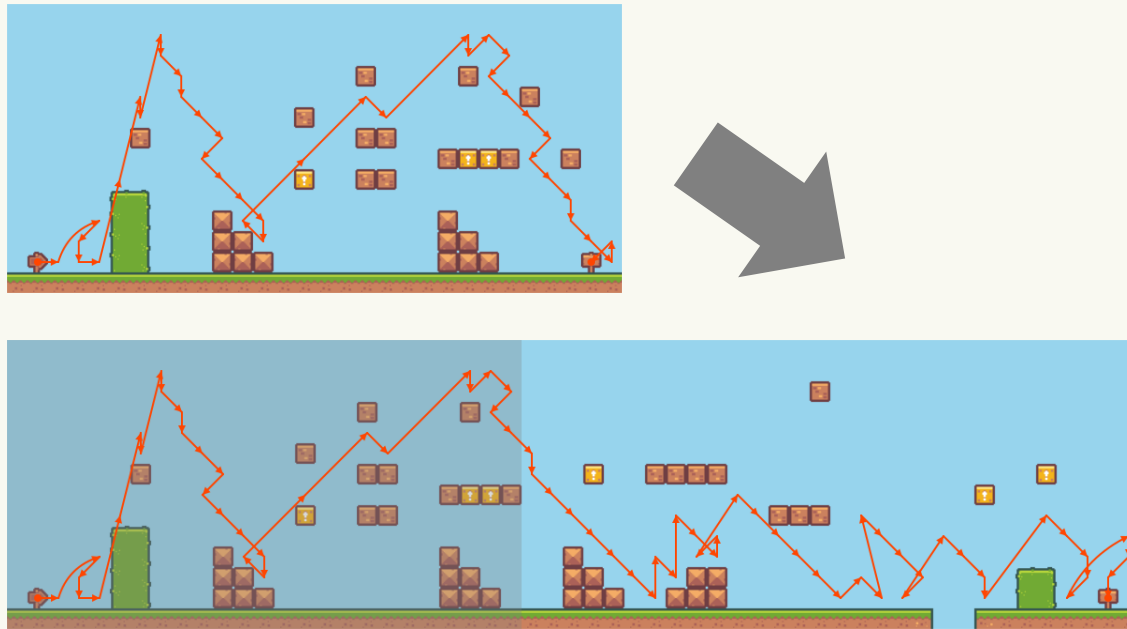
# Applications



Extension

# Applications

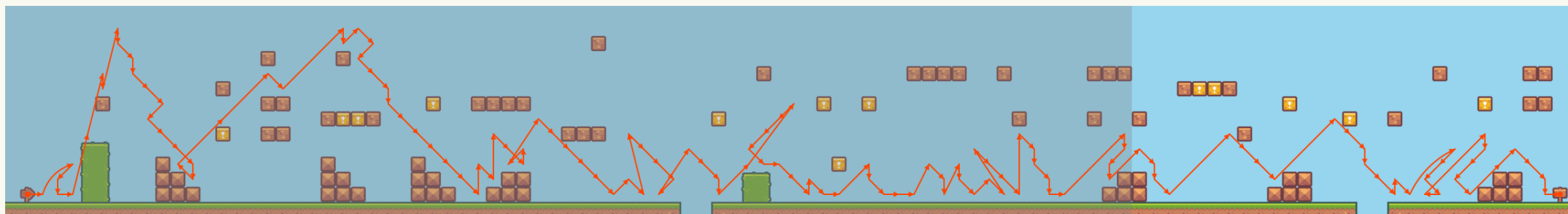
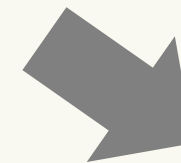
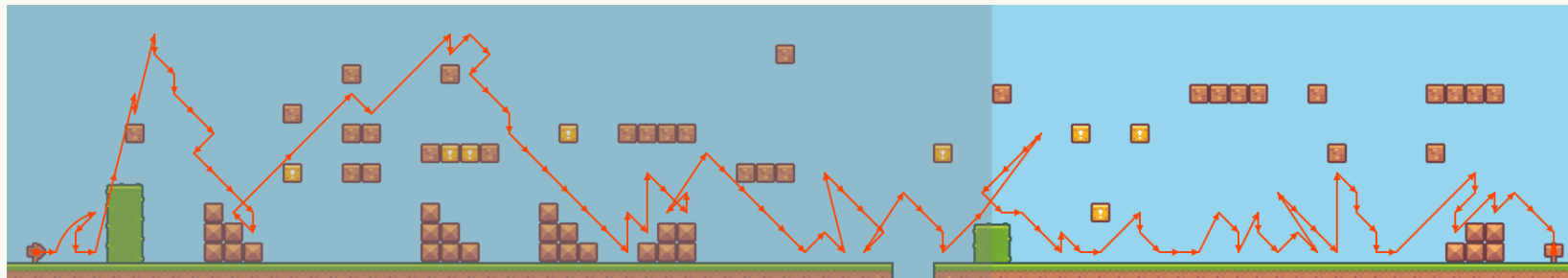
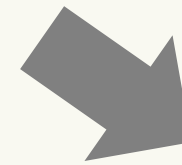
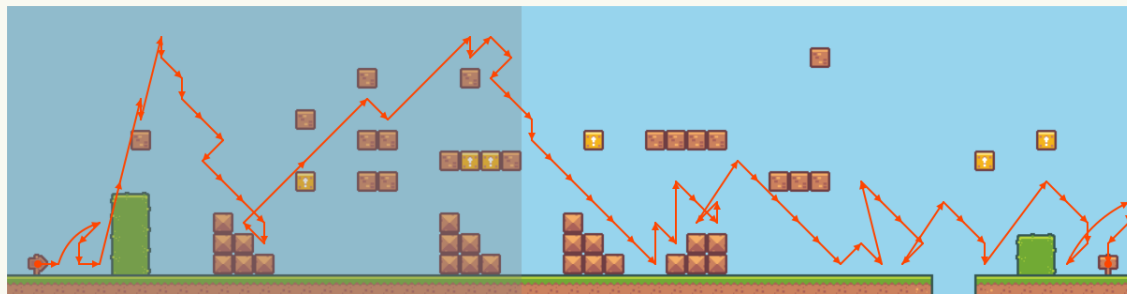
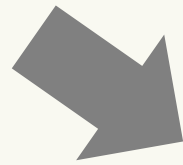
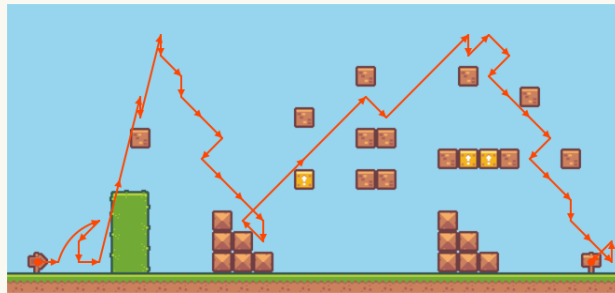
Extension





# Applications

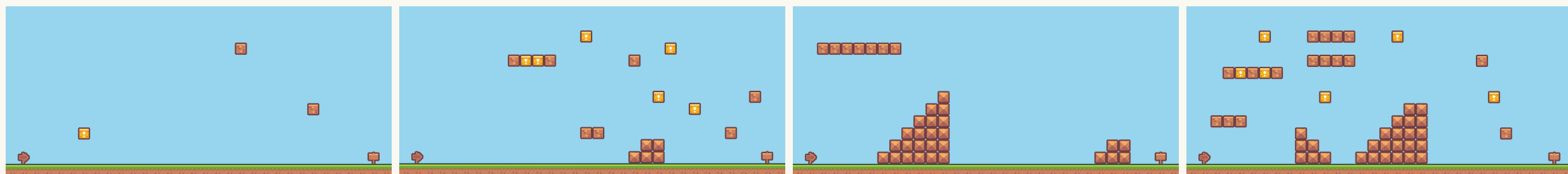
Extension



# Applications

Range

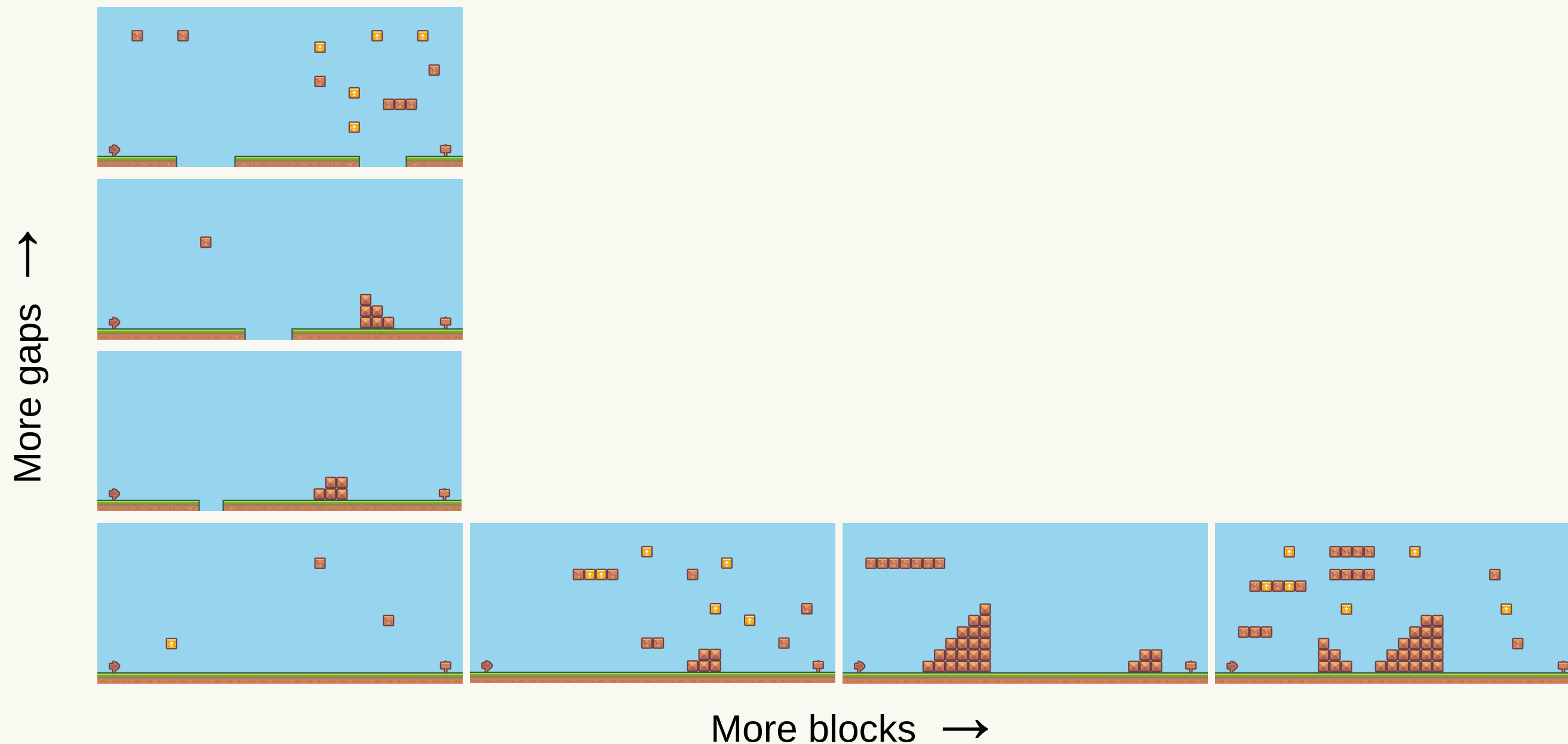
↑  
More gaps



More blocks →

# Applications

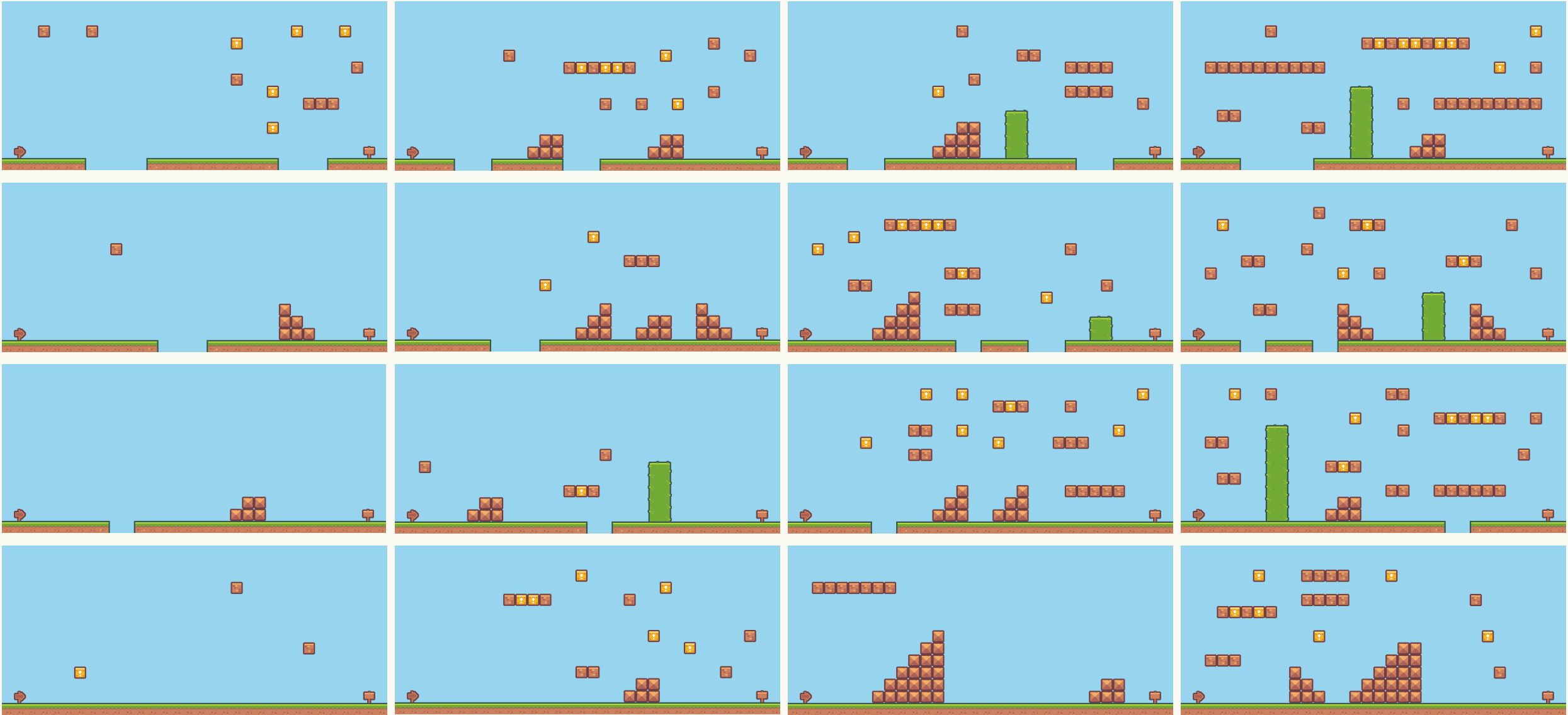
Range



# Applications

Range

More gaps ↑



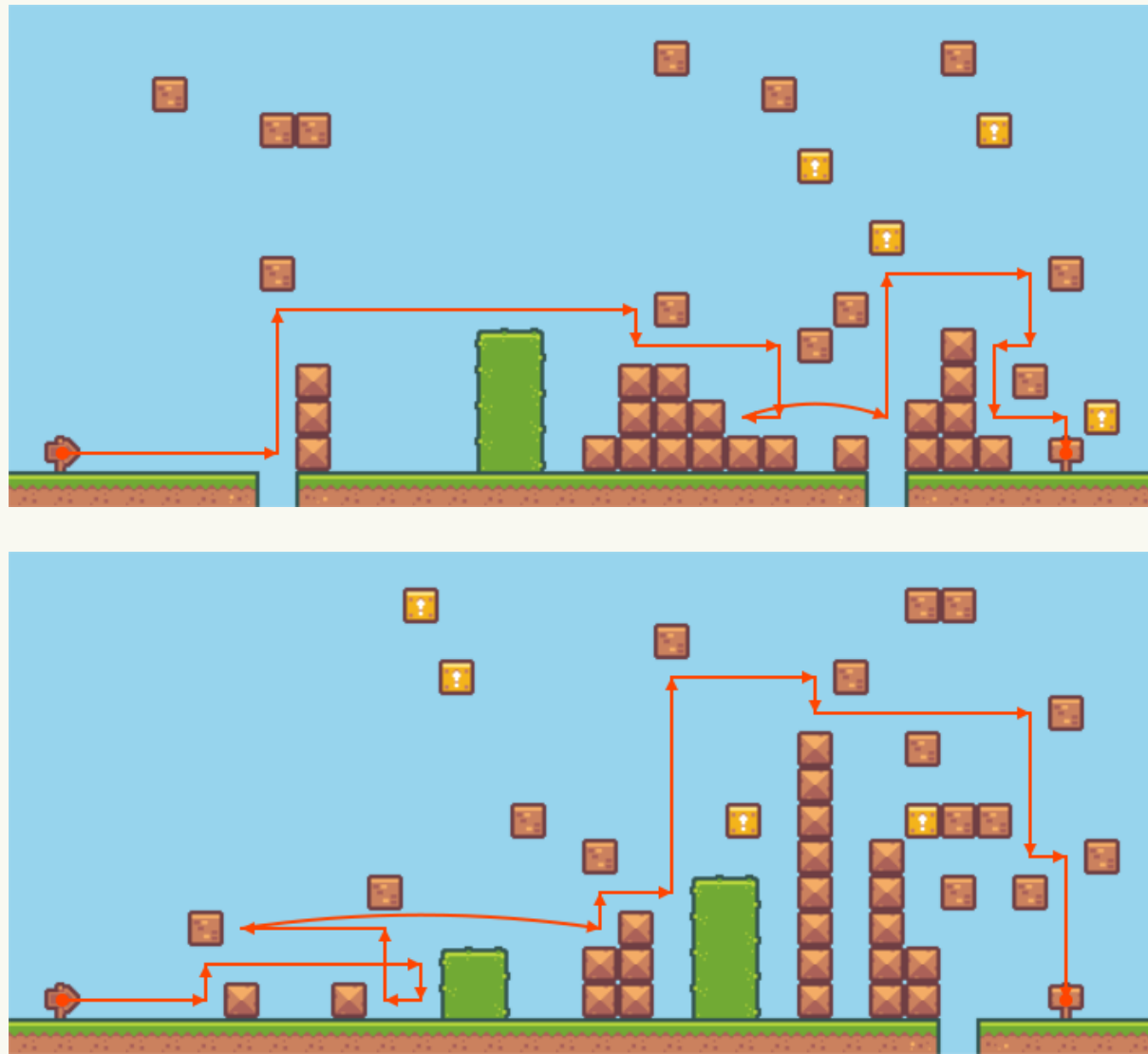
More blocks →



# Blending

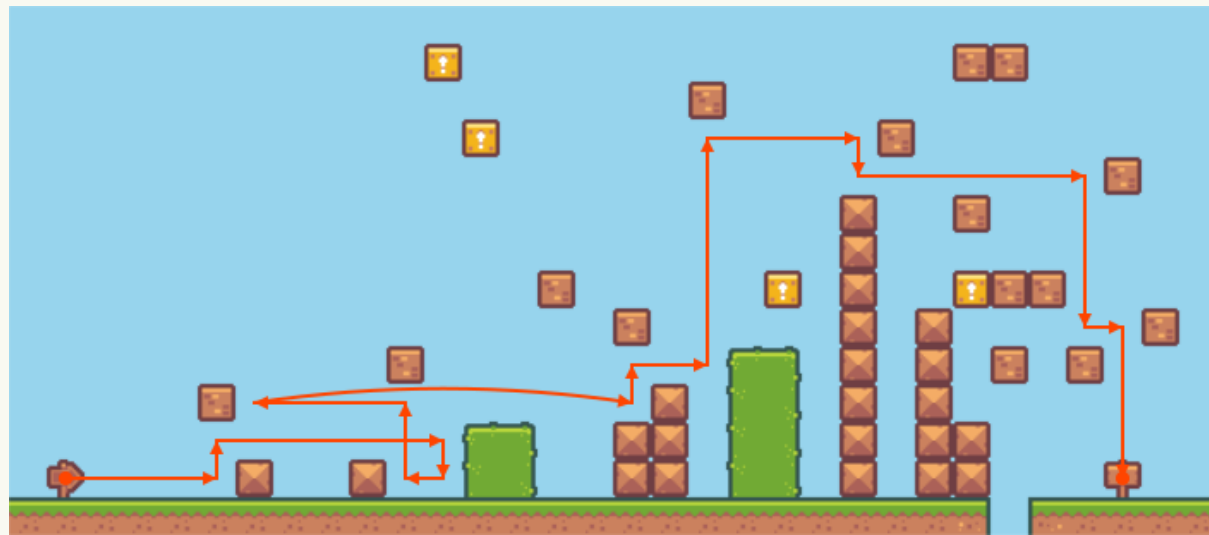
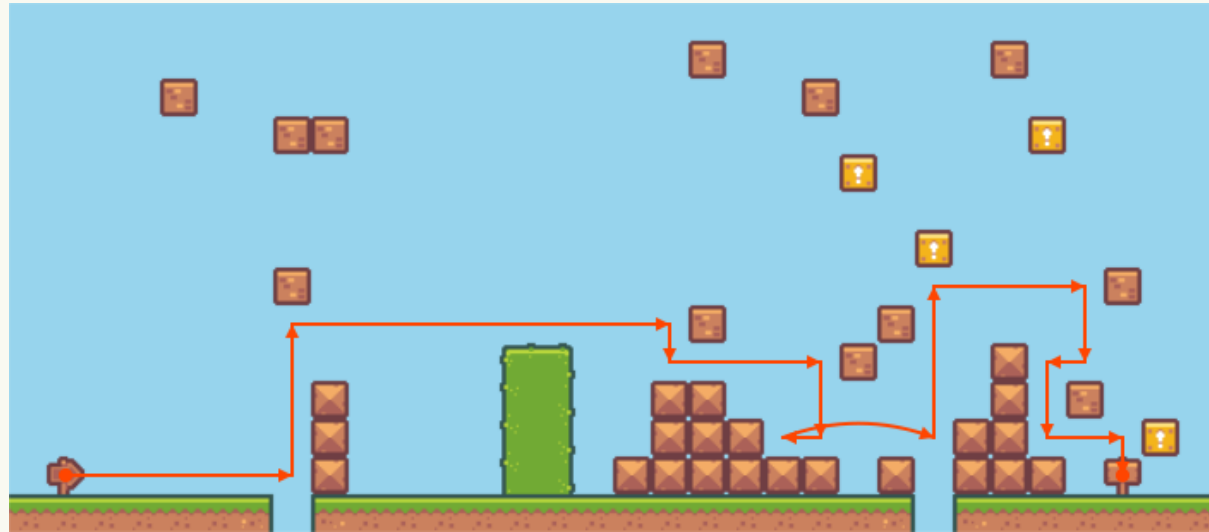
## Patterns and Movement

Platformer played by sliding



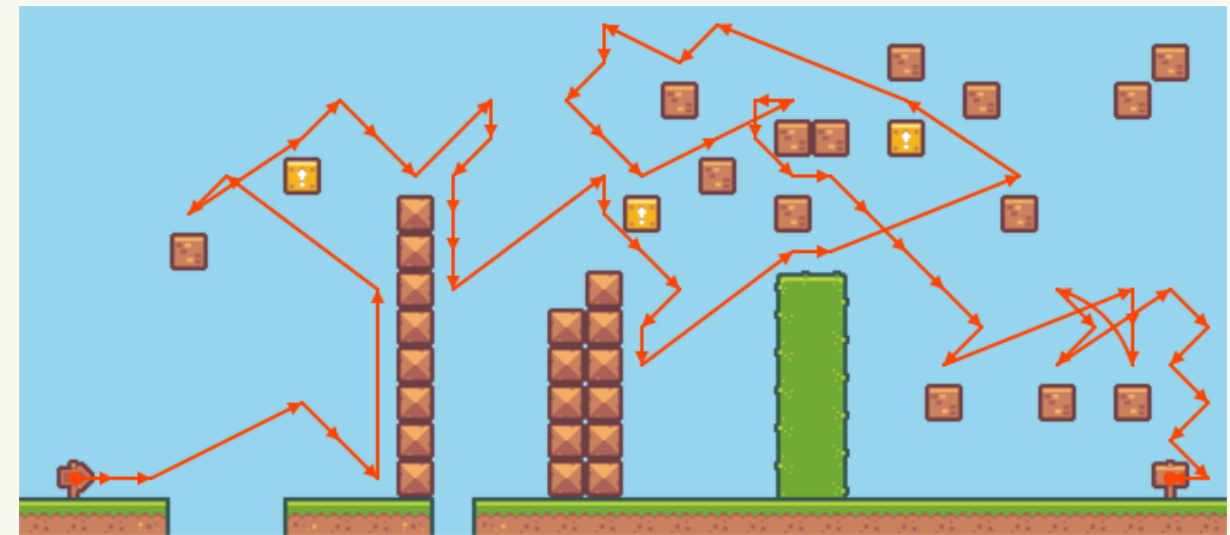
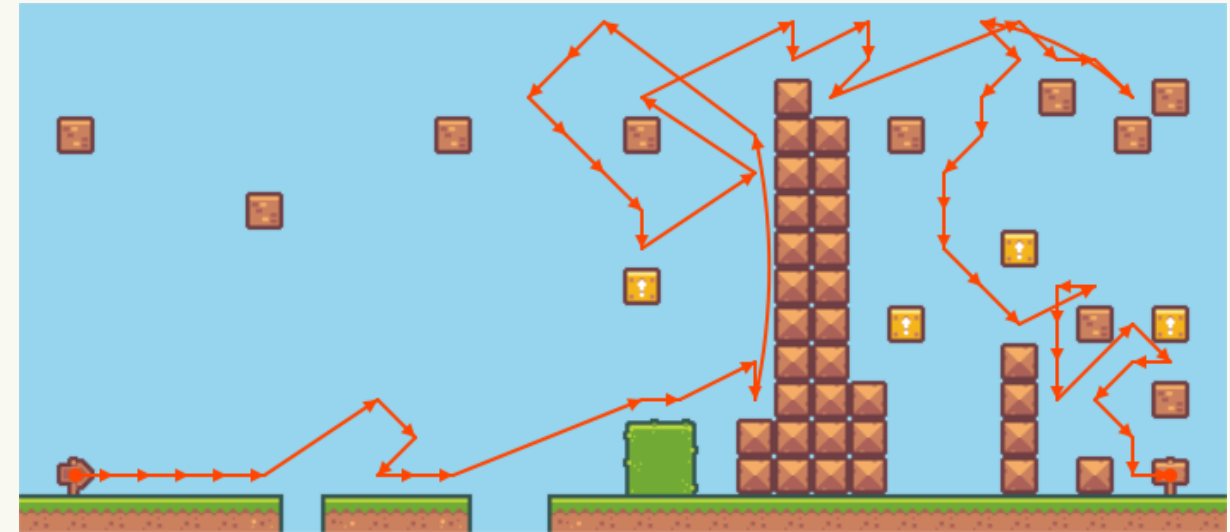
# Blending

Platformer played by sliding



# Patterns and Movement

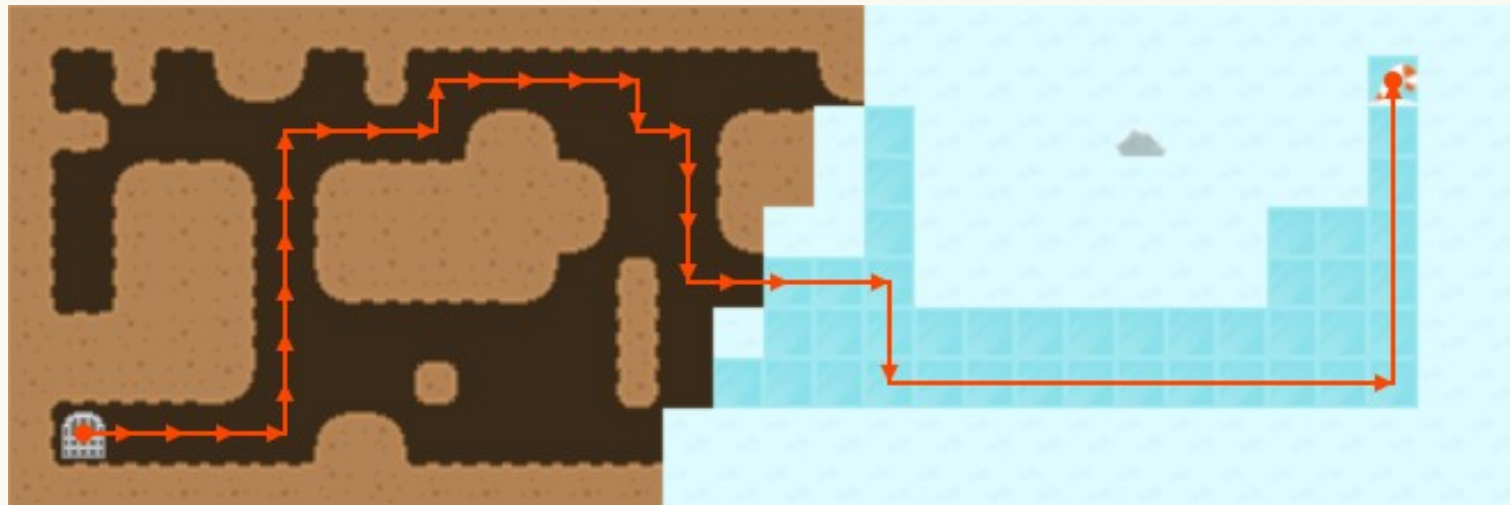
Platformer played by vertical platformer



# Blending

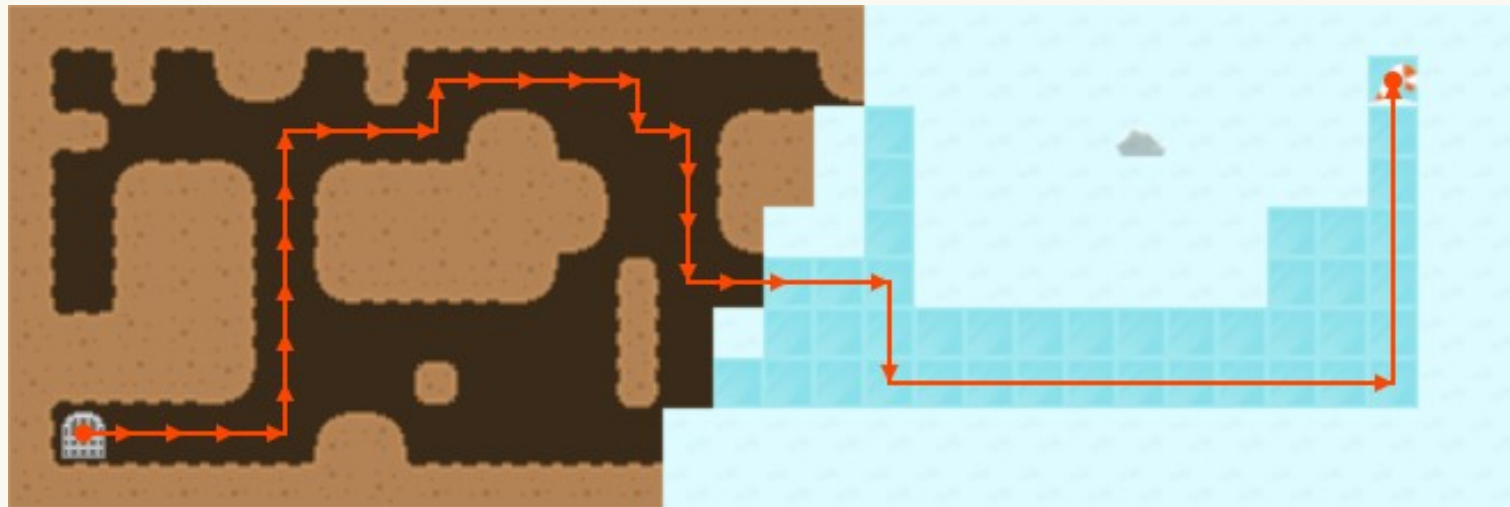
## Multi-Game Levels

Cave to sliding



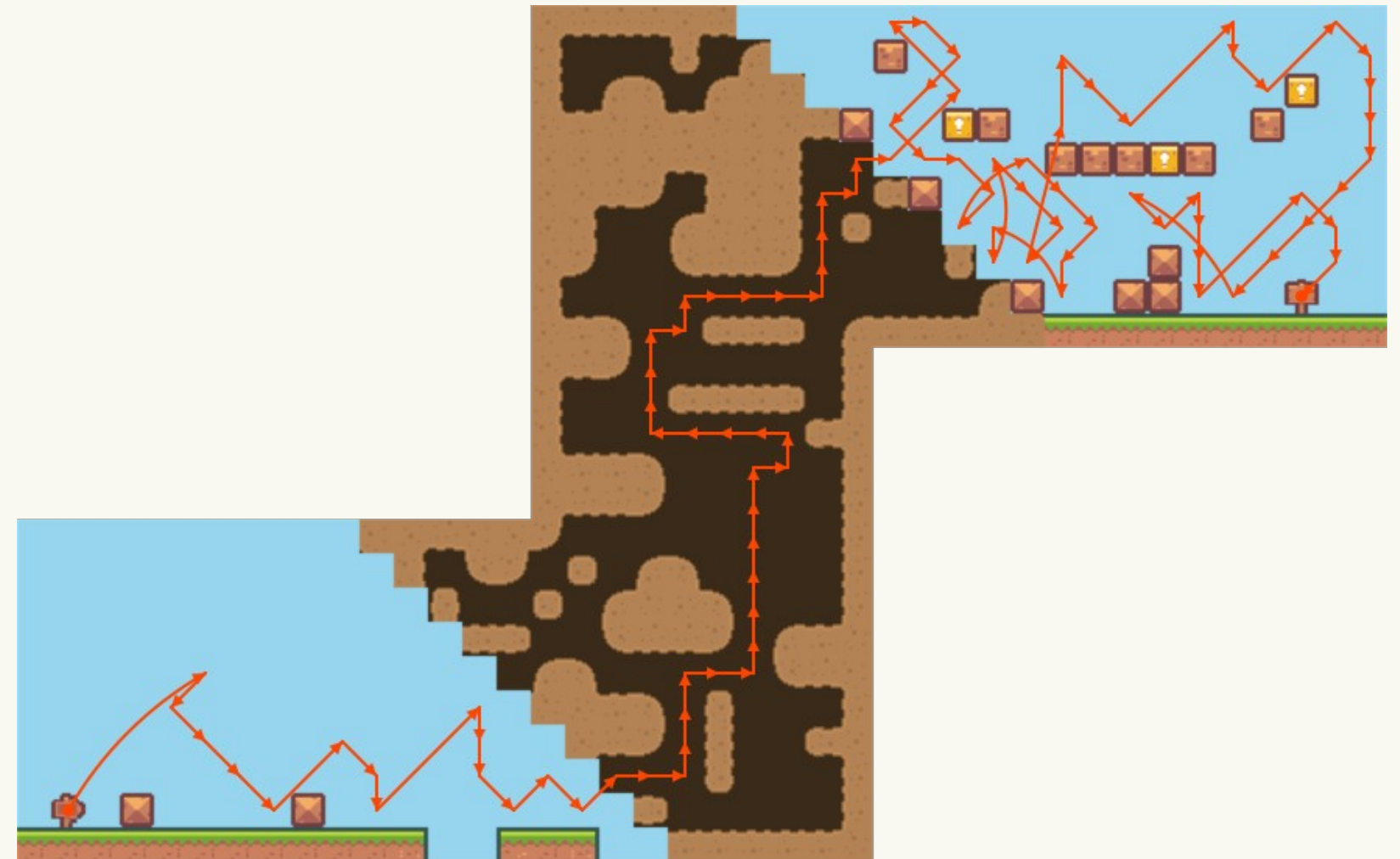
# Blending

Cave to sliding



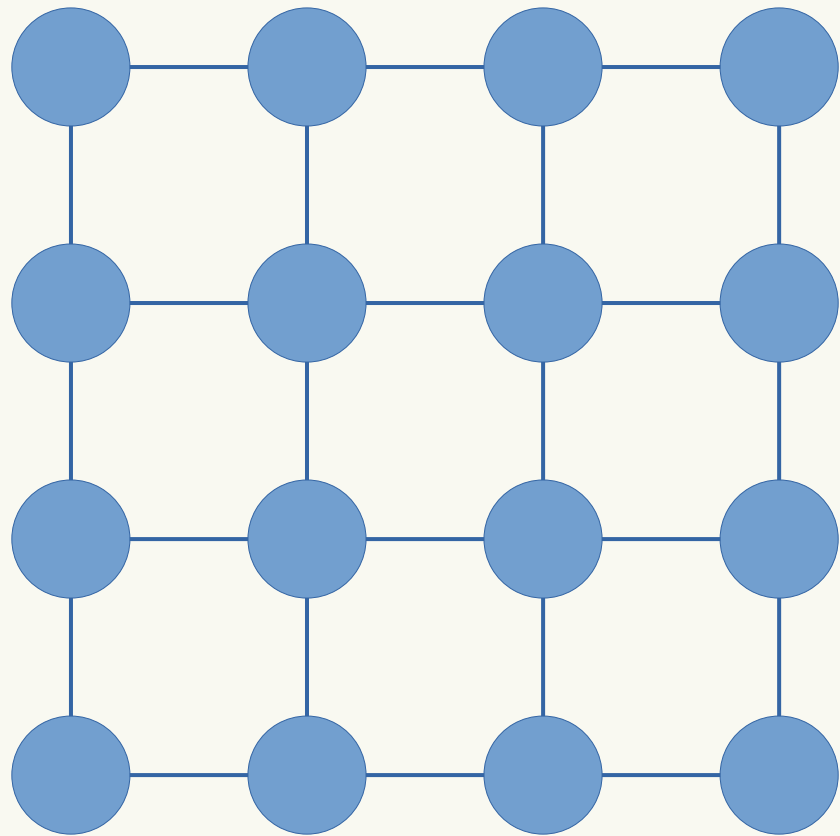
## Multi-Game Levels

Platformer to cave to platformer





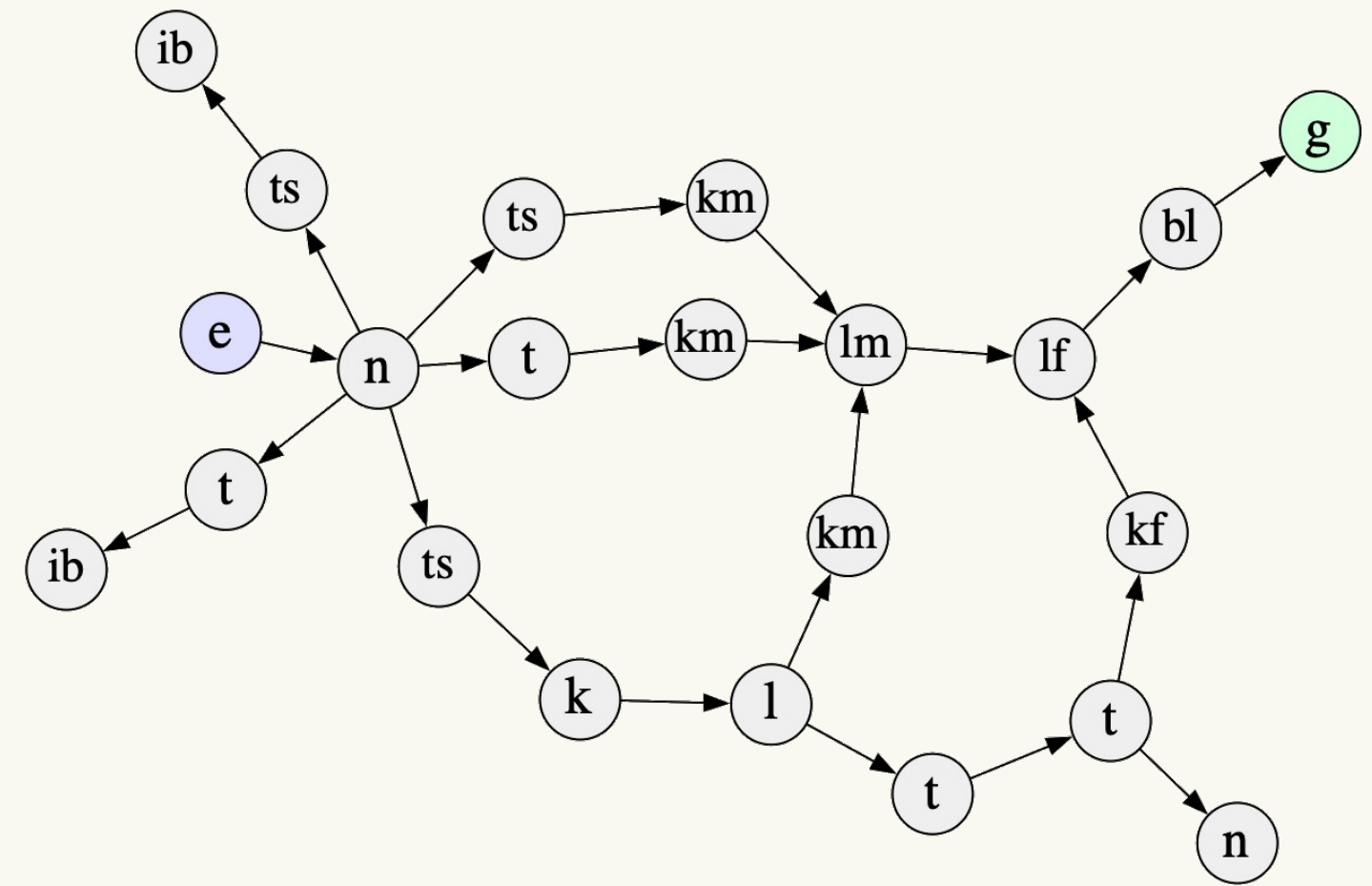
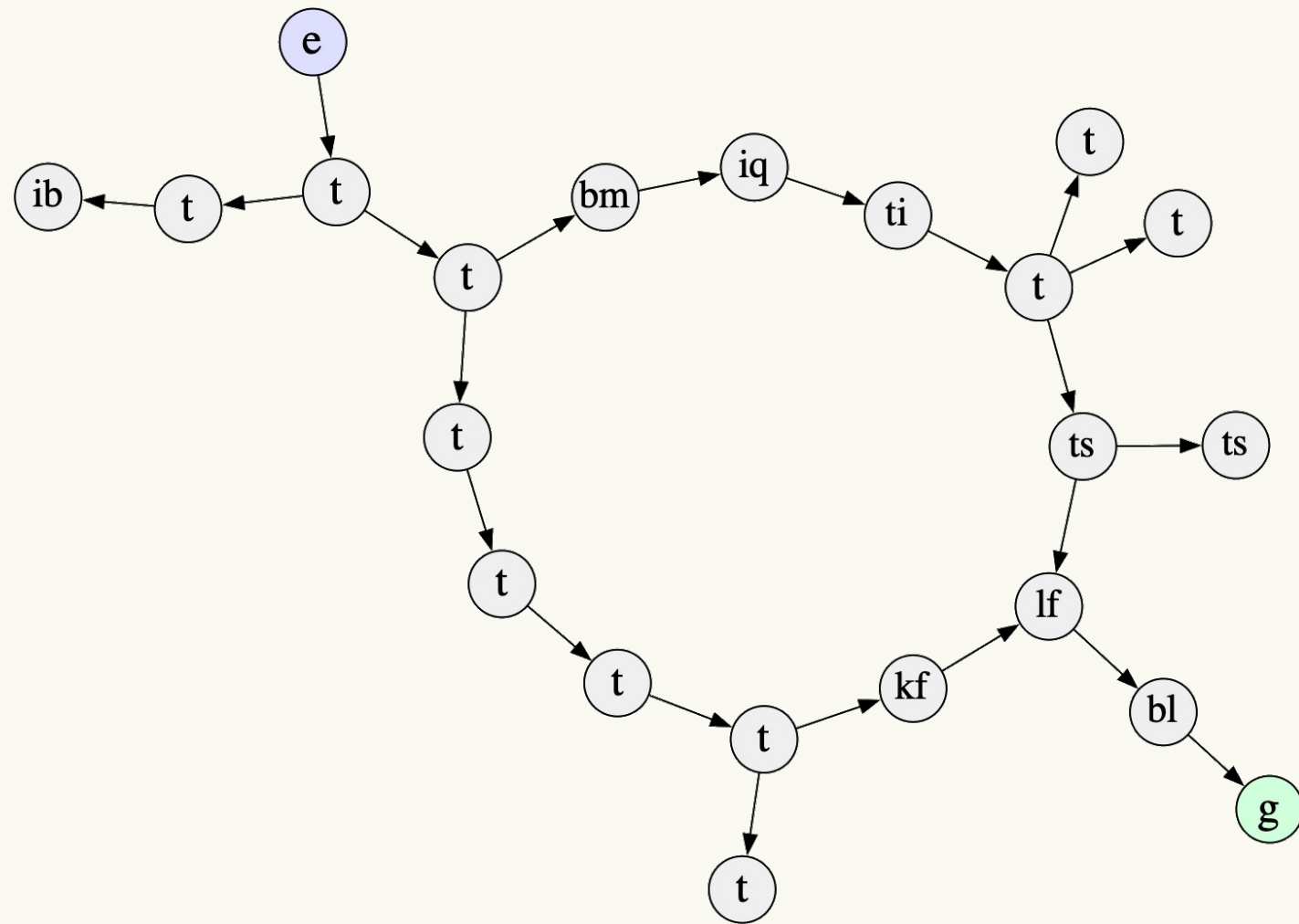
# Off the Grid - Graph Generation



- 2D grid is a special case of graphs
  - (Note: this is not the reachability graph but the tile grid)
- Same general concept:
  - learn local patterns from example(s)
  - generate graphs with only with those patterns
- To learn from / generate graphs:
  - Variables for the graph structure (e.g. how “tiles” neighbor each other)
  - Constraints on structure (e.g. what local connectivity can be, must be connected, be a tree, etc)

# Graph Generation

# Abstract Missions

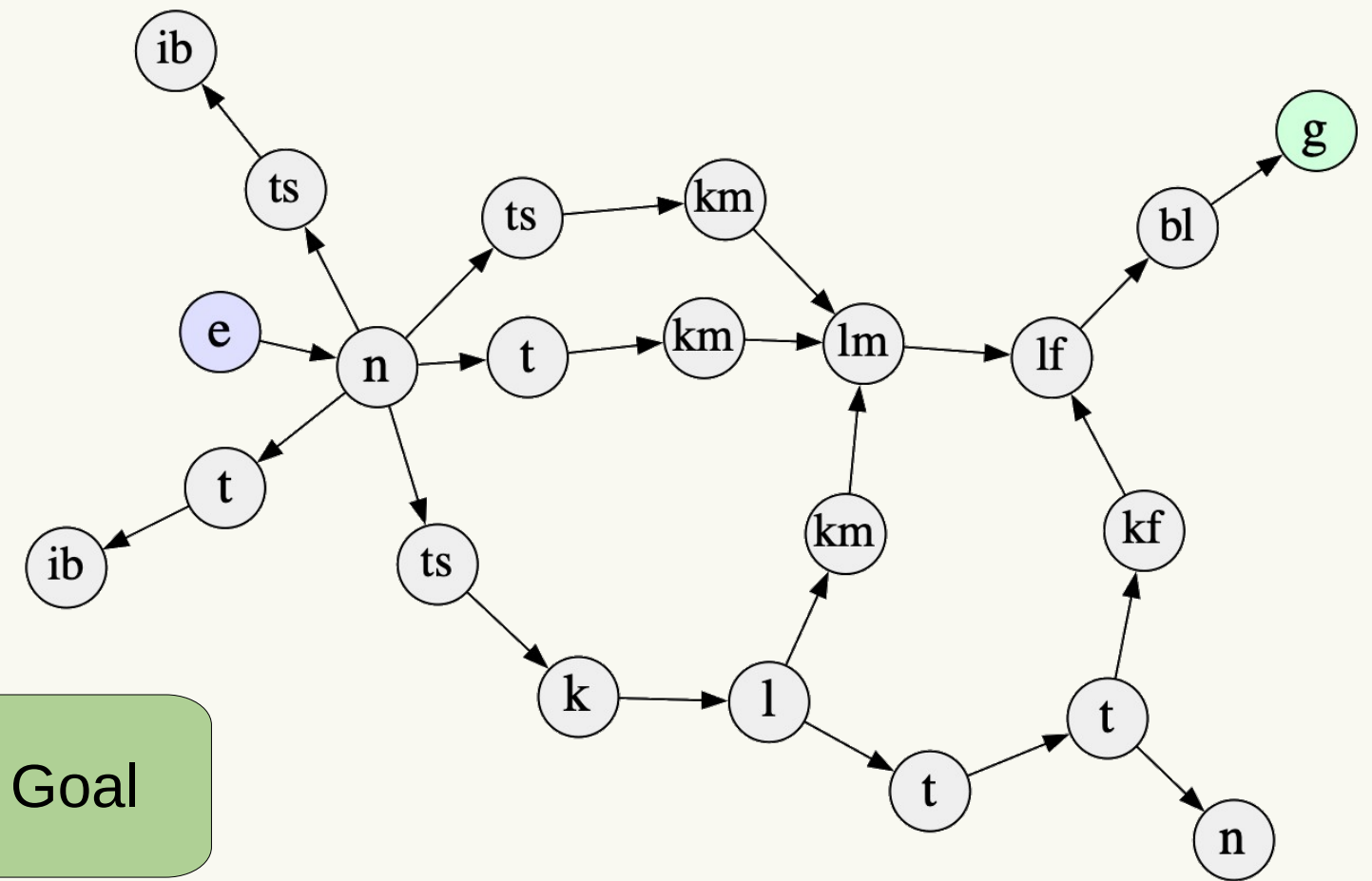
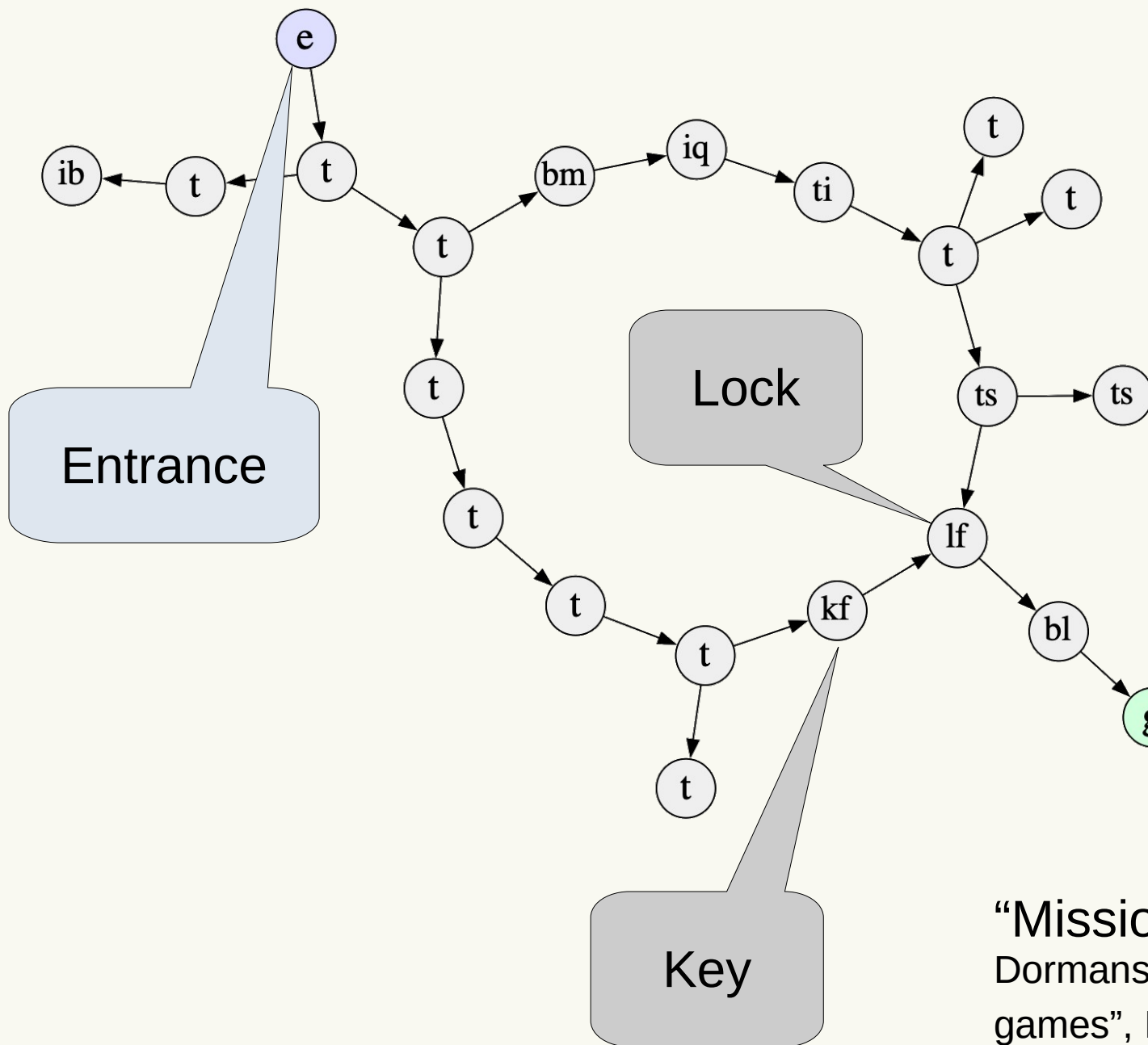


## “Mission” graphs

Dormans “Adventures in level design: generating missions and spaces for action adventure games”, Proceedings of the FDG Workshop on Procedural Content Generation (2010)

# Graph Generation

## Abstract Missions

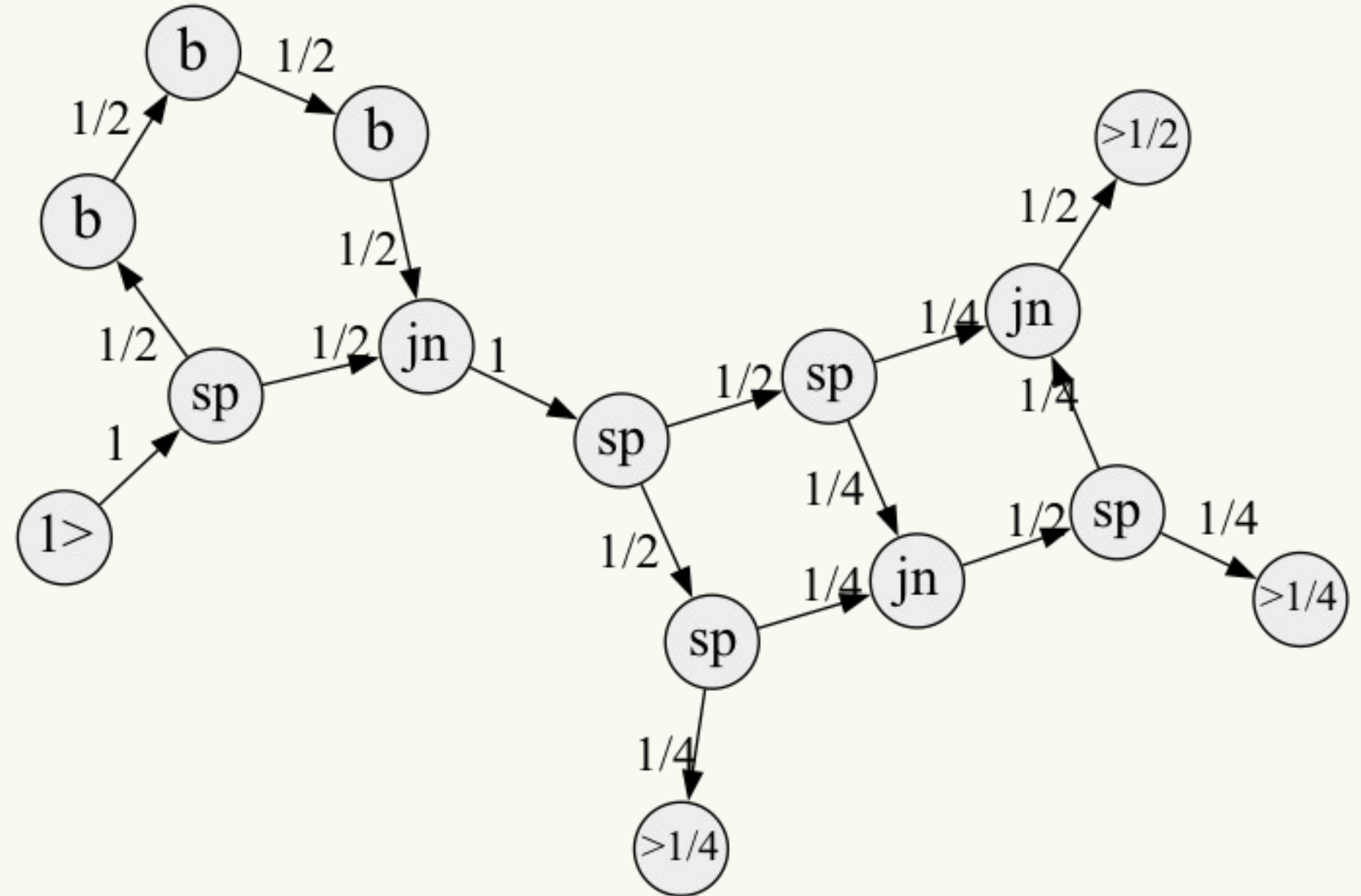
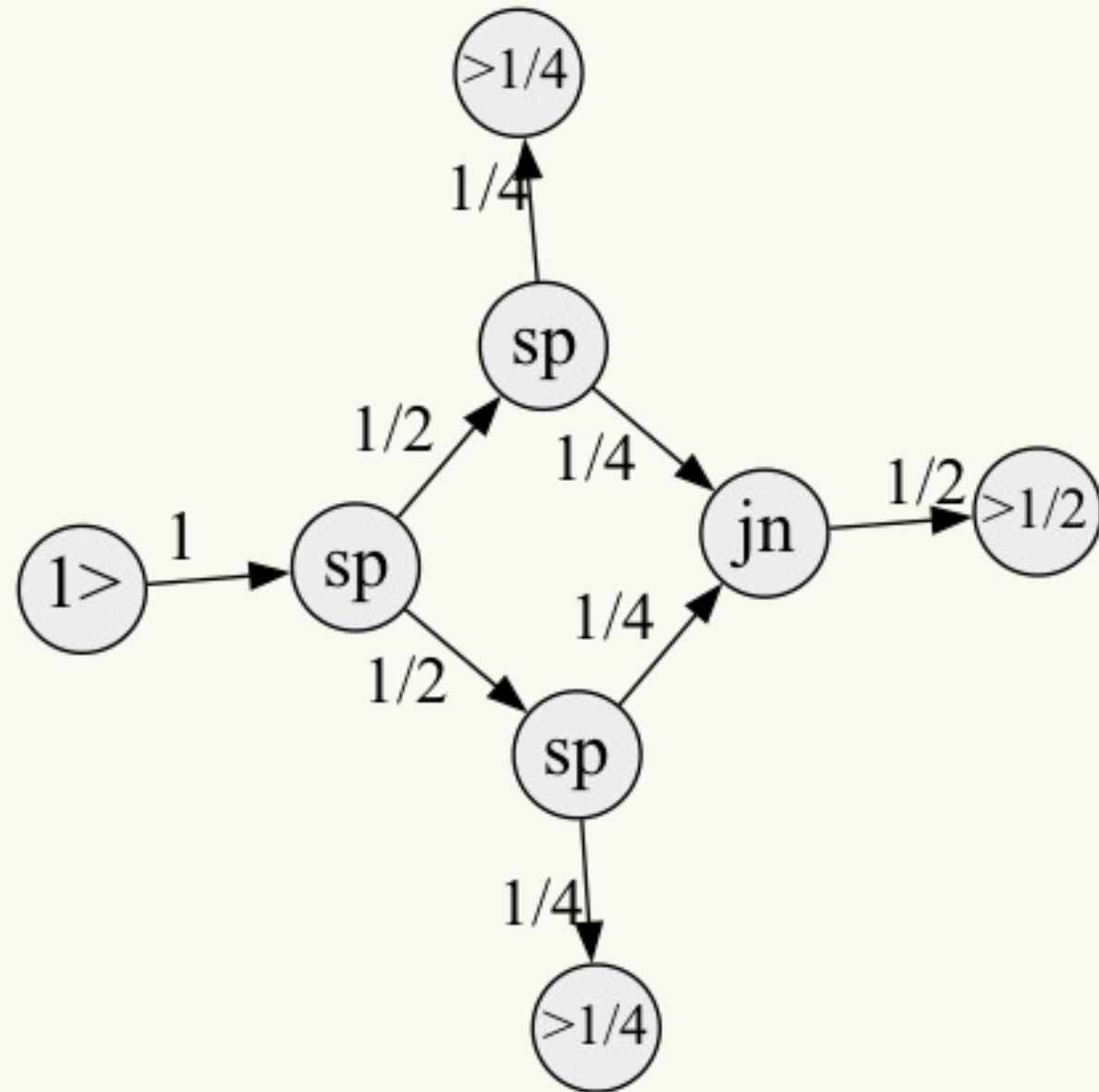


### “Mission” graphs

Dormans “Adventures in level design: generating missions and spaces for action adventure games”, Proceedings of the FDG Workshop on Procedural Content Generation (2010)

# Graph Generation

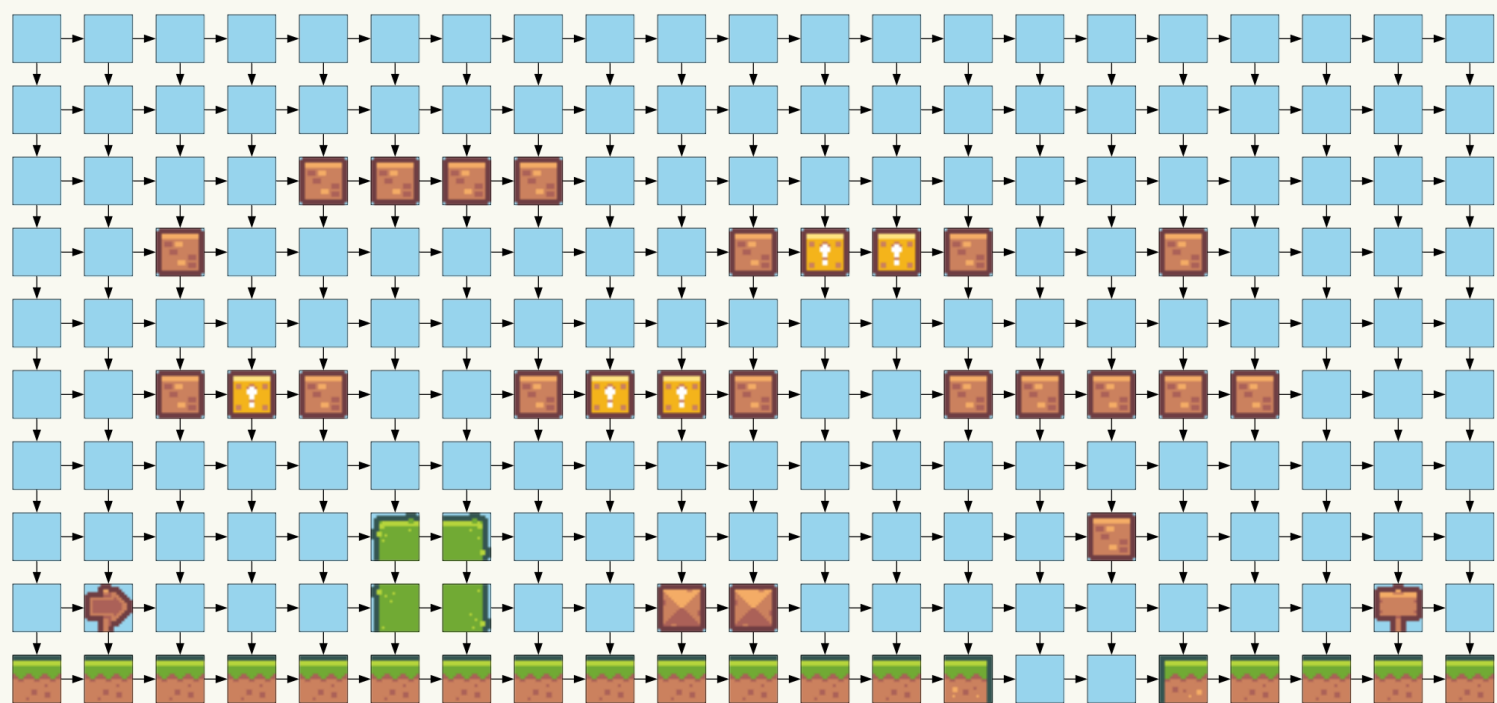
Fractions





# Graph Generation

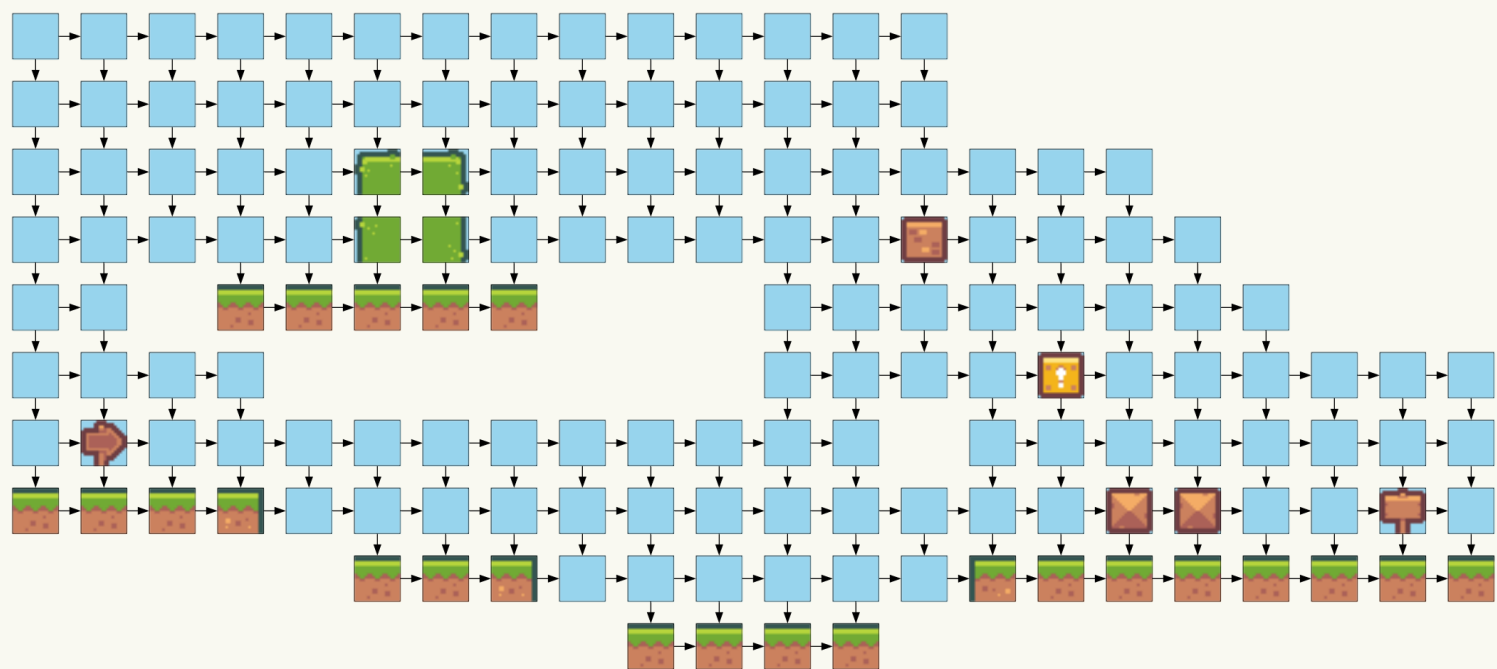
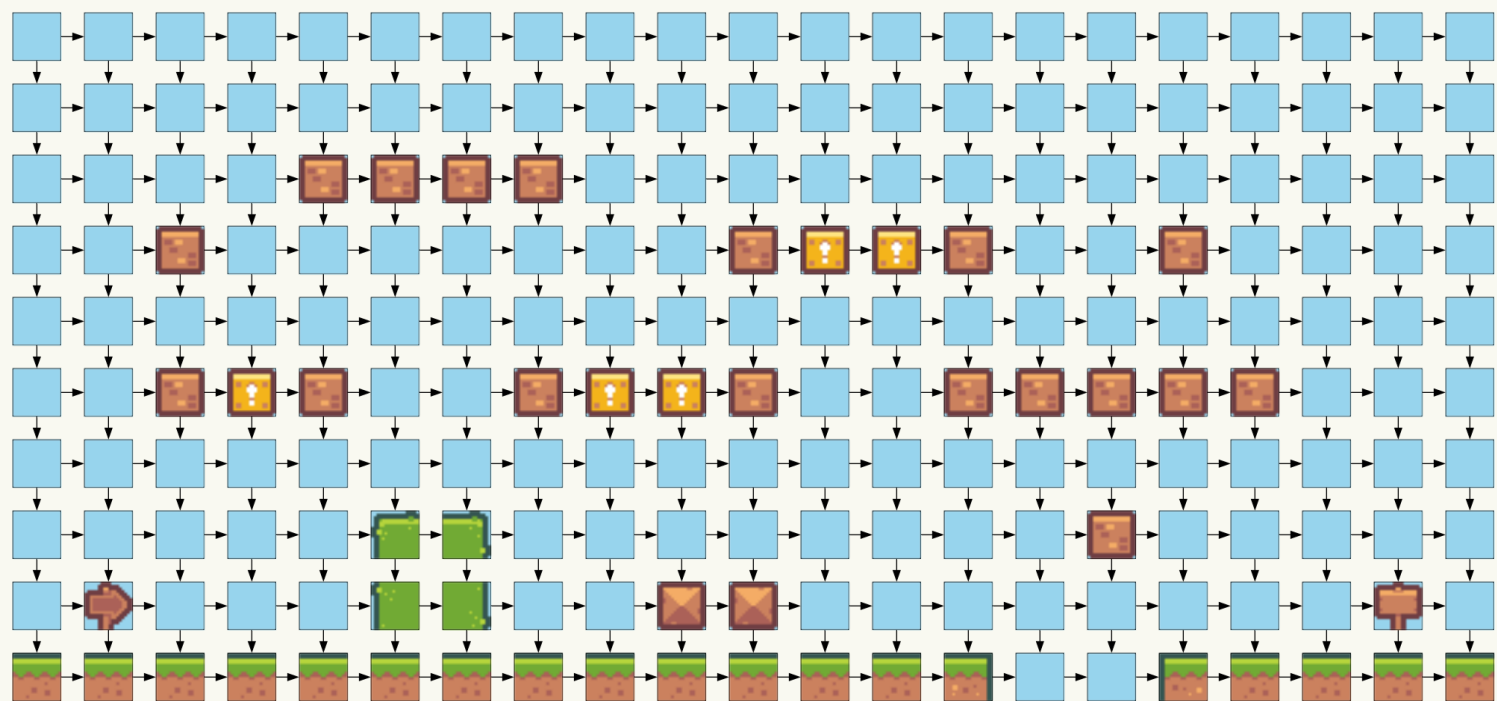
Flexible “Grids”





# Graph Generation

Flexible “Grids”

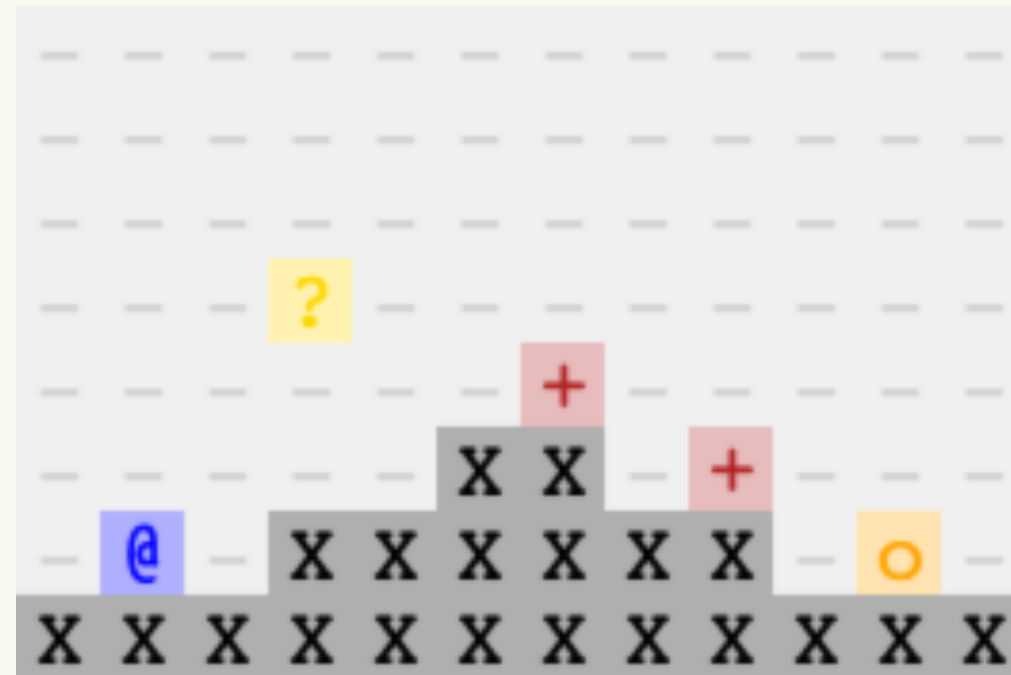
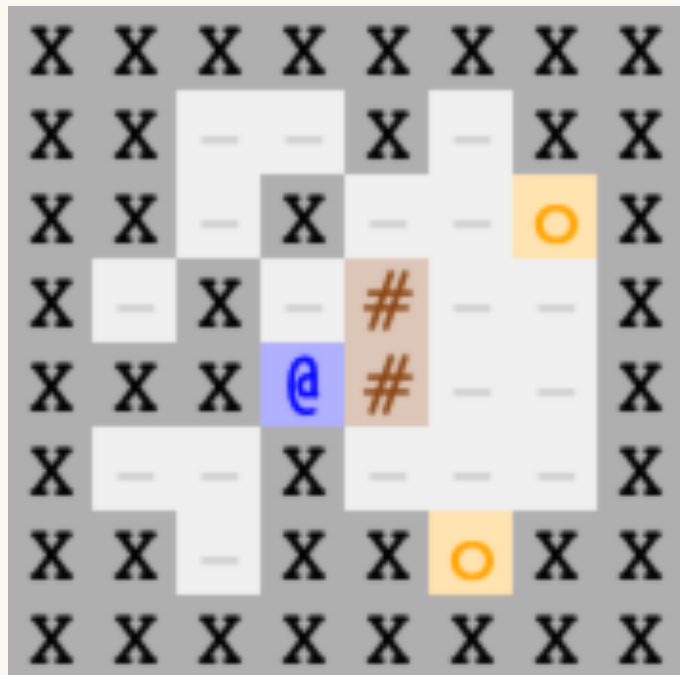


# Game Mechanics

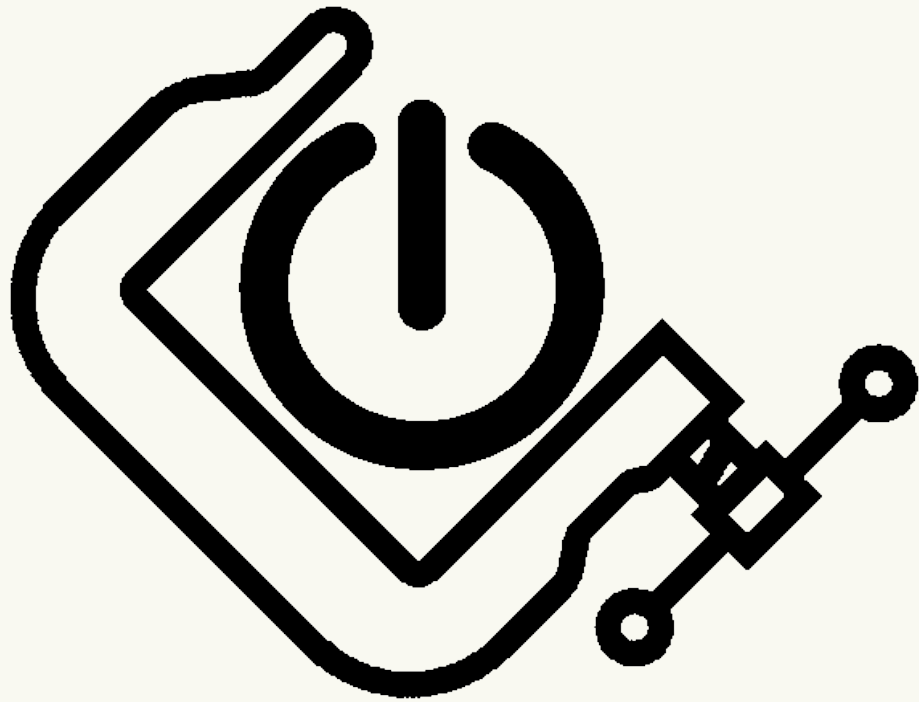


- Add another dimension - Time
- Model game mechanics as tile replacement rules
  - Inspired by Gumin's Markov Junior
  - Various ways of grouping and ordering
- Basic setup:
  - Level generation constrains timestep 0
  - Replacement rules constrain changes between timesteps
  - Level must be solved by the last timestep
- Solution is a level **and** example playthrough that level is completable!

# Game Mechanics



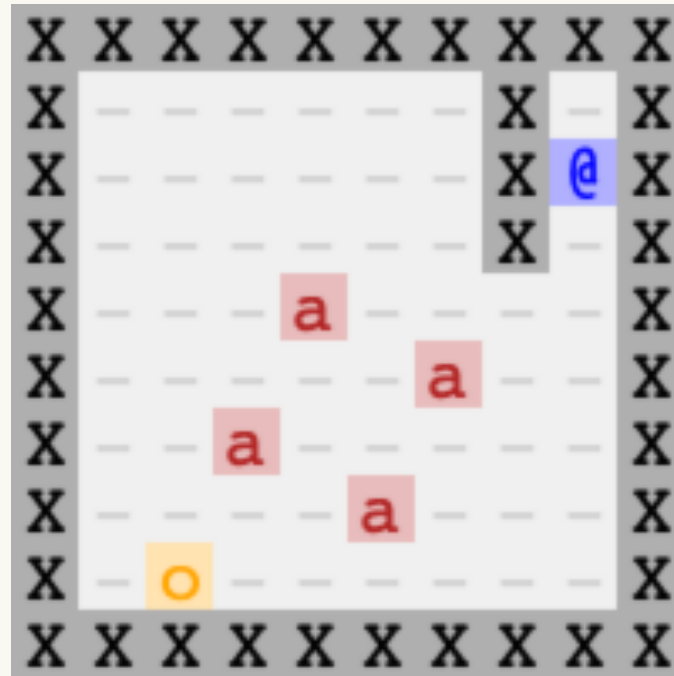
# Summary



- Constraint solving can be a powerful and flexible technique for level generation (and editing)
- Can learn from few examples and provide guarantees on generated content (e.g. path through level)
- Application of general solvers allows a variety of design constraints to be expressed, may benefit from general improvements



# Thanks!



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<https://www.kenney.nl/>

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Adam Smith, Pete Manolios, Andrew Walter,  
Northeastern Game Research Seminar

<https://github.com/crowdgames/sturgeon-pub>

- Seth Cooper. **Sturgeon: tile-based procedural level generation via learned and designed constraints.** Proceedings of the Eighteenth AAAI Conference on Artificial Intelligence and Interactive Digital Entertainment (2022).
- Seth Cooper. **Constraint-based 2D tile game blending in the Sturgeon system.** Proceedings of the Experimental AI in Games Workshop (2022).
- Seth Cooper. **Sturgeon-GRAPH: Constrained Graph Generation from Examples.** Proceedings of the 17th International Conference on the Foundations of Digital Games (2023, to appear).
- Seth Cooper. **Sturgeon-MKIII: Simultaneous Level and Example Playthrough Generation via Constraint Satisfaction with Tile Rewrite Rules.** Proceedings of the FDG Workshop on Procedural Content Generation (2023, accepted)